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TEACHERS' KEY

Plane Geometry

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KRICKENBERGER
PEARSON

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TEACHERS' KEY

Plane Geometry

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TEACHERS' KEY

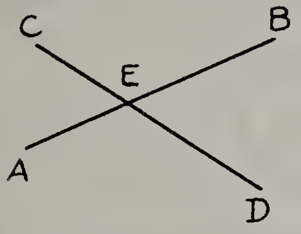
Chapter 1. Lines and Angles

PAGES 8-9

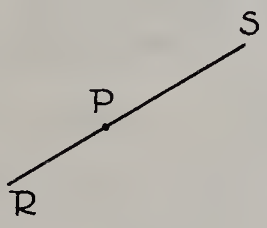
1. The point E has position but no size. See Fig. (1) below.

2. (1) An infinite number. (2) An infinite number. See Fig. (2) below.

3. Only 1 line can be drawn through 2 given points. See Fig. (3) below.

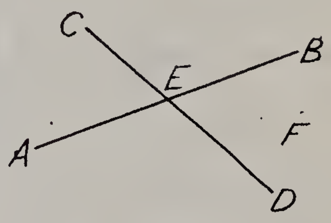


(1)



(2)

(3)

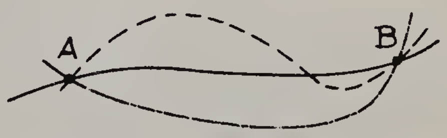


(4)

4. No. Because only 1 st. line can be drawn through E and F. See Fig. (4).

5. To determine the line of the fence, and because an error made at either end would cause the fence to deviate from the desired line less than the same error made between the ends.

6. (1) Yes (see dotted lines in figure). (2) An infinite number.



7. A st. line.

8. AO, AC, OC (any two). 9. n. 10. 6.

PAGE 10

1. (1) 2. (2) 1. (3) None.

2. AD, AB, BC, AC, AO, OC, BO, OD, BD.

3.

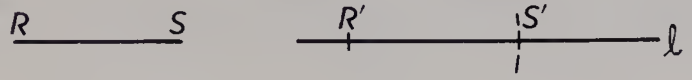
	Smallest Class	Distinguishing Characteristics
(1)	branch of geometry	deals with plane figures
(2)	school room	used as a place for study
(3)	writing instrument	makes its inscription with graphite
(4)	mathematical statement	expresses the equality of two quantities
(5)	coin	worth 10 cents

4. a. A closed broken line is a polygon.
   b. A polygon having three sides is a triangle.
   c. A triangle having two equal sides is an isosceles triangle

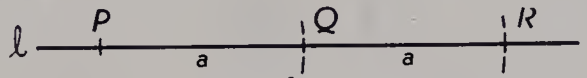
PAGE 14

1. (1) 9 inches. (2) 9 1/8 inches. (3) Yes.
2. 4 4/8 inches.
3. (1) AB is approximately equal to EF. (2) CD. (3) GH.
4. AB appears to equal DC and AD appears equal to BC.

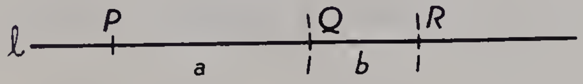
PAGES 15-16

1. 

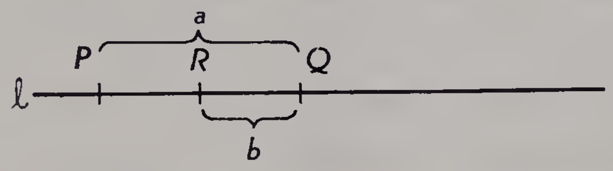
Draw a line ℓ of indefinite length. Choose a point R' on ℓ. Adjust the compasses so that its points rest on R and S. With RS as radius and R' as center make an arc cutting ℓ at S'. R'S' is the required segment.

2. (1) 

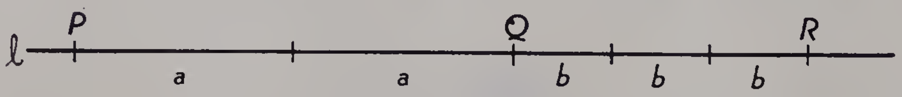
Draw a line ℓ of indefinite length and on it choose a point P. With P as center and a as radius make an arc cutting ℓ at Q. With Q as center and a as radius make an arc cutting ℓ at R, beyond PQ. PR = 2a.

(2) 

Same as (1) except that the second radius is of length b, instead of a. PR = a + b.

(3) 

Draw a line ℓ of indefinite length and on it choose a point P. With P as center and a as radius make an arc cutting ℓ at Q. With Q as center and b as radius make an arc cutting ℓ at R between P and Q. Then PR = a − b.

(4) 

Draw a line ℓ of indefinite length and on it choose a point P. On ℓ make PQ = 2a as in (1). In similar manner make QR = 3b. Then PR = 2a + 3b.

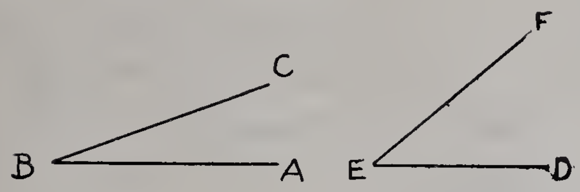
3. AB, AC, BC, BD, CD. 4. BC. 5. BC, CD. 6. CD.
7. (1) OA; OD; AD. (2) OB; OE; BE. 8. EF. 9. AD. 10. OB.
11. EF. 12. OC.

PAGES 18-20

1. (1) No. (2) Yes. (3) Yes. 2. (1) ∠a, ∠b, ∠c, ∠d. (2) ∠TOH, ∠ROT, ∠MOR, ∠HOM. 3. The student should observe that ∠TOH = ∠MOR.

4. See diagram below.

5. (1) ∠DHE. (2) ∠BKE. (3) ∠FKB. (4) ∠AKF. (5) ∠CHF.



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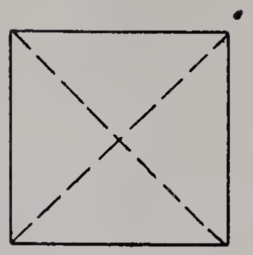

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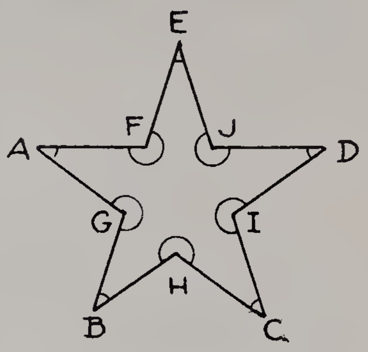
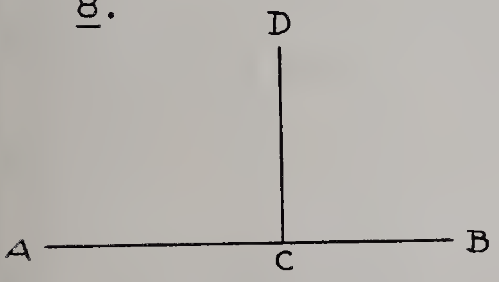


(7)

6. (1) 2. (2) Because there are 2 pairs of vertices not already connected, and each pair of vertices determines a line. (3) 16.

7. (1) 10. (2) ∠A = ∠B = ∠C = ∠D = ∠E;
∠F = ∠G = ∠H = ∠I = ∠J.

8.

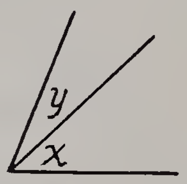


9. ∠ODC.
10. ∠OCB.
11. ∠OBC.
12. ∠OAD.
13. ∠AOD.

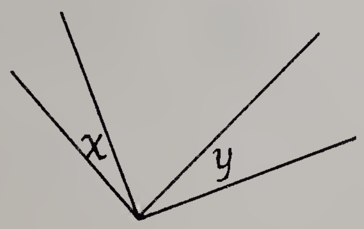
14. BO + OD.
15. (1) OB, AD, BC.
(2) BC, OB, DO.
(3) OC.
(4) DC.

16. (1) ∠BOC [Also ∠OAD, ∠OCB].
(2) ∠DOC

18.



17.

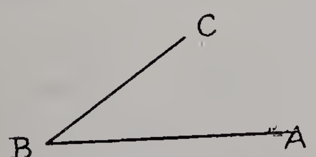


19. ∠ABF.
20. ∠DBF.

21. ∠FBD.
22. ∠ABE.

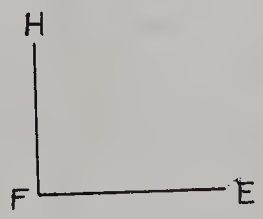
PAGES 21-22

1. (1)



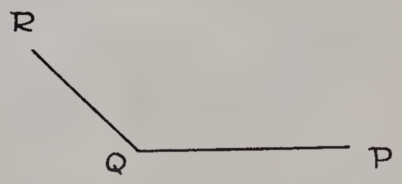
Acute

(2)



Right

(3)

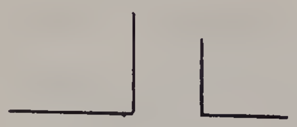


Obtuse

2. Half. 3. Rugs, radio, door, chair, book, window, and the like. 4. a. rt. ∠ ; acute ∠ . b. ∠ ACD, ∠ BCD; ∠ ADC, ∠ BDC. c. Point D. d. CD. e. ∠BDC. f. CD. 5. Acute; right;

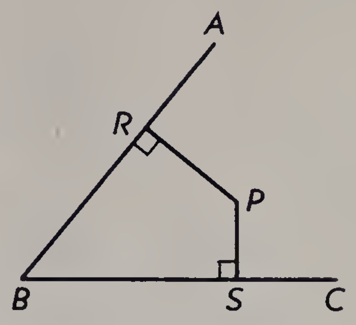
straight; obtuse. 6. ∠ABC, ∠BCD, ∠BFE, and ∠CFE; ∠CDE and ∠FEA; ∠EAB and ∠DEF. 7. ∠WST; ∠SWR. 8. No. WT is not a common side between them.

9. (1)



(2) Yes.

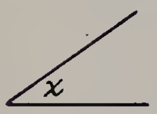
10.



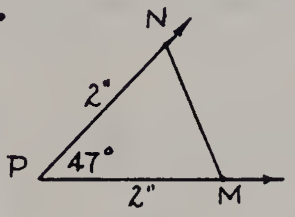
PR and PS are perpendicular to AB and BC, respectively.

PAGES 23-24

1.



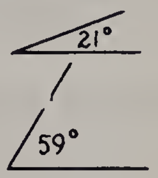
2.



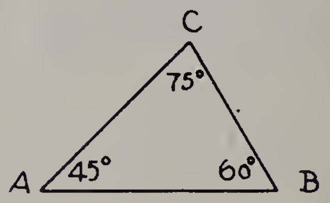
∠PMN = $66\frac{1}{2}^\circ$

∠PNM = $66\frac{1}{2}^\circ$

3.

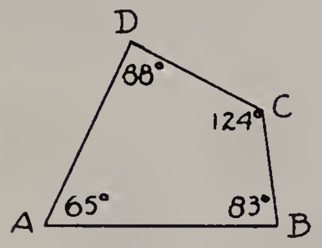


4. One example:



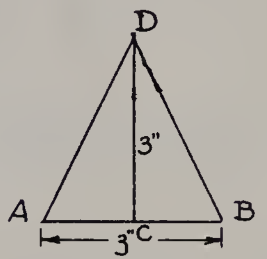
∠A + ∠B + ∠C = 180°

5. One example:



∠A + ∠B + ∠C + ∠D = 360°

6.



∠CAD = 64°

∠CBD = 64°

7. 51°15'
8. 22°35'15"
9. 62°13'57"
10. 18°19'
11. 14°3'35"
12. 71°27'35"

PAGE 25

1. Subtracting each ∠ from 90°: 60°; 50°; 70.8°; 57°42'; $(90 - x)^\circ$; $(108 - 2x)^\circ$; $(83 - 3x)^\circ$.

2. Subtracting each ∠ from 180°: 120°; 135°; 143°; 62°; 149°15'; $(180 - x)^\circ$; $140^\circ - x^\circ$; $(135 - 3a)^\circ$.

3. $x + 2x = 90^\circ$; whence $3x = 90^\circ$, and $x = 30^\circ$. Then $2x = 60^\circ$.
30°, 60°. Ans.

4. $x + 4x = 90$; whence $5x = 90$, and $x = 18$. Then $4x = 72$.
72°. Ans.

5. We have the equations $x + y = 90$ and $x - 2y = 8$. Subtracting these equations, $3y = 82$; whence $y = 27\frac{1}{3}$. Then $x = 90 - 27\frac{1}{3} = 62\frac{2}{3}$.
$62\frac{2}{3}^\circ$. Ans.

6. (1) $\angle c = 40°$. (2) $\angle d = 40°$. (3) $\angle b = 140°$. (4) No. (5) Yes.

7. $180 - x = 3(90 - x)$; whence $2x = 90$, and $x = 45$. 45°. Ans.

PAGE 26

1. (1) Yes. (2) Yes.

2. A line segment can be bisected at only 1 point.

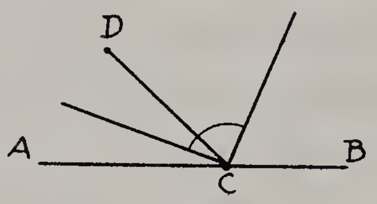
3. Yes. They are usually bisected each time they are folded.

4. (1) By placing the 2 end points together and finding the point at which the double taut string is folded.

(2) By bisecting it and then bisecting each half as in (1).

5. 1.

9. (1)



(2) 90°.

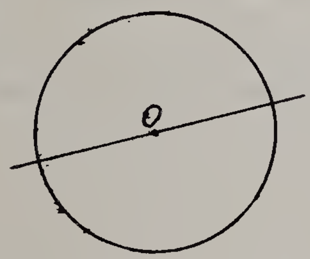
10. The student should conclude that the bisectors of 2 supp. adj. $\angle$s are probably ⊥ each other.

11. $x + \frac{7}{8}x = 90$. $8x + 7x = 720$; $15x = 720$; $x = 48$, and $\frac{7}{8}x = 42$. 42° and 48°. Ans.

12. $x + y = 40$ and $x - y = 8$. Adding, $2x = 48$, and $x = 24$. Subtracting, $2y = 32$ and $y = 16$. 24° and 16°. Ans.

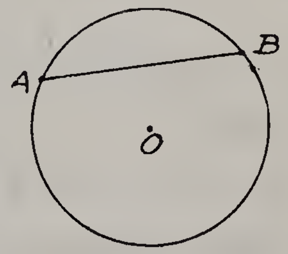
PAGE 27

1. (1)



(2) 2.
(3) 2.
(4) 2.

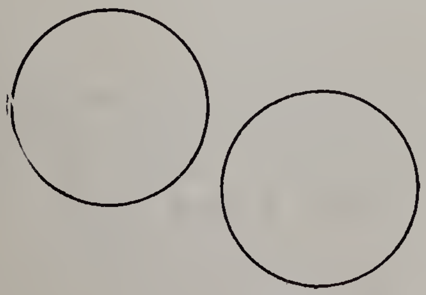
2. (1)



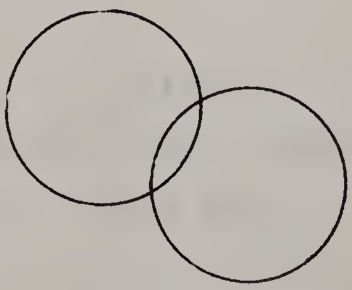
(2) No (in this figure)
(3) Diameter.

3. A line can be drawn touching a ⊙ in 1 point.

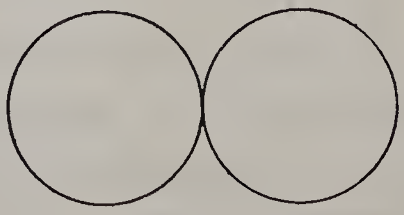
4. (a)



(b)

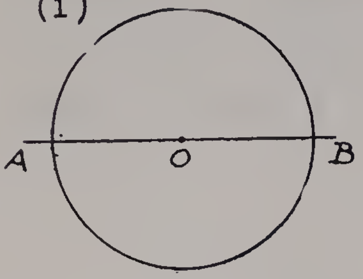


(c)



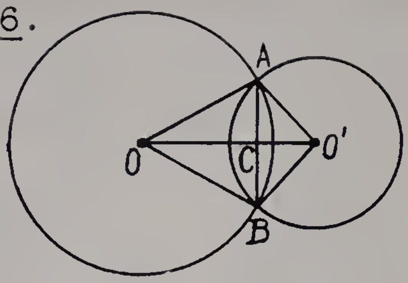
(d) No.

5. (1)



(2) Radius.
(3) Radius.
(4) Diameter.
(5) An infinite number.

6.



(1) Yes.
(2) Yes.
(3) No.
(4) Yes.

PAGES 27-28 (Review Questions)

1. (1) None. (2) One. (3) Two.

2. (1) ∠CAB or ∠BAC. (2) ∠ACB or ∠BCA. (3) Line segment.

3. Bisect.

4. ∠x is larger than ∠y.

5. (1) An infinite number. (2) One. (3) None if the points are not in a straight line

6. (1) AC. (2) OB. (3) ∠DAB. (4) Adjacent. (5) Vertical.

7. An angle less than a right angle is an acute angle.

8. A good definition (a) names the term to be defined, (b) places it in the smallest class to which it belongs, (c) gives the definite characteristics that distinguish it from other members of the class, and (d) is reversible.

9. (1) Equal line segments are line segments that can be made to coincide. (2) Equal angles are angles that can be made to coincide.

10. A line segment is a definite part of a line while a ray is an infinite part.

11. (1) An infinite number. (2) Only one.

12. (1) Complementary. (2) Supplementary.

13. Adjacent.

14. (1) Right. (2) Acute. (3) Obtuse.

15. (1) None. (2) Two. (3) None.

16. Bisector.

17. Yes.

18. (1) Two lines are perpendicular to each other if one of them forms two equal adjacent angles with the other. (2) If two lines are perpendicular to each other, one of them forms equal adjacent angles with the other.

19. (1) The equal adjacent angles which are formed by a line and its perpendicular are called right angles. (2) If the ad-

jacent angles formed by two intersecting lines are right angles, the lines are perpendicular to each other.

20. Midpoint.

PAGE 29 (Test 1)

1. T. 2. T. 3. T. 4. F. 5. T. 6. F. 7. F.
8. T. 9. F. 10. F. 11. F. 12. T. 13. F. 14. T.
15. T. 16. T. 17. F. 18. T. 19. F. 20. F.

Chapter 2. Inductive Reasoning

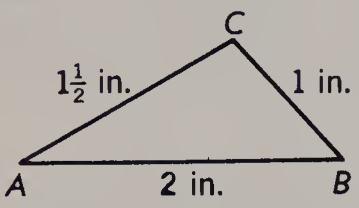
PAGES 33-35

1. (1) 50°; 60°; 100°; 117°; 33°. (2) 360°. (3) 360°.

2. (1) 180°. (2) 180°. (3) No.

3. (1) Two sides appear to be equal. (2) Two sides appear to be equal. (3) In a triangle the sides opposite the equal sides are equal.

4. (1) One example:



(2) $AB = 2$ in.
$BC = 1$ in.
$AC = 1\ 1/2$ in.
$BC + AC = 2\ 1/2$ in.

(3) The sum of the lengths of the two shorter segments is greater than the length of the longest. (4) greater; side. (5) No.

5. (1) $\angle A + \angle B + \angle C + \angle D = 360°$. (2) The sum of the angles of another quadrilateral is 360°. (3) The sum of the angles of any quadrilateral is 360°.

6. (1) AE appears to equal BE. (2) $\angle AEC$ appears to equal $\angle BEC$ (and each of the other angles). (3) For each drawing it appears that the segments are equal and the angles equal.

7. (1) $\angle AOE$ appears to equal $\angle EOB$. (2) The construction produces the bisector of an angle. (3) No.

8. (1) A right angle. (2) It appears true that line segments joining a point on a circle to the end points of a diameter of the circle form a right angle.

9. (1) $BC < AC$. (2) $AC < BC$. (3) It seems that for each triangle, the side opposite the smaller angle is less than the side opposite the larger angle.

PAGES 37-38

1. Not necessarily.

2. (1) Someone, observing that it happened one or more times, concluded that it will always happen. (2) No.

3. The pupil should conclude that the chicken was the most probable cause of the illness by reasoning as follows:

The possible causes of illness were the chicken and the water because all the boys who were sick had some of both. (The other items can probably be eliminated because some of the sick boys had partaken of them and some had not.) The water was probably not the cause since some of the boys who drank it were not ill. All the boys who ate the chicken were ill and those who were not ill did not have chicken. Hence, from the facts known, the logical conclusion is that the chicken caused the illness.

4. (1) No. (2) Not a sure preventative. (3) Several hundred, or better, several thousand. (4) No. (5) Because the compound could never be given to all people. As long as there is even one person who has not tried the compound, there is the possibility that he might develop a cold even after taking the compound.

PAGES 40-41 (Review Questions)

1. Through observation.

2. a. (1) False. (2) We have no way to make a perfect measuring device since instruments themselves are affected by such conditions as temperature and moisture. Neither do we have an absolute measure to use as a standard of reference.

b. (1) False. (2) Our eyes do not always give us accurate impressions.

c. (1) False. (2) Experiments can help us to see probable relationships, but since our observations and measurements are imperfect, conclusions based upon our experiments cannot be regarded as absolute truth.

3. (1) No. (2) Even though it had rained for four or more days in succession it would not follow that the rain would continue forever.

4. (1)

	I	II	III
$\angle a$	30°	37°	45°
$\angle c$	83°	35°	45°
$\angle a + \angle c$	113°	72°	90°
$\angle r$	113°	72°	90°

(2) It appears that $\angle r = \angle a + \angle c$. (3) Yes. (4) No.

5. Egypt; Babylonia. 6. measurement; observation.

7. Euclid. 8. inductively.

PAGE 42 (Test 2)

1. Determining possible new truths. 2. Cannot. 3. Cannot.

4. (1) Probably not. (2) It is impossible to make exact measurements.

5. (1)

	I	II	III
∠x	122°	90°	148°
∠y	122°	90°	148°

(2) Supplements of the same angle appear to be equal.

(3) No.

6. (1) 90°. (2) 90°. (3) The sum of the two angles which are not right angles appears to be 90°. (4) No.

PAGE 43 (Comprehensive Test)

1. line segment. 2. induction. 3. diameter. 4. radius.
5. vertical angles. 6. equal angles. 7. degree. 8. Euclid.
9. ray. 10. supplementary angles. 11. complementary angles.
12. bisect. 13. adjacent angles. 14. perpendicular.
15. straightedge.

Chapter 3. Deductive Reasoning

PAGES 47-48

1. Conclusion: A robin has feathers.

2. Conclusion: John is passing in three one-credit subjects.

3. Conclusion: Our dog is not well.

4. Conclusion: Richard Johnson has at least one study period.

5. Conclusion: Line segment AB is bisected.

6. Conclusion: $\angle m$ and $\angle n$ are equal.

7. No conclusion can be made because the specific statement does not satisfy the condition of the general statement. The specific statement satisfies the conclusion of the general statement.

8. Conclusion: Robert locked his father's car last Thursday.

9. No conclusion can be made as the specific statement does not satisfy the condition of the general statement.

10. No valid conclusion can be made because the specific statement does not satisfy all conditions of the general statement. It is not known that Mr. Black and his son went fishing at Jasper Lake.

11. Conclusion: Philip Duncan is at least six feet tall.

12. Conclusion: DE is one-half AB.

13. Conclusion: AB and CD will not intersect.

PAGE 51

1. The hypothesis is "If you can run the 100-yard dash in 10 seconds" and the conclusion is "you are a good sprinter."

2. The hypothesis is "if you wish to enjoy it" and the conclusion is "You must study a subject."

3. The hypothesis is "If it rains" and the conclusion is "I shall not go to the football game."

4. The hypothesis is "If air is heated" and the conclusion is "it will expand."

5. The hypothesis is "when the weather is clear" and the conclusion is "Radio reception is best."

6. The hypothesis is "An isosceles triangle" and the conclusion is "has two equal sides."

7. The hypothesis is "The diagonals of a rectangle" and the conclusion is "are equal."

8. The hypothesis is "If two sides of a triangle are equal" and the conclusion is "the angles opposite these sides are equal."

9. The hypothesis is "Two lines perpendicular to a third line" and the conclusion is "are parallel."

10. The hypothesis is "the opposite sides of a parallelogram" and the conclusion is "are equal."

11. If angles are right angles, then they are equal.

12. If two angles are vertical angles, they are equal.

13. If a person is a public official, he should be honest.

14. The line segment joining the midpoints of two sides of a triangle is half the third side.

15. A line intersecting one of two parallel lines intersects the other.

PAGES 54-55

1. Asmt. 2. 2. Asmt. 1. 3. Asmt. 3. 4. Asmt. 4.
5. Asmt. 1. 6. Asmt. 2. 7. Asmt. 3. 8. Asmt. 3.
9. Asmt. 4. 10. Asmt. 6. 11. Asmt. 3. 12. Asmt. 6, or
Asmt. 7. 13. Asmt. 8. 14. Asmt. 8. 15. Asmt. 9.
16. Asmt. 9. 17. Asmt. 9. 18. Asmt. 9. 19. Asmt. 9.
20. Asmt. 1. 21. Asmt. 2. 22. Asmt. 2. 23. Asmt. 6, or
Asmt. 7. 24. Asmt. 6. 25. Asmt. 6, or Asmt. 7. 26. Asmt. 1.
27. Asmt. 2. 28. Asmt. 2. 29. Asmt. 1. 30. Asmt. 1.
31. Asmt. 6. 32. Asmt. 6. 33. Asmt. 3. 34. Asmt. 2.

PAGES 57-59

1. Asmt. 4. 2. Asmt. 28. 3. Asmt. 3. 4. §19. 5. §19.
6. §13. 7. §21. 8. Asmt. 30. 9. Asmt. 6, or Asmt. 7.
10. Asmt. 6, or Asmt. 7. 11. Asmt. 6. 12. Asmt. 6, or Asmt. 7.
13. Asmt. 6, or Asmt. 7. 14. Asmt. 1. 15. Asmt. 9. 16. § 22.
17. Asmt. 18. 18. § 14. 19. § 14. 20. Asmt. 11.
21. Asmt. 9. 22. Asmt. 14. 23. Asmt. 9. 24. Asmt. 13.
25. Asmt. 2. 26. § 22. 27. Asmt. 12. 28. Asmt. 24.
29. Asmt. 9; Asmt. 9. 30. Asmt. 14. 31. Asmt. 24.
32. Asmt. 6, or Asmt. 7. 33. Asmt. 20. 34. Asmt. 21.
35. §14. 36. § 14. 37. Asmt. 4. 38. Asmt. 2. 39. Asmt. 2.
40. Asmt. 24.

PAGES 60-61

1.

Statements	Reasons
1. A bass is a fish.	
2. A bass lives in water.	2. All fish live in water.

2.

Statements	Reasons
1. Marcia is a member of the XYZ Club.	
2. Marcia has studied mathematics.	2. All members of the XYZ Club have studied mathematics.

3.

Statements	Reasons
1. In triangle ABC, AC = BC.	
2. Triangle ABC is isosceles.	2. A triangle having two equal sides is isosceles.

4.

Statements	Reasons
1. Triangle ABC has $\angle C$ a right angle.	
2. Triangle ABC is a right triangle.	2. A triangle having a right angle is a right triangle.

5.

Statements	Reasons
1. $\angle ABE$ and $\angle EBC$ have a common vertex and a common side between them.	
2. $\angle ABE$ and $\angle EBC$ are adjacent angles.	2. Two angles which have a common vertex and a common side between them are adjacent angles.

PAGES 62-63

1.

Statements	Reasons
1. $4x = 40$.	1. Given
2. $x = 10$.	2. Asmt. 4.

2.

Statements	Reasons
1. AF bisects DE.	1. Given.
2. DA = AE.	2. §22.

3.

Statements	Reasons
1. In line segment DE, DA = AE.	1. Given.
2. AF bisects DE.	2. §22.

4.

Statements	Reasons
1. $x + 9 = 75$.	1. Given.
2. $x = 66$.	2. Asmt. 2.

5.

Statements	Reasons
1. In Fig. ABCD, BD $\perp$ ABC.	1. Given.
2. $\angle ABD = \angle CBD$.	2. § 14.

6.

Statements	Reasons
1. $\angle x + \angle y = 1$ rt. $\angle$.	1. Given.
2. $\angle x + \angle y$ are complementary.	2. § 20.

7.

Statements	Reasons
1. $y - 3 = 12$.	1. Given.
2. $y = 15$.	2. Asmt. 1.

8.

Statements	Reasons
1. $\frac{y}{4} = 5$.	1. Given.
2. $y = 20$.	2. Asmt. 3.

9.

Statements	Reasons
1. AB $\perp$ BC.	1. Given.
2. AC is not $\perp$ BC.	2. Asmt. 20.

10.

Statements	Reasons
1. $\angle x$ is the supp. of $\angle y$.	1. Given.
2. $\angle m$ is the supp. of $\angle y$.	2. Given.
3. $\angle x = \angle m$.	3. Asmt. 30.

11.

Statements	Reasons
1. $\angle x + \angle y = 1$ st. $\angle$.	1. Given.
2. $\angle x = \angle m$.	2. Given.
3. $\angle m + \angle y = 1$ st. $\angle$.	3. Asmt. 6.

12.

Statements	Reasons
1. $AO = OC$.	1. Given.
2. $BO = OD$.	2. Given.
3. $AO + BO = DO + CO$.	3. Asmt. 1.

13.

Statements	Reasons
1. $\angle m$ is the supp. of $\angle y$.	1. Given.
2. $\angle m$ is the supp. of $\angle x$.	2. Given.
3. $\angle x = \angle y$.	3. Asmt. 30.

14.

Statements	Reasons
1. $\angle A = 30^\circ$.	1. Given.
2. $\angle B = 22^\circ$.	2. Given.
3. $\angle C = 128^\circ$.	3. Given.
4. $\angle A + \angle B + \angle C = 180^\circ$.	4. Asmt. 1.

PAGES 64-65

1. Solution.

Statements	Reasons
1. $2x - 1 = 15$.	1. Given.
2. $2x = 16$.	2. Asmt. 1.
3. $x = 8$.	3. Asmt. 4.

2. Solution.

Statements	Reasons
1. $7y + 2 = 37$.	1. Given.
2. $7y = 35$.	2. Asmt. 2.
3. $y = 5$.	3. Asmt. 4.

3. Solution.

Statements	Reasons
1. $5x + 4 = -26$.	1. Given.
2. $5x = -30$.	2. Asmt. 2.
3. $x = -6$.	3. Asmt. 4.

4. Solution.

Statements	Reasons
1. $4x = 12 - 2x$.	1. Given.
2. $6x = 12$.	2. Asmt. 1.
3. $x = 2$.	3. Asmt. 4.

5. Solution.

Statements	Reasons
1. $3p = 30 + 5p$.	1. Given.
2. $-2p = 30$.	2. Asmt. 2.
3. $p = -15$.	3. Asmt. 4.

6. Solution.

Statements	Reasons
1. $4x - x = 16 + x$.	1. Given.
2. $4x - x - x = 16$.	2. Asmt. 2.
3. $2x = 16$.	3. Asmt. 6.
4. $x = 8$.	4. Asmt. 4.

7. Solution.

Statements	Reasons
1. $2(y - 1) + (y + 3) = 10.$	1. Given.
2. $2y - 2 + y + 3 = 10.$	2. Asmt. 6.
3. $y + 1 = 10.$	3. Asmt. 6.
4. $y = 9.$	4. Asmt. 2.

8. Solution.

Statements	Reasons
1. $x - 4(x - 1) = 7.$	1. Given.
2. $x - 4x + 4 = 7.$	2. Asmt. 6.
3. $-3x + 4 = 7.$	3. Asmt. 6.
4. $-3x = 3.$	4. Asmt. 2.
5. $x = -1.$	5. Asmt. 4.

9. Solution.

Statements	Reasons
1. $4x = 10 - (x + 5).$	1. Given.
2. $4x = 10 - x - 5.$	2. Asmt. 6.
3. $4x = 5 - x.$	3. Asmt. 6.
4. $5x = 5.$	4. Asmt. 1.
5. $x = 1.$	5. Asmt. 4.

10. Solution.

Statements	Reasons
1. $3(x - 1) = 5(x - 3).$	1. Given.
2. $3x - 3 = 5x - 15.$	2. Asmt. 6.
3. $-2x - 3 = -15.$	3. Asmt. 2.
4. $-2x = -12.$	4. Asmt. 1.
5. $x = 6.$	5. Asmt. 4.

11. Solution.

Statements	Reasons
1. $\frac{m}{4} - \frac{m}{5} = 4.$	1. Given.
2. $5m - 4m = 80.$	2. Asmt. 3.
3. $m = 80.$	3. Asmt. 6.

12. Solution.

Statements	Reasons
1. $\frac{5x}{6} + 4 = \frac{x - 6}{3}$.	1. Given.
2. $5x + 24 = 2x - 12$.	2. Asmt. 3.
3. $3x + 24 = -12$.	3. Asmt. 2.
4. $3x = -36$.	4. Asmt. 2.
5. $x = -12$.	5. Asmt. 4.

13. Solution.

Statements	Reasons
1. $\frac{y + 6}{4} = 4\frac{1}{2}$.	1. Given.
2. $y + 6 = 18$.	2. Asmt. 3.
3. $y = 12$.	3. Asmt. 2.

14. Solution.

Statements	Reasons
1. $\frac{x - 4}{9} - \frac{x}{3} = \frac{x + 2}{6}$.	1. Given.
2. $2x - 8 - 6x = 3x + 6$.	2. Asmt. 3.
3. $-4x - 8 = 3x + 6$.	3. Asmt. 6.
4. $-7x - 8 = 6$.	4. Asmt. 2.
5. $-7x = 14$.	5. Asmt. 1.
6. $x = -2$.	6. Asmt. 4.

PAGES 66-67

1. Reasons: 1. Given. 2. Given. 3. § 14. 4. § 14. 5. Asmt. 1. 6. Asmt. 6. 7. § 21.

2. Proof: 1. $\angle CAB = \angle CBA$ (Given). 2. $\angle DAB = \angle DBA$ (Given). 3. $\angle CAB - \angle DAB = \angle CBA - \angle DBA$ (Asmt. 2). 4. Asmt. 6.

3. Proof: 1. Given. 2. Asmt. 24. 3. Asmt. 24. 4. Given. 5. $\angle x = \angle y$ (Asmt. 31).

4. Proof: 1. Given. 2. Asmt. 24. 3. Given. 4. $\angle y$ is the supp. of $\angle m$ (Asmt. 24). 5. (Asmt. 30).

PAGE 71

1. (1) $\angle s = 100°$ (§49). (2) $\angle m = 80°$ (Asmt. 24).
(3) $\angle t = 80°$ (§49).

2. (1) $\angle m = 55°$ (§49). (2) $\angle s = 125°$ (Asmt. 24).
(3) $\angle n = 125°$ (§49).

3. (1) $\angle t = 30°45'$ (§49).
(2) $\angle n = 149°15'$ (Asmt. 24).

4. (1) $\angle m = x°$ (§49). (2) $\angle s = (180 - x)°$ (Asmt. 24).

5. (1) $\angle COD = (x - 10)°$ (§49).
(2) $\angle AOD = (190 - x)°$ (Asmt. 24).

6. Reasons: 1. §49. 2. §49. 3. Given. 4. Asmt. 7.

7. (1) $x + 2y = 180$ (Asmt. 24).
(2) $x = 2y + 60$ (§49).
(3) From (2), $x - 2y = 60$.
(4) Adding (1) and (3),
$2x = 240$.
(5) $x = 120$.
(6) Subtracting (1) and (3),
$4y = 120$.
(7) $y = 30$.
(8) $2y = 60$.
(9) $2y + 60 = 120$.

The angles contain 120°, 60°, 120°, and 60°.

PAGES 71-72

1. The implication is that teachers lose their effectiveness when they reach the age of 65.

2. The implication is that a pupil who makes good grades in high school will make good grades in college.

3. The assumption is that a person who is tall and fast, and can shoot well, will make the basketball team.

4. It is assumed that a house should be painted every four years.

5. It is assumed that a good salesman does not argue with customers.

6. It is assumed that Dick's car failed to start because of defective sparkplugs.

7. It is assumed that a person has plenty of money if he does not spend any.

8. It is assumed that women who dress well are wealthy.

9. It is assumed that a falling barometer is an indication of rain.

10. It is assumed that an arrest is a conviction.

11. It is assumed that love includes helpfulness.

12. It is assumed that a study of geometry is a prerequisite for college entrance.

13. It is assumed that eating apples prolongs life.

14. It is assumed that anyone can obtain a driver's license upon becoming 16 years old.

15. The implication is that overweight causes shortness of breath.

16. It is assumed that a man 58 years old can give better advice than a man 48 years old.

PAGES 72-74 (Review Questions)

1. One. 2. 1 st. ∠, or 180°. 3. Hypothesis. 4. Conclusion. 5. Yes. 6. Yes. 7. No. 8. Adjacent and supplementary. 9. No. 10. No. 11. A reason. 12. Yes. 13. Not necessarily. 14. (1) No. (2) No. (3) No. (4) Yes. 15. 162°. 16. One. 17. One. 18. 360°, or 2 st. ∠s.

19. The angles are supplementary.

20. Perpendicular.

21. They lie in a straight line.

22. The hypothesis is "If it rains this morning" and the conclusion is "I shall wear a raincoat."

23. The hypothesis is "if we win this football game" and the conclusion is "We shall not have school Monday."

24. specific, conclusion.

25. assumptions, definitions.

26. (1) Given. (2) Asmt. 1.

27. (1) Given. (2) Given. (3) Given. (4) Asmt. 29.

PAGE 75 (Test 3)

1. Asmt. 4. 2. Asmt. 2. 3. Asmt. 1. 4. Asmt. 3. 5. Asmt. 6, or Asmt. 7. 6. Asmt. 6. 7. Asmt. 18. 8. Asmt. 17. 9. §14. 10. Asmt. 8. 11. §20. 12. §49. 13. Asmt. 14. 14. Asmt. 9.

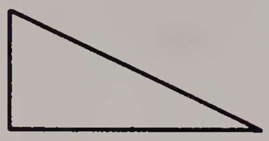
Chapter 4. Triangles

PAGE 78

1. (1) △ 4 is scalene. (2) ⟁ 1, 2, and 5 are isosceles. (3) ⟁ 2 and 5 are equilateral. (4) △ 3 is a rt. △. (5) ⟁ 1, 2, and 5 are acute. (6) ⟁ 1, 2, 4, and 5 are oblique. (7) ⟁ 2 and 5 are equiangular.

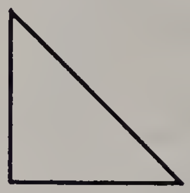
2.

(1)



A right scalene △.

(2)



A right isosceles △.

PAGES 80-81

1. (1) The hypotenuse of △ABC is AB and the legs are AC and BC; (2) of △ ADC the hypotenuse is AC and the legs are AD and DC; (3) of △ DBC the hypotenuse is BC and the legs are DB and DC.

2. (1) RS and RT; (2) RS and ST; (3) RT and ST; (4) ∠ R and ∠S; (5) ∠S and ∠T; (6) ∠R and ∠T.

3. (1) ∠A and ∠ABD; (2) ∠A and ∠ADB; (3) ∠ABD and ∠ADB.

4. (1) AB and AD; (2) AB and BD; (3) AD and BD.

5. BC.

6. (1) ∠A and ∠C; (2) ∠A and ∠ADC.

7. SK.

8. (1) RW and RT; (2) RW and TW.

9. (1) △HKR is isos. (2) HK. (3) ∠R.

10. (1) Right △. (2) Hypotenuse.

PAGE 82

1. (1) Yes; (2) Yes; (3) Yes; (4) Yes.

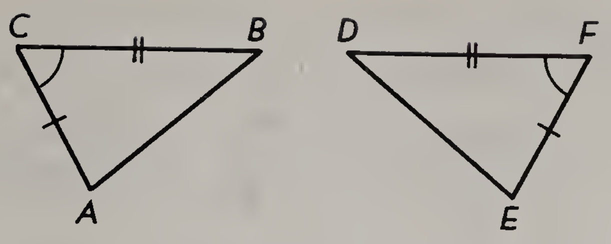
2. 1 ≅ 5; 4 ≅ 6; 3 ≅ 7.

3. Yes. 4. Yes. 5. Yes. 6. Yes.

PAGES 85-86

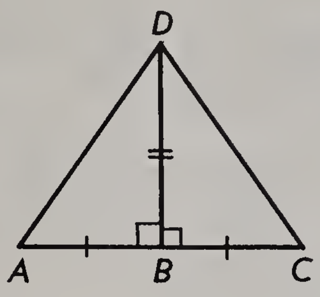
1. ⟁ ≅ by Asmt. 32.
2. ⟁ not ≅ by Asmt. 32.
3. ⟁ ≅ by Asmt. 32.
4. ⟁ not ≅ by Asmt. 32.
5. ⟁ not ≅ by Asmt. 32.
6. ⟁ ≅ by Asmt. 32.
7. ⟁ not ≅ by Asmt. 32.
8. ⟁ ≅ by Asmt. 32.
9. ⟁ not ≅ by Asmt. 32.
10. ⟁ ≅ by Asmt. 32.

11.



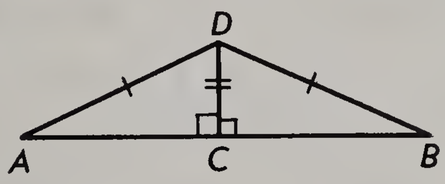
△ABC ≅ △EDF (Asmt. 32).

12.



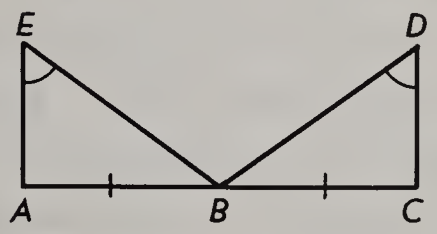
△ABD ≅ △BCD (Asmt. 32).

13.



△ACD not ≅ △BCD by Asmt. 32.

14.



△ABE not ≅ △BCD by Asmt. 32.

15. (1) ∠x = ∠y (§50d). (2) △AOC ≅ △BOD (Asmt. 32).

PAGE 87

1. EG = FH (Asmt. 1). △EGK ≅ △FHK (Asmt. 32).
2. △ACN ≅ △BCM (Asmt. 32).

PAGE 89

1. Planning the Proof: 1. We can prove ⟁ ≅ by §65 (Asmt. 32). 2. We shall use §65.

Proof: 1. In △ABC, AC = BC (Given). 2. CD bisects ∠ACB (Given). 3. ∠ACD = ∠DCB (§22). 4. CD = CD (Iden.). 5. △ADC ≅ △DBC (§65).

2. Planning the Proof: (Same as in Ex. 1).

Proof: 1. AO = OC (Given). 2. DO = OB (Given). 3. ∠x = ∠y (§50d). 4. △AOB ≅ △DOC (§65).

3. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $CA \perp AB$ (Given). 2. $\angle A$ is a rt. $\angle$ (§14). 3. $DB \perp AB$ (Given). 4. $\angle B$ is a rt. $\angle$ (§14). 5. $\angle A = \angle B$ (§50a). 6. $CA = DB$ (Given). 7. $AO = OB$ (Given). 8. $\triangle AOC \cong \triangle OBD$ (§65).

4. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $AB = CD$ (Given). 2. $BE = CE$ (Given). 3. $\angle EBA = \angle ECD$ (Given). 4. $\triangle ABE \cong \triangle CED$ (§65).

5. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $AD \perp AB$ (Given). 2. $\angle DAB$ is a rt. $\angle$ (§14). 3. $CB \perp AB$ (Given). 4. $\angle ABC$ is a rt. $\angle$ (§14). 5. $\angle DAB = \angle CBA$ (§50a). 6. $AB = AB$ (Iden.). 7. $AD = CB$ (Given). 8. $\triangle ABD \cong \triangle ABC$ (§65).

6. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $AB = AD$ (Given). 2. $BC = DC$ (Given). 3. $\angle x = \angle y$ and $\angle m = \angle n$ (Given). 4. $\angle x + \angle m = \angle y + \angle n$ (Asmt. 1). 5. $\angle x + \angle m = \angle ADC$ and $\angle y + \angle n = \angle ABC$ (Asmt. 9). 6. $\angle ADC = \angle ABC$ (Asmt. 7). 7. $\triangle ADC \cong \triangle ABC$ (§65).

7. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $AC = CB$ (Given). 2. $\angle C = \angle C$ (Iden.). 3. E is the midpoint of AC and D is the midpoint of BC (Given). 4. $2CE = 2CD$ (Asmt. 6). 5. $CE = CD$ (Asmt. 4). 6. $\triangle ADC \cong \triangle BEC$ (§65).

8. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $MR = NS$ (Given). 2. $MN = MN$ (Iden.). 3. $\angle MNP = \angle NMP$ (Given). 4. $\angle SNP = \angle RMP$ (Given). 5. $\angle MNS = \angle NMR$ (Asmt. 2). 6. $\triangle MNR \cong \triangle NMS$ (§65).

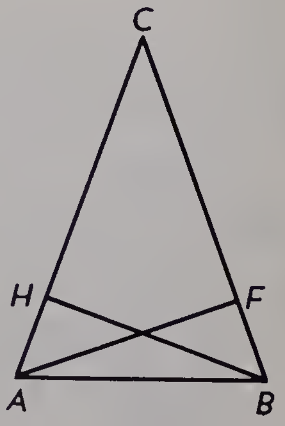
PAGE 91

1. Proof: 1. $CA = DB$ (Given). 2. $AO = OB$ (Given). 3. $CA \perp AO$ (Given). 4. $\angle A$ is a rt. $\angle$ (§14). 5. $BD \perp OB$ (Given). 6. $\angle B$ is a rt. $\angle$ (§14). 7. AOC and OBD are rt. $\triangle$s (§57). 8. $\triangle AOC \cong \triangle OBD$ (§71).

2. Proof: 1. $AD = CB$ (Given). 2. $AB = AB$ (Iden.). 3. $AD \perp AB$ (Given). 4. $\angle DAB$ is a rt. $\angle$ (§14). 5. $CB \perp AB$ (Given). 6. $\angle ABC$ is rt. $\angle$ (§14). 7. ABD and ABC are rt. $\triangle$s (§57). 8. $\triangle ABD \cong \triangle ABC$ (§71).

3. Planning the Proof: 1. We can prove $\triangle$s = by §§65, 71. 2. We shall use §71.

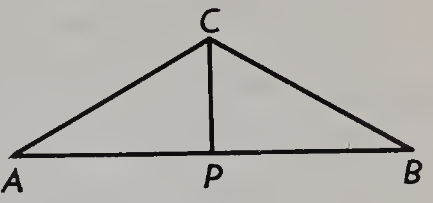
Proof: 1. $CF = CH$ (Given). 2. $AF = BH$ (Given). 3. $BH \perp AC$ (Given). 4. $\angle BHC$ is



Ex. 3

a rt. ∠ (§14). 5. AF ⊥ BC (Given). 6. ∠AFC is a rt. ∠ (§14). 7. △BHC and △AFC are rt. △s (§57). 8. △AFC ≅ △BHC (§71).

4. Planning the Proof: 1. We can prove △s ≅ by §§ 65, 71. 2. We shall use § 71.



Proof: 1. AP = PB (Given, §22). 2. CP = CP (Iden.). 3. CP ⊥ AB (Given). 4. ∠CPA = ∠CPB (§14). 5. △APC and △PBC are rt. △s (§57). 6. △APC ≅ △BPC (§71).

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1. △s not ≅ by Asmt. 33.
2. △s ≅ by Asmt. 33.
3. △s ≅ by Asmt. 33.
4. △s ≅ by Asmt. 33.
5. △s not ≅ by Asmt. 33.
6. △s ≅ by Asmt. 33.
7. (1) Yes. (2) Yes.
   (3) △s ≅ by Asmt. 33.

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1. Planning the Proof: 1. We can prove △s ≅ by §§ 65, 71, 73. 2. We shall use § 73.

Proof: 1. ∠x = ∠y (Given). 2. ∠r = ∠s (Given). 3. AB = AB (Iden.). 4. △ABC ≅ △ADB (§73).

2. Planning the Proof: (Same as in Ex. 1).

Proof: 1. ∠x = ∠y (Given). 2. ∠r = ∠s (Given). 3. BD = BD (Iden.). 4. △ABD ≅ △DBC (§73).

3. Planning the Proof: (Same as in Ex. 1).

Proof: 1. AB ⊥ BC (Given). 2. ∠x is a rt. ∠ (§14). 3. CD ⊥ BC (Given). 4. ∠y is a rt. ∠ (§14). 5. ∠x = ∠y (§50a). 6. ∠m = ∠n (§50d). 7. O is the midpoint of BC (Given). 8. BO = OC (§22). 9. △AOB ≅ △CDO (§73).

4. Planning the Proof: (Same as in Ex. 1).

Proof: 1. ∠A = ∠B (Given). 2. AC = BC (Given). 3. ∠x = ∠y (Given). 4. △ADC ≅ △EBC (§73).

5. Planning the Proof: (Same as in Ex. 1).

Proof: 1. ∠A = ∠B (Given). 2. AC = BC (Given). 3. ∠c = ∠c (Iden.) 4. △CEB ≅ △CAD (§73).

6. Planning the Proof: (Same as in Ex. 1).

Proof: 1. ∠x = ∠y (Given). 2. AO = AO (Iden.) 3. AOB is a st. line (Given). 4. ∠r = ∠s (Given). 5. ∠m is the supp. of ∠r and ∠n is the supp. of ∠s (Asmt. 24). 6. ∠m = ∠n (§50c). 7. △AOC ≅ △AOD (§73).

PAGES 95-96

1. $\triangle$s $\cong$ by Asmt. 32.
2. $\triangle$s $\cong$ by Asmt. 32.
3. $\triangle$s $\cong$ by Asmt. 33 or by Asmt. 32.
4. $\triangle$s not $\cong$ either by Asmt. 32 or by Asmt. 33.
5. $\triangle$s not $\cong$ either by Asmt. 32 or by Asmt. 33.
6. $\triangle$s not $\cong$ either by Asmt. 32 or by Asmt. 33.

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1. Planning the Proof: 1. We can prove $\triangle$s $\cong$ by §§ 65, 71, 73. 2. We shall use § 65.

Proof: 1. AC bisects $\angle BAD$ (Given). 2. $\angle BAC = \angle CAD$ (§22). 3. $AB = AD$ (Given). 4. $AC = AC$ (Iden.). 5. $\triangle ABC \cong \triangle ADC$ (§65).

2. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $BD \perp AC$ (Given). 2. $\angle ADB = \angle BDC$ (§14). 3. $AD = DC$ (Given). 4. $BD = BD$ (Iden.). 5. $\triangle ADB \cong \triangle CDB$ (§65).

3. Planning the Proof: 1. We can prove $\triangle$s $\cong$ by §§ 65, 71, 73. 2. We shall use §73.

Proof: 1. $DB \perp AC$ (Given). 2. $\angle ABD = \angle CBD$ (§14). 3. $\angle A = \angle C$ (Given). 4. $AB = BC$ (Given). 5. $\triangle ABD \cong \triangle CBD$ (§73).

4. Planning the Proof: 1. (Same as in Ex. 3).

Proof: 1. AOB and COD are st. lines (Given). 2. $\angle x = \angle y$ (§50d). 3. $\angle A = \angle B$ (Given). 4. $AO = OB$ (Given). 5. $\triangle AOC \cong \triangle BOD$ (§73).

5. Planning the Proof: 1. (Same as in Ex. 1).

Proof: 1. $AC = BC$ (Given). 2. $AD = DB$ (Given). 3. $\angle x = \angle y$ (Given). 4. $\angle m$ is the supp. of $\angle x$ and $\angle n$ is the supp. of $\angle y$ (§53b). 5. $\angle m = \angle n$ (§50c). 6. $\triangle ADC \cong \triangle BDC$ (§65).

6. Planning the Proof: (Same as in Ex. 1).

Proof: 1. AB and CD are diameters of $\odot$ O (Given). 2. AO, BO, CO, and DO are radii of $\odot$ O (§23). 3. $AO = DO$ (Asmt. 26). 4. $CO = BO$ (Asmt. 26). 5. $\angle AOC = \angle BOD$ (§§23, 50d). 6. $\triangle AOC \cong \triangle BOD$ (§65).

7. Planning the Proof: (Same as in Ex. 1).

Proof: 1. $AB = AD$ (Given). 2. $CD = CB$ (Given). 3. $\angle x = \angle y$ (Given). 4. $\angle m$ is the supp. of $\angle x$ and $\angle n$ is the supp. of $\angle y$ (§53b). 5. $\angle m = \angle n$ (§50c). 6. $\triangle ABC \cong \triangle ADC$ (§65).

8. Planning the Proof: (Same as in Ex. 3).

Proof: 1. AB and CD are st. lines (Given). 2. $EO = OF$ (Given).

3. ∠EOC = ∠BOF (§50d). 4. ∠x = ∠y (Given). 5. ∠m is the supp. of ∠x and ∠n is the supp. of ∠y (§53b). 6. ∠m = ∠n (§50c). 7. △CEO ≅ △BFO (§73).

9. Planning the Proof: (Same as in Ex. 3).

Proof: 1. ACDB is a st. line (Given). 2. ∠A = ∠B (Given). 3. AC = DB (Given). 4. CD = CD (Iden.). 5. AC + CD = CD + DB (Asmt. 1). 6. AC + CD = AD and CD + DB = CB (Asmt. 9). 7. AD = CB (Asmt. 7). 8. ∠x = ∠y (Given). 9. △ADE ≅ △BCF (§73).

10. Planning the Proof: (Same as in Ex. 1).

Proof: 1. AC = BD (Given). 2. BC = BC (Iden.). 3. AB = CD (Asmt. 2). 4. ∠x = ∠y (Given). 5. ∠m is the supp. of ∠x and ∠n is the supp. of ∠y (§53b). 6. ∠m = ∠n (§50c). 7. BE = CE (Given). 8. △ABE ≅ △DCE (§65).

11. Planning the Proof: (Same as in Ex. 3).

Proof: 1. DEB is a st. line (Given). 2. CD = DE (Given). 3. ∠x = ∠C (Given). 4. BD ⊥ AC (Given). 5. ∠ADE = ∠BDC (§14). 6. △ADE ≅ △BDC (§73).

12. Planning the Proof: (Same as in Ex. 1).

Proof: 1. ACDB is a st. line (Given). 2. CE = DF (Given). 3. ∠x = ∠y (Given). 4. ∠ECD is supp. ∠x and ∠CDF is supp. ∠y (§53b). 5. ∠ECD = ∠CDF (§50c). 6. CD = CD (Iden.). 7. △CDE ≅ △DCF (§65).

13. Planning the Proof: (Same as in Ex. 1).

Proof: 1. DF bisects AC and BC (Given). 2. CD = AD (§22). 3. BF = AD (Given). 4. CD = BF (Asmt. 7). 5. ∠C = ∠EBF (Given). 6. ∠DEC = ∠BEF (§50d). 7. CE = BE (§22). 8. △CDE ≅ △EBF (§65).

Note. §73 can be used to prove the △s ≅.

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1. CO and DO; AD and BC; ∠A and ∠B.
2. EG and FG; GH and GH; ∠E and ∠F; ∠EGH and ∠HGF.
3. AB and AB; AD and AC; DB and BC; ∠C and ∠D.
4. BD and BD; AD and BC; AB and DC; ∠A and ∠C.
5. OP and OP; NO and MO; ∠M and ∠N; ∠NOP and ∠POM.
6. ∠D and ∠B; ∠DCE and ∠ACB; DC and CB.

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1. (1) §§65, 73. (2) §§65, 71, 73.
2. (1) §§50, 78. (2) Asmt. 1-8, 26, §78.

3. *Planning the Proof*: 1. We can prove line segments = by Asmt. 1-8, 26, § 78. 2. We shall use §78.

Proof: 1. AC and DB are st. lines (Given). 2. DE = EB (Given). 3. AE = EC (Given). 4. ∠ AEB = ∠ DEC (§50d). 5. △ABE ≅ △DEC (§65). 6. DC = AB (§78).

4. *Planning the Proof*: 1. We can prove ∠s = by §§ 50, 78. 2. We shall use §78.

Proof: 1. BC and AD are st. lines (Given). 2. DO = OA (Given). 3. BO = OC (Given). 4. ∠DOC = ∠AOB (§50d). 5. △DOC ≅ △AOB (§ 65). 6. ∠D = ∠A (§78).

5. *Planning the Proof*: (Same as Ex. 3).

Proof: 1. CD bisects AB at D (Given). 2. AD = DB (§22). 3. ∠m = ∠n (Given). 4. CD = CD (Iden.). 5. △ADC ≅ △CDB (§65). 6. AC = BC (§78).

6. *Planning the Proof*: (Same as in Ex. 4).

Proof: 1. AC bisects ∠BCD (Given). 2. ∠DCA = ∠ACB (§22). 3. AC bisects ∠DAB (Given). 4. ∠DAC = ∠CAB (§22). 5. AC = AC (Iden.). 6. △DCA ≅ △CAB (§73). 7. ∠B = ∠D (§78).

7. *Planning the Proof*: (Same as in Ex. 4).

Proof: 1. AD = DC (Given). 2. ∠x = ∠y (Given). 3. DB = DB (Iden.). 4. △DBA ≅ △DCB (§65). 5. ∠m = ∠n (§78).

8. *Planning the Proof*: 1. We can prove lines ⊥ by §14. 2. We shall use §14.

Proof: 1. CD bisects ∠ ACB (Given). 2. ∠ x = ∠ y (§ 22). 3. AC = BC (Given). 4. CD = CD (Iden.). 5. △ ADC ≅ △ DBC (§65). 6. ∠ ADC = ∠ BDC (§78). 7. CD ⊥ AB (§14).

9. *Planning the Proof*: (Same as in Ex. 3).

Proof: 1. BD bisects ∠ ABC (Given). 2. ∠ ABD = ∠ DBC (§22). 3. BA = BC (Given). 4. BD = BD (Iden.). 5. △ BAD ≅ △ BDC (§65). 6. AD = CD (§78).

Note: § 73 can be used to prove the △s ≅.

10. *Planning the Proof*: (Same as in Ex. 4).

Proof: 1. ∠x = ∠y (Given). 2. ∠m = ∠n (Given). 3. BD = BD (Iden.). 4. △ABD ≅ △BCD (§73). 5. ∠A = ∠C (§78).

11. *Planning the Proof*: (Same as in Ex. 3).

Proof: 1. ∠x = ∠y (Given). 2. AD ⊥ DC (Given). 3. ∠ ADC = 1 rt. ∠ (§14). 4. CB ⊥ AB (Given). 5. ∠ ABC is a rt. ∠ (§ 14). 6. ∠ m is comp. ∠ x and ∠ n is comp. ∠ y (§20). 7. ∠ m = ∠ n

(§50b). 8. BD = BD (Iden.). 9. △ ABD ≅ △ BDC (§73). 10. AD = BC (§78).

12. Planning the Proof: (Same as in Ex. 3).

Proof: 1. AB and CD bisect each other (Given). 2. CE = ED and AE = EB (§22). 3. ∠ AEC = ∠ BED (§50d). 4. △ ACE ≅ △ BED (§65). 5. AC = BD (§78).

13. Planning the Proof: (Same as in Ex. 3).

Proof: 1. CA = CD (Given). 2. CE bisects ∠ ACB (Given). 3. ∠ ACE = ∠ ECD (§22). 4. CE = CE (Iden.). 5. △ ACE ≅ △ ECD (§65). 6. AE = DE (§78).

14. Planning the Proof: (Same as in Ex. 3).

Proof: 1. ∠ DAB = ∠ ABC (Given). 2. AD = BC (Given). 3. AB = AB (Iden.). 4. △ DAB ≅ △ ABC (§65). 5. AC = BD (§78).

15. Planning the Proof: (Same as in Ex. 3).

Proof: 1. OA, OB, OC, and OD are radii of ⊙ O (Given). 2. OA = OB = OC = OD (Asmt. 26). 3. ∠ AOB = ∠ COD (Given). 4. △ ABO ≅ △ CDO (§65). 5. AB = CD (§78).

16. Planning the Proof: (Same as in Ex. 4).

Proof: 1. AC = BC (Given). 2. AE = BD (Given). 3. CE = CD (Asmt. 2). 4. ∠C = ∠C (Iden.). 5. △ACD ≅ △BCE (§65). 6. ∠ A = ∠ B (§78).

17. Planning the Proof: (Same as in Ex. 4).

Proof: 1. AD ⊥ AB (Given). 2. ∠DAB is a rt. ∠ (§14). 3. CB ⊥ AB (Given). 4. ∠ABC is a rt. ∠ (§14). 5. ∠DAB = ∠ABC (§50a). 6. ∠x = ∠y (Given). 7. ∠BAC = ∠ABD (Asmt. 2). 8. AB = AB (Iden.). 9. △DAB ≅ △ ABC (§73). 10. ∠D = ∠C (§78).

18. Planning the Proof: (Same as in Ex. 4).

Proof: 1. CE ⊥ AB (Given). 2. ∠w is a rt. ∠ (§14). 3. BF ⊥ CD (Given). 4. ∠z is a rt. ∠ (§14). 5. ∠w = ∠z (§50a). 6. BE = CF and CE = BF (Given). 7. △CEB ≅ △CFB (§65). 8. ∠ OCB = ∠ OBC (§78).

Note: §71 can be used to prove the △s ≅.

19. Planning the Proof: (Same as in Ex. 3).

Proof: 1. ACE and BCD are st. lines (Given). 2. F is the midpoint of AB (Given). 3. AF = BF (§22). 4. ∠ A = ∠ B and ∠ x = ∠ y (Given). 5. ∠ BFD is the supp. of ∠ x and ∠ AFE is the supp. of ∠ y (§53b). 6. ∠ BFD = ∠ AFE (§50c). 7. △ AFE ≅ △ BFD (§73). 8. AE = BD (§78).

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1. 1. PE = PB (§79). 2. ℓ bisects EB (§22). 3. $\triangle EHP \cong \triangle HBP$ (§65). 4. $\angle E = \angle B$ and $\angle EPH = \angle BPH$ (§78). 5. PH bisects $\angle EPB$ (§22).

2. 1. MN $\perp$ RS (§14). 2. MN bisects RS (§22). 3. MR = MS = 3 (§79). 4. SN = RN = 10 (§79). 5. $\triangle TRN \cong \triangle TNS$ and $\triangle TRM \cong \triangle TMS$ (§65). 6. $\angle MNR = \angle MNS$, $\angle MRS = \angle MSR$, and $\angle TRN = \angle TSN$ (§78). 7. $\angle MRN = \angle MSN$ (Asmt. 1, 9, 7).

The student will probably find other conclusions.

3. 1. $\triangle BFA \cong \triangle CFA$ (§65). 2. $\angle ABF = \angle ACF$, $\angle AFB = \angle AFC$, and BF = CF (§78). 3. $\angle BFD = \angle CFD$ (§50c). 4. $\triangle BDF \cong \triangle CDF$ (§65). 5. BD = DC, $\angle DBF = \angle DCF$, and $\angle BDF = \angle CDF$ (§78). 6. FD $\perp$ BC (§14). 7. $\angle ABC = \angle ACB$ (Asmt. 1, 9, 7).

4. 1. AO = OC and DO = OB (§22). 2. $\angle AOD = \angle BOC$ and $\angle DOC = \angle AOB$ (§50d). 3. $\triangle AOD \cong \triangle BOC$ and $\triangle DOC \cong \triangle AOB$ (§65). 4. AB = DC and AD = BC (§78). 5. $\angle BDC = \angle ABD$ and $\angle ADB = \angle DBC$ (§78). 6. $\angle ADC = \angle ABC$ (Asmt. 1, 9, 7). 7. $\angle DAC = \angle ACB$ and $\angle BAC = \angle DCA$ (§78). 8. $\angle BAD = \angle BCD$ (Asmt. 1, 9, 7).

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1. §§65, 73.
2. Asmt. 1-8, 26; §§ 78, 79.
3. §§50, 78, 83, 84.
4. §14.
5. §56.
6. $\angle S = 36°$ (§83).

7. $\angle R = \angle S$ (§83). $\angle R + \angle S = 78°$.
$\angle R + \angle R = 78°$. $2\angle R = 78°$. $\angle R = 39°$.
$\angle S = 39°$. $\angle R = \angle S = 39°$. Ans.

8. $\angle R = \angle S$ (§83). $3x + 6 = x + 46$ (Asmt. 6).
$2x = 40$. $x = 20$. $\angle R = 60 + 6 = 66$.
$\angle S = 66$. $\angle R = \angle S = 66°$.

9. BC, AC, AB; $\angle C$, $\angle A$, $\angle B$. 10. QN; MN; MQ.

11. Planning the Proof: 1. We can prove $\angle$s = by §§50, 78, 83, 84. 2. We shall use §§83, 50c.

Proof: 1. AC, AF, and ED are st. lines (Given). 2. AB = AH (Given). 3. $\angle ABH = \angle AHB$ (§83). 4. $\angle x = \angle ABH$ and $\angle y = \angle AHB$ (§50d). 5. $\angle x = \angle y$ (Asmt. 7).

12. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

Proof: 1. AD $\perp$ DC (Given). 2. $\angle ADC$ is a rt. $\angle$ (§14). 3. BC $\perp$ DC (Given). 4. $\angle BCD$ is a rt. $\angle$ (§14). 5. $\angle ADC$ =

$\angle BCD$ (§50a). 6. $\angle x = \angle y$ (Given). 7. $DC = DC$ (Iden.). 8. $\triangle ADC \cong \triangle BDC$ (§73). 9. $AD = BC$ (§78).

13. Planning the Proof: (Same as in Ex. 12).

Proof: 1. $AC = BC$ (Given). 2. $\angle A = \angle B$ (§83). 3. D is the midpoint of AB (Given). 4. $AD = DB$ (§22). 5. $\angle x = \angle y$ (Given). 6. $\triangle ADE \cong \triangle BDF$ (§73). 7. $DE = DF$ (§78).

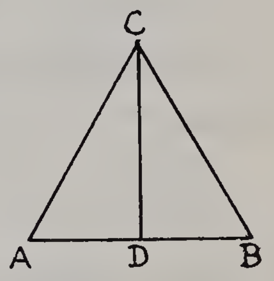
14. Planning the Proof: 1. We can prove: $\triangle$s $=$ by §§50, 78, 83, 84; $\triangle$s $\cong$ by §§65, 71, 73. 2. We shall use: §83 for (a) and (b); Asmt. for (c); and §65 for (d).

Proof: 1. $AC = BC$ (Given). 2. $\angle x = \angle y$ (§83). 3. $AD = DB$ (Given). 4. $\angle m = \angle n$ (§83). 5. $\angle x + \angle m = \angle y + \angle n$ (Asmt. 1). 6. $\angle x + \angle m = \angle CAD$ and $\angle y + \angle n = \angle CBD$ (Asmt. 9). 7. $\angle CAD = \angle CBD$ (Asmt. 7). 8. $\triangle ACD \cong \triangle BCD$ (§65).

15. Planning the Proof: 1. We can prove angles = by §§50, 78, 83, 84. 2. We shall use §83 and Asmt. 2.

Proof: 1. $HF = KF$ (Given). 2. $\angle KHF = \angle HKF$ (§83). 3. $HP = KP$ (Given). 4. $\angle PHK = \angle HKP$ (§83). 5. $\angle x = \angle y$ (Asmt. 2).

16. Given isos. $\triangle ABC$, $AC = BC$, and CD bisecting vertex $\angle ACB$.



To prove that CD bisects base AB.

Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26 and §§78, 79. 2. We shall use §78.

Proof: 1. $AC = BC$ (Given). 2. CD bisects $\angle ACB$ (Given). 3. $\angle ACD = \angle DCB$ (§22). 4. $CD = CD$ (Iden.). 5. $\triangle ACD \cong \triangle BCD$ (§65). 6. $AD = DB$ (§78). 7. CD bisects AB (§22).

17. See Fig. for Ex. 16.

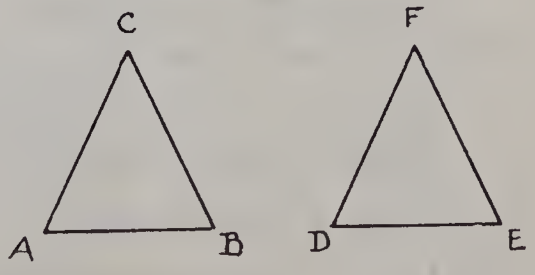
Given isos. $\triangle ABC$, $AC = BC$, and CD bisecting $\angle ACB$.

To prove that $CD \perp AB$.

Planning the Proof: 1. We can prove lines $\perp$ by §14. 2. We shall use §14.

Proof: 1. $AC = BC$ (Given). 2. CD bisects $\angle ACB$ (Given). 3. $\angle ACD = \angle DCB$ (§22). 4. $CD = CD$ (Iden.). 5. $\triangle ACD \cong \triangle BCD$ (§65). 6. $\angle ADC = \angle BDC$ (§78). 7. $CD \perp AB$ (§14).

18. Given isos. $\triangle$s ABC and DEF, having vertex $\angle C =$ vertex $\angle F$, and side $AC =$ side DF.



To prove that $\triangle ABC \cong \triangle DEF$.

Planning the Proof: 1. We can prove $\triangle$s $\cong$ by §§65, 71, 73. 2. We shall use §65.

Proof: 1. △s ABC and DEF are isos. having vertices C and F respectively (Given). 2. Then AC = BC and DF = FE (§§56, 58). 3. AC = DF (Given). 4. BC = EF (Asmt. 7). 5. ∠C = ∠F (Given) 6. △ABC ≅ △DEF (§65).

19. Let x = one side of the △. Then each of the other sides is x (§56). Then 3x = 168 (§55). x = 56. 56 ft. Ans.

20. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26 and §§78, 79. 2. We shall use §78.

Proof: 1. AC = BC (Given). 2. ∠A = ∠B (§83). 3. AD = EB (Given). 4. △ADC ≅ △EBC (§65). 5. CD = CE (§78).

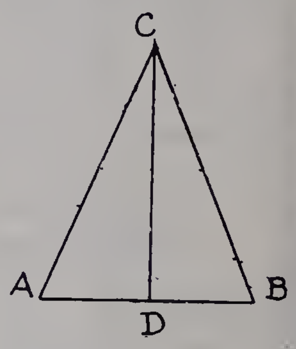
21. 1. CD = CE (Given). 2. ∠EDC = ∠DEC (§83). 3. ∠ ADC is the supp. of ∠EDC and ∠BEC is the supp. of ∠DEC (§53b). 4. ∠ADC = ∠BEC (§50c). 5. AD = EB (Given). 6. △ADC ≅ △BEC (§65). 7. ∠A = ∠B, AC = BC, and ∠ACD = ∠BCE (§78). 8. ∠DCE = ∠DCE (Iden.). 9. ∠ACE = ∠DCB (Asmt. 1, 7, 9).

22. Given DC bisecting ∠ACB, and AB ⊥ CD at D.

To prove that △ABC is isos.

Planning the Proof: 1. We can prove that a triangle is isos. by §56. 2. We shall use §56.

Proof: 1. DC bisects ∠ACB (Given). 2. ∠ACD = ∠DCB (§22). 3. CD ⊥ AB (Given). 4. ∠ ADC = ∠BDC (§14). 5. CD = CD (Iden.). 6. △ ADC ≅ △BDC (§73). 7. AC = BC (§78). 8. △ABC is isos. (§56).



23. Given isos. △ ADE having AE = DE, AB = BC = CD, and line segments EB and EC.

To prove that EB = EC.

Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26 and §§78, 79. 2. We shall use §78.

Proof: 1. AE = DE (Given). 2. ∠A = ∠D (§83). 3. AB = CD (Given). 4. △ABE ≅ △CDE (§65). 5. EB = CE (§78).

24. Given △ABC, having CD bisecting ∠ACB and ⊥ AB.

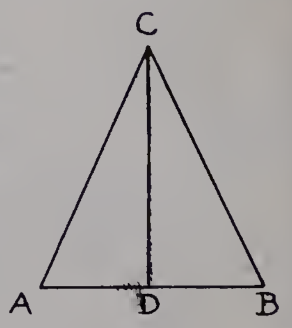
To prove that ABC is isos.

Planning the Proof: 1. We can prove that a triangle is isos. by §56. 2. We shall use §56.

Proof: 1. CD bisects ∠ACB (Given).

2. ∠ACD = ∠BCD (§22). 3. CD ⊥ AB (Given).

4. ∠ADC = ∠CDB (§14). 5. CD = CD (Iden). 6. △ADC ≅ △DBC (§73). 7. AC = BC (§78). 8. △ABC is isos. (§56).

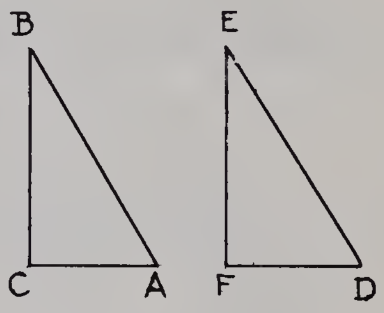


25. Planning the Proof: 1. We can prove: (a) △s = by §§ 65, 71, 73; (b) ∠s = by §§ 50, 78, 83, 84; (c) ∠s ≠ by Asmt. 9, 10. 2. We shall use: (a) §65; (b) § 78; (c) Asmt. 9, 6.

Proof: 1. AD = DE and CD = DB (Given). 2. ∠ CDA = ∠ BDE (§50d). 3. ∴ △ ADC ≅ △ DBE (§65). 4. ∠ E = ∠ m (§ 78). 5. ∠ BAC > ∠ m (Asmt. 9). 6. ∠ BAC > ∠ E (Asmt. 6).

26. Planning the Proof: 1. We can prove: (a) line segments = by Asmt. 1-8, 26, and §§ 78, 79; (b) ∠s = by §§50, 78, 83, 84. 2. We shall use: (a) § 78; (b) § 78, and Asmt.

Proof: 1. AB = HF and ∠x = ∠y (Given). 2. HB = HB (Iden.). 3. ∴ △ ABH ≅ △BHF (§65). 4. AH = BF (§ 78). 5. ∠ AHB = ∠FBH (§78). 6. ∠x + ∠AHB = ∠y + ∠FBH (Asmt. 1). 7. ∠x + ∠AHB = ∠AHF and ∠y + ∠FBH = ∠ABF (Asmt. 9). 8. ∠AHF = ∠FBA (Asmt. 7).



27. Given the rt. △s ABC and DEF with ∠s C and F the rt. ∠s, BC = EF, and ∠ B = ∠ E.

To prove that △ ABC ≅ △DEF.

Planning the Proof: 1. We can prove △s ≅ by §§65, 71, 73. 2. We shall use § 73.

Proof: 1. BC = EF and ∠B = ∠E (Given). 2. ∠C and ∠F are rt. ∠s (Given). 3. ∠C = ∠F (§50a). 4. ∴ △ ABC ≅ △DEF (§ 73).

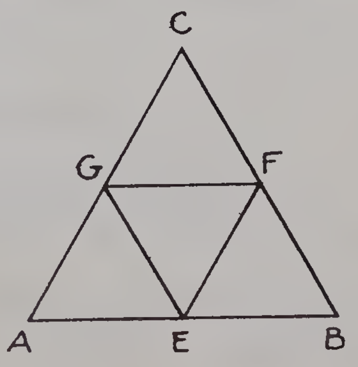
28. Given the supp. adj. ∠s AOC and COB with DO bisecting ∠AOC and EO bisecting ∠COB.

To prove that DO ⊥ EO.

Planning the Proof: 1. We can prove lines ⊥ by §14. 2. We shall use § 14.

Proof: 1. DO bisects ∠AOC and EO bisects ∠BOC (Given). 2. ∠ AOD = ∠ DOC and ∠COE = ∠BOE (§ 22). 3. ∠ AOC + ∠COB = 1 st. ∠ (Asmt. 9). 4. 2∠DOC + 2∠COE = 2 rt. ∠s (Asmt. 9, 6, §16). 5. ∠DOC + ∠COE = 1 rt. ∠ (Asmt. 4). 6. ∠DOC + ∠COE = ∠DOE (Asmt. 9). 7. ∠DOE = 1 rt. ∠ (Asmt. 6). 8. DO ⊥ EO (§14).

29. Given the equilateral △ABC with EF, FG, and GE joining the midpoints of the sides AB, BC, and CA.



To prove that △GEF is equilateral.

Planning the Proof: 1. We can prove that a △ is equilateral by § 56. 2. We shall use § 56.

Proof: 1. E, F, and G are the midpoints of AB, BC, and AC respectively (Given). 2. AB = BC = CA (Given). 3. AE = EB, BF = FC, and

CG = GA (§22). 4. 2AE = 2BF = 2GC (Asmt. 6). 5. AE = BF = CG (Asmt. 4). 6. In like manner, AG = BE = CF (Statements 4, 5). 7. $\angle A = \angle B = \angle C$ (§84). 8. $\triangle AEG \cong \triangle BFE \cong \triangle CGF$ (§65). 9. GE = EF = FG (§78). 10. $\therefore \triangle GEF$ is equilateral.

30. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

Proof: 1. AC = BC (Given). 2. $\angle CBA = \angle CAB$ (§83). 3. $\angle m = \angle n$ (Given). 4. AB = AB (Iden.). 5. $\triangle ABD \cong \triangle ABE$ (§73). 6. AE = BD (§78).

31. Planning the Proof: (Same as in Ex. 30).

Proof: 1. AO = BO and $\angle p = \angle q$ (Given). 2. $\angle AOD = \angle BOE$ (§50d). 3. $\triangle AOD \cong \triangle BOE$ (§73). 4. $\therefore$ AD = BE (§78).

32. Planning the Proof: (Same as in Ex. 30).

Proof: 1. AC = BC and CD = CE (Given). 2. $\angle C = \angle C$ (Iden.). 3. $\triangle AEC \cong \triangle BDC$ (§65). 4. AE = BD (§78).

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1. Planning the Proof: We can prove $\angle$s = by §§50, 78, 83, 84. 2. We shall use §78.

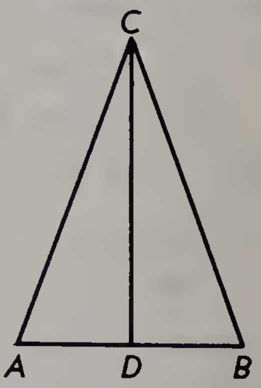
Proof: 1. AD = AB and DC = BC (Given). 2. AC = AC (Iden.). 4. $\triangle ADC \cong \triangle ABC$ (§85). 5. $\angle ACB = \angle ACD$ (§78). 6. AC bisects $\angle BCD$ (§22).

2. Given the isos. $\triangle ABC$ with AC = BC, D the midpoint of AB, and CD drawn.

To prove that $\triangle ACD \cong \triangle BCD$.

Planning the Proof: 1. We can prove $\triangle$s $\cong$ by §§65, 71, 73, 85. 2. We shall use §85.

Proof: 1. AC = BC (Given). 2. AD = DB (Given and §22). 3. CD = CD (Iden.). 4. $\triangle ACD \cong \triangle BCD$ (§85).



3. Planning the Proof: 1. We can prove: (a) $\triangle$s $\cong$ by §§65, 71, 73, 85; (b, c, d, e) $\angle$s = by §§50, 78, 83, 84. 2. We shall use: (a) §85; (b, c, d) §78; (e) Asmt. 1, 9, 7.

Proof: 1. AD = BC and AB = DC (Given). 2. DB = DB (Iden.). 3. $\triangle ABD \cong \triangle DBC$ (§85). 4. $\angle A = \angle C$, $\angle x = \angle y$, $\angle m = \angle n$ (§78). 5. $\angle x + \angle m = \angle y + \angle n$ (Asmt. 1). 6. $\angle x + \angle m = \angle ADC$ and $\angle y + \angle n = \angle ABC$ (Asmt. 9). 7. $\angle ADC = \angle ABC$ (Asmt. 7).

4. Planning the Proof: 1. We can prove: (a) $\angle$s = by §§50, 78, 83, 84; (b) $\triangle$s $\cong$ by §§65, 71, 73, 85; (c) line segments = by Asmt. 1-8, 26, §§78, 79; (d) lines $\perp$ by §14. 2. We shall use: (a) §78; (b) §65; (c) §78; (d) §14.

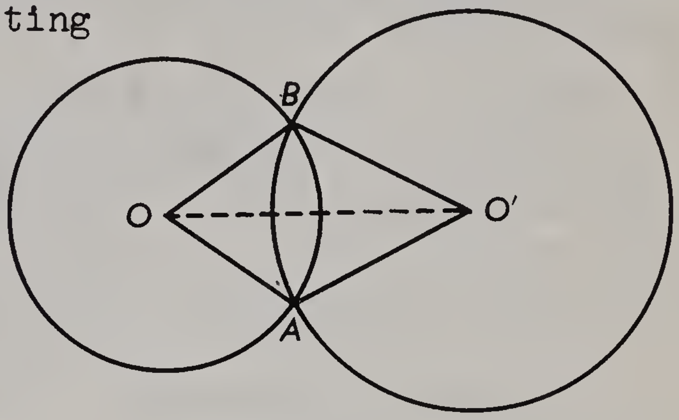
Proof: 1. AB = BC and AD = DC (Given). 2. BD = BD (Iden.). 3. $\triangle ABD \cong \triangle CBD$ (§85). 4. $\angle x = \angle y$ (§78). 5. BE = BE (Iden.). 6. $\triangle ABE \cong \triangle CBE$ (§65). 7. AE = EC (§78). 8. $\angle AEB = \angle CEB$ (§78). 9. BE $\perp$ AC (§14).

5. (1) The brace forms a triangle with the other supports, thereby making the fence more rigid. (2) Theorem 4.

6. Given $\odot$O and $\odot$O', intersecting in points A and B.

Prove that $\angle OAO' = \angle OBO'$.

Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use §78.



Proof: 1. OB = OA and O'B = O'A (Asmt. 26). 2. Draw OO' (Asmt. 11). 3. $\triangle OBO' \cong \triangle OAO'$ (§85). 4. $\angle$ OAO' = OBO' (§78).

7. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

Proof: 1. $\angle AEB = \angle CED$ (Given). 2. AE = DE (Given). 3. $\angle A = \angle D$ (§83). 4. $\triangle ABE \cong \triangle DCE$ (§73). 5. BE = CE (§78).

8. Proof: 1. AC = BC, and AD = DB (Given). 2. $\angle A = \angle B$ (§83). 3. $\angle x = \angle y$ (Given). 4. $\triangle ADE \cong \triangle BDF$ (§73). 5. AE = BF, DE = DF, and $\angle AED = \angle BFD$ (§78). 6. $\angle DEC$ is the supp. of $\angle AED$ and $\angle CFD$ is the supp. of $\angle BFD$ (§53b). 7. $\angle DEC = \angle CFD$ (§50c). 8. CE = CF (Asmt. 2).

9. Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use §84 and Asmt. 7.

Proof: 1. AB = BC = AC (Given). 2. $\angle C = \angle BAC$ (§84). 3. $\angle x = \angle BAC$ (§50d). 4. $\angle x = \angle C$ (Asmt. 7).

10. Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. CD is the $\perp$ bisector of AB (Given). 2. AC = BC and DA = DB (§79). 3. CD = CD (Iden.). 4. $\triangle ACD \cong \triangle BCD$ (§85). 5. $\angle CAD = \angle CBD$ (§78).

11. Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use Asmt. 7.

Proof: 1. EB bisects $\angle ABC$ and EC bisects $\angle BCD$ (Given). 2. $\angle x = \angle y$ and $\angle m = \angle n$ (§22). 3. EB = EC (Given). 4. $\angle y = \angle m$ (§83). 5. $\therefore \angle x = \angle n$ (Asmt. 7).

12. Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. MP = NP (Given). 2. ∠M = ∠N (§83). 3. RS bisects MP and RT bisects NP (Given). 4. MS = SP and NT = TP (§22). 5. 2MS = 2NT (Asmt. 6). 6. MS = NT (Asmt. 4). 7. R is the midpoint of MN (Given). 8. MR = RN (§22). 9. △MRS ≅ △NRT (§65). 10. ∠x = ∠y (§78).

13. Planning the Proof: 1. We can prove lines ⊥ by §14. 2. We shall use §14.

Proof: 1. AC = BC (Given). 2. ∠ABC = ∠BAC (§83). 3. AD = BD (Given). 4. ∠ABD = ∠BAD (§83). 5. ∠CBD = ∠CAD (Asmt. 2) 6. △ACD ≅ △BCD (§65). 7. ∠CDA = ∠CDB (§78). 8. ∠ADE = ∠BDE (§§53b, 50c). 9. DE = DE (Iden.). 10. △ADE ≅ △BDE (§65) 11. ∠AED = ∠BED (§78). 12. CD ⊥ AB (§14).

14. Planning the Proof: 1. We can prove ∠s = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. AB = DC (Given). 2. DE = BF and AE = FC (Given.). 3. EF = EF (Iden.). 4. AE + EF = EF + FC (Asmt. 1). 5. AE + EF = AF and EF + FC = EC (Asmt. 9). 6. AF = EC (Asmt. 7). 7. △ABF ≅ △DCE (§85). 8. ∠x = ∠y (§78).

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1. Given the st. lines AD, EB, AB, and ED; CD = CA, CE = CB, and ED = 48 feet.

To prove that AB = 48 feet.

Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78 and Asmt. 6.

Proof: 1. CD = CA and EC = CB (Given). 2. ∠ECD = ∠ACB (§50d). 3. △CED ≅ △CAB (§65). 4. ∴ ED = AB (§78). 5. ED = 48 feet (Given). 6. ∴ AB = 48 feet (Asmt. 6).

2. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

Proof: 1. ∠BAC = 90° (Given). 2. ∠BAC is a rt. ∠ (§17). 3. DE ⊥ AD (Given). 4. ∠CDE is a rt. ∠ (§14). 5. ∠BAC = ∠CDE (§50a). 6. AC = CD (Given). 7. ∠ACB = ∠FCD (§50d). 8. △ACB = △DCF (§73). 9. DF = AB (§78).

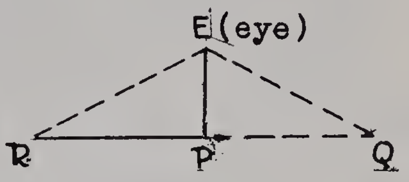
3. (1) The triangle is more rigid than a quadrilateral.
(2) No.

4. (1) The diagram at the left. (2) In the diagram at the left, the middle of the bridge is supported by means of the iron-wire cable attached at A and C. In the diagram at the right, the

middle of the bridge is supported only by the wood girder. If that sags, the wire would sag with it instead of giving support.

5. Given the st. lines RQ, RE, EQ, and EP; $EP \perp RQ$; and $\angle REP = \angle QEP$.

To prove that $PR = PQ$.



Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§ 78, 79. 2. We shall use § 78.

Proof: 1. $\angle REP = \angle QEP$ (Given). 2. $EP \perp RQ$ (Given). 3. $\angle RPE = \angle QPE$ (§ 14). 4. $PE = PE$ (Iden.). 5. $\triangle RPE \cong \triangle QPE$ (§ 73). 6. $\therefore PR = PQ$ (§ 78).

6. Given the isos. $\triangle ABC$ having $AC = BC$, M the midpoint of AB, and line segment CM.

To prove that $AB \perp CM$.

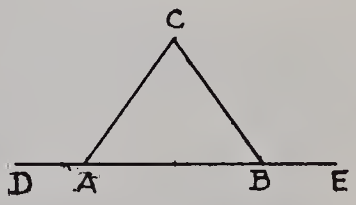
Planning the Proof: 1. We can prove lines $\perp$ by § 14. 2. We shall use § 14.

Proof: 1. $AC = BC$ (Given). 2. M is the midpoint of AB (Given). 3. $AM = MB$ (§ 22). 4. $CM = CM$ (Iden.). 5. $\triangle AMC \cong \triangle BMC$ (§ 85). 6. $\angle AMC = \angle BMC$ (§ 78). 7. $AB \perp CM$ (§ 14). 8. $\therefore$ AB is level (by definition of level given in exercise).

8. MQ.

9. Given the isos. $\triangle ABC$ with $AC = CB$ and AB extended in both directions to E and D.

To prove that $\angle DAC = \angle EBC$.



Planning the Proof: 1. We can prove $\angle$s = by §§ 50, 78, 83, 84. 2. We shall use §§ 83, 50c.

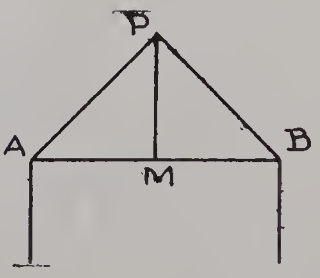
Proof: 1. $AC = BC$ (Given). 2. $\angle CAB = \angle CBA$ (§ 83). 3. $\angle DAC$ is the supp. of $\angle BAC$ and $\angle EBC$ is the supp. of $\angle ABC$ (§ 53b). 4. $\angle DAC = \angle EBC$ (§ 50c).

10. In the figure take M as the midpoint of AB and draw PM.

Given the $\triangle PAB$ with M the midpoint of AB and $PM \perp AB$.

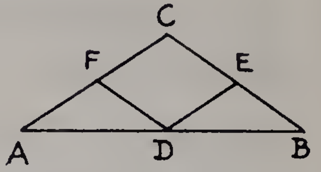
To prove that $PA = PB$.

Planning the Proof: (Same as in Ex. 5).



Proof: 1. $AM = BM$ (Given and § 22). 2. AB is horizontal and PM is vertical, making $\angle PMA = \angle PMB$ (§ 14). 3. $PM = PM$ (Iden.). 4. $\triangle PMA \cong \triangle PBM$ (§ 65). 5. $PA = PB$ (§ 78).

11. Given the isos. $\triangle ABC$ with $AC = BC$, DE joining the midpoints of the sides AB and BC, and DF joining the midpoints of the sides AB and AC.



To prove that $\angle ADF = \angle BDE$.

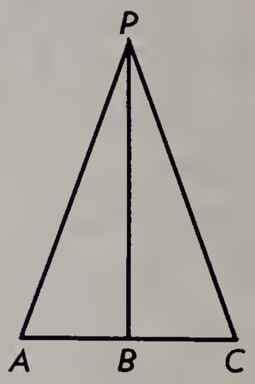
Planning the Proof: (Same as in Ex. 9).

Proof: 1. D is the midpoint of AB, E the midpoint of BC, and F the midpoint of AC (Given). 2. $AD = DB$ (§22). 3. $AC = BC$ (Given). 4. $\angle A = \angle B$ (§83). 5. $AF = FC$ and $BE = CE$ (§22). 6. $2AF = 2BE$ (Asmt. 6). 7. $AF = BE$ (Asmt. 4). 8. $\triangle ADF \cong \triangle BDE$ (§65). 9. $\angle ADF = \angle BDE$ (§78).

12. The rigidity of the house on the left is due to the sum of the rigidities of the sheathing boards.

The house on the right is more rigid because use is made of many rigid triangles. These triangles are rigid by §85.

13. Given oblique line segments PA and PC cutting off equal line segments AB and BC and $PB \perp AC$.



To prove $PA = PC$.

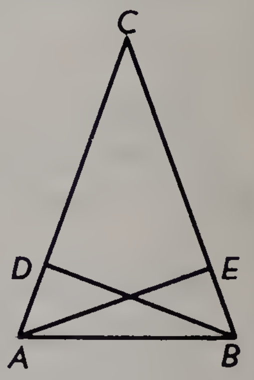
Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §79.

Proof: 1. $PB \perp AC$ (Given). 2. $AB = BC$ (Given). 3. $PA = PC$ (§79).

14. Given isos. $\triangle ABC$, $AC = BC$, AE bisecting $\angle BAC$ and intersecting BC in E, and BD bisecting $\angle ABC$ and intersecting AC in D.

To prove that $CD = CE$.

Planning the Proof: (Same as in Ex. 5).



Proof: 1. $AC = BC$ (Given). 2. $\angle BAC = \angle ABC$ (§83). 3. AE bisects $\angle BAC$ and BD bisects $\angle ABC$ (Given). 4. $\angle CAE = \angle EAB$ and $\angle CBD = \angle DBA$ (§22). 5. $2\angle CAE = 2\angle DBC$ (Asmt. 6). 6. $\angle CAE = \angle CBD$ (Asmt. 4). 7. $\angle C = \angle C$ (Iden.). 8. $\triangle ACE \cong \triangle BDC$ (§73). 9. $CD = CE$ (§78).

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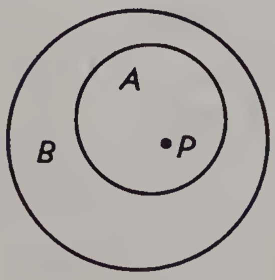
1.



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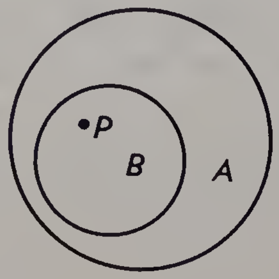

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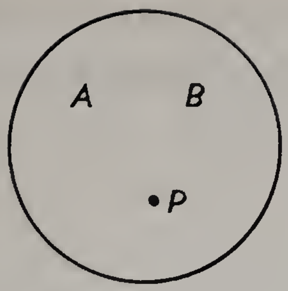

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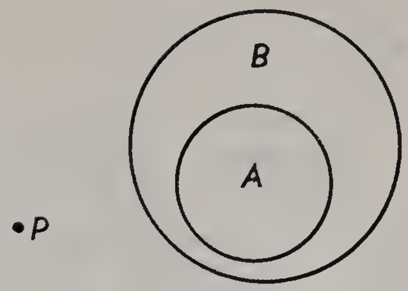

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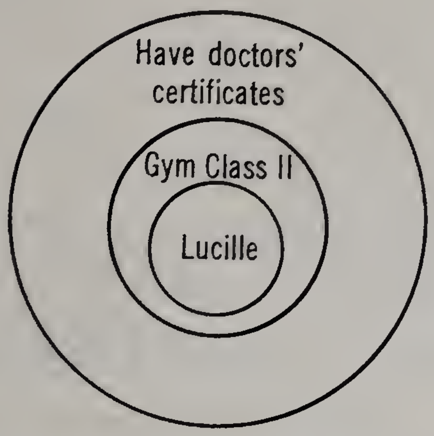
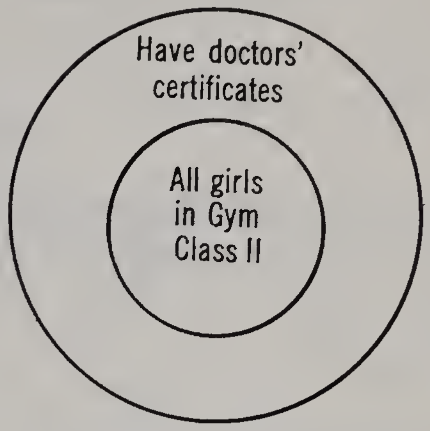


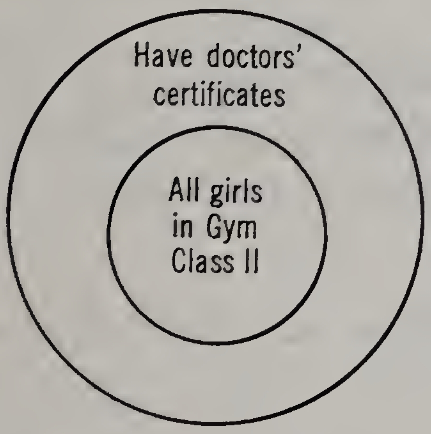
Conclusion is valid

6.



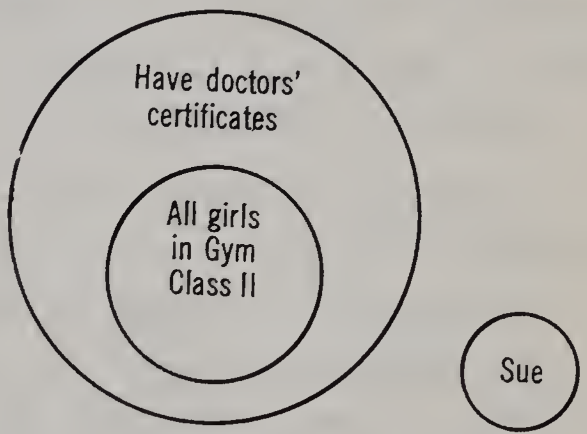
Conclusion is not valid because Frances' circle may be inside the larger circle but not inside the smaller circle.

7.



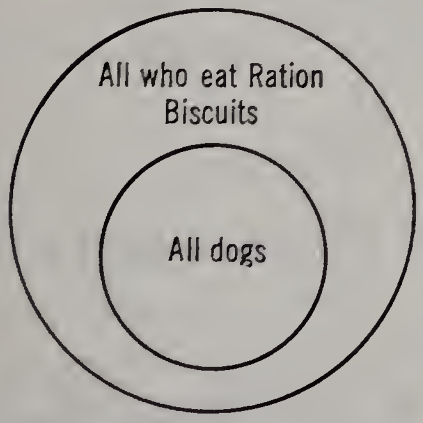
Conclusion is not valid as Caroline's circle may be inside larger circle but not inside the smaller circle.

8.



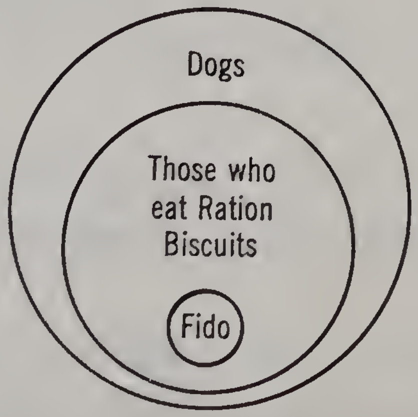
Conclusion is not valid.

9.



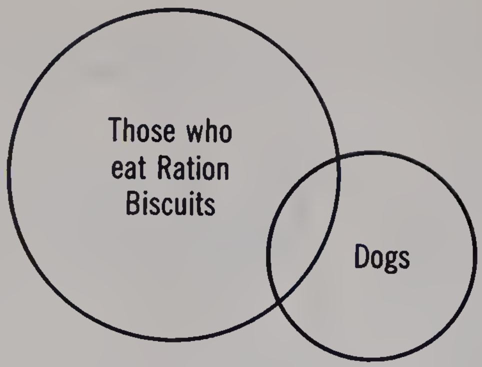
Conclusion is not valid. Susie may be a dog or she may not be a dog.

10.



Conclusion is valid.

11.



Conclusion is not valid as Blimpy's circle may be any place outside the larger circle.

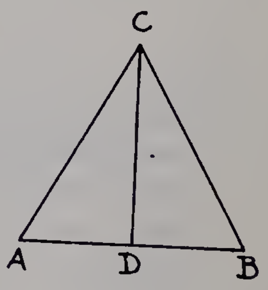
PAGES 119-120

1. (1) In the figure at the left $\angle x = 125°$, $\angle A = 77°$, and $\angle C = 48°$; In the figure at the right $\angle x = 75°$, $\angle A = 48°$, and $\angle C = 27°$. (2) $\angle x > \angle A$. (3) $\angle x > \angle C$. (4) $\angle x$ appears to $= \angle A + \angle C$.

2. Planning the Proof: 1. We can prove: (a) $\triangle$s = by §§50, 78, 83, 84; (b, c) $\triangle$s $\neq$ by Asmt. 9, 10. 2. We shall use: (a) §78; (b, c) Asmt. 9.

Proof: 1. AD and BC bisect each other (Given). 2. CO = OB and AO = OD (§22). 3. $\angle AOC = \angle BOD$ (§50d). 4. $\triangle AOC \cong \triangle BOD$ (§65). 5. $\angle x = \angle C$ (§78). 6. $\angle CBE > \angle x$ (Asmt. 9). 7. $\angle CBE > \angle C$ (Asmt. 6).

3. Given the $\triangle ABC$ with CD both a median and an altitude to side AB.



To prove that $\triangle ABC$ is isos.

Planning the Proof: 1. We can prove a $\triangle$ isos. by §56. 2. We shall use §56.

Proof: 1. In $\triangle ABC$, CD is a median and an altitude (Given). 2. AD = DB (§89). 3. $CD \perp AB$ (§90). 4. $\angle CDA = \angle CDB$ (§14). 5. CD = CD (Iden.). 6. $\triangle CDA \cong \triangle CDB$ (§65). 7. AC = BC (§78). 8. $\triangle ABC$ is isos. (§56).

4. (For Fig., see Ex. 3.)

Given the isos. $\triangle ABC$ with AC = BC and CD the median to the base AB.

To prove that $\angle ACD = \angle BCD$.

Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. CD is the median to base AB of isos. $\triangle ABC$ (Given.) 3. AC = BC (Given). 3. AD = DB (§89). 4. CD = CD (Iden.). 5. $\triangle ADC \cong \triangle BDC$ (§85). 6. $\angle ACD = \angle BCD$ (§78).

5. (For Fig., see Ex. 3.)

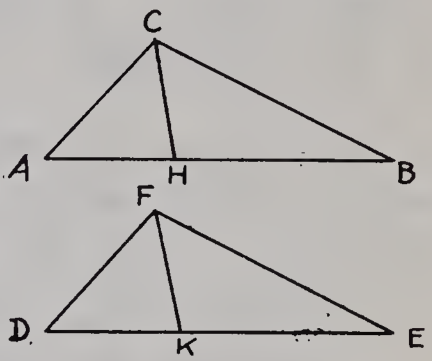
Given the isos. △ABC with AC = BC and CD the median to the base AB.

To prove that CD is an altitude of △ABC.

Planning the Proof: 1. We can prove that a line segment is an altitude of a △ by §90. 2. We shall use §90.

Proof: 1. CD is a median of isos. △ABC (Given). 2. AD = DB (§89). 3. AC = BC (Given). 4. CD = CD (Iden.). 5. △ADC ≅ △BDC (§85). 6. ∠ADC = ∠BDC (§78). 7. CD ⊥ AB (§14). 8. CD is an altitude of △ABC (§90).

6. Given △ABC ≅ △DEF with CH bisecting ∠C and FK bisecting the corr. ∠F.

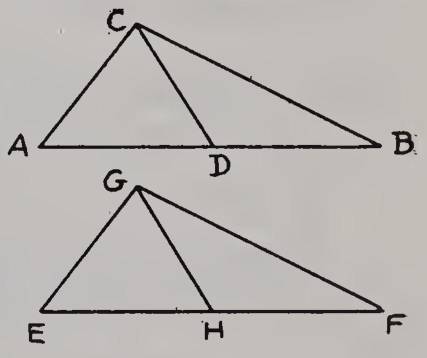


To prove that CH = FK.

Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

Proof: 1. △ACB ≅ △DEF (Given). 2. AC = DF and ∠ACB = ∠DFE (§78). 3. CH bisects ∠ACB and KF bisects ∠DFE (Given). 4. ∠ACH = ∠HCB and ∠DFK = ∠KFE (§22). 5. 2∠ACH = 2∠DFK (Asmt. 6). 6. ∠ACH = ∠DFK (Asmt. 4). 7. ∠A = ∠D (§78). 8. △ACH ≅ △DFK (§73). 9. CH = FK (§78).

7. Given △ABC ≅ △EFG, CD the median to side AB of △ABC, and GH the median to the corr. side EF of △EGF.

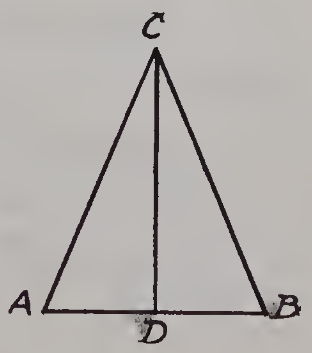


To prove that CD = GH.

Planning the Proof: (Same as in Ex. 6).

Proof: 1. △ABC ≅ △EGF (Given). 2. AC = EG and AB = EF (§78). 3. CD is a median of △ABC and GH is a median of △EFG (Given). 4. AD = DB and EH = HF (§§89, 22). 5. 2AD = 2EH (Asmt. 6). 6. AD = EH (Asmt. 4). 7. ∠A = ∠E (§78). 8. △ACD ≅ △EGH (§65). 9. CD = GH (§78).

8. Given the isos. △ABC with AC = BC, D the midpoint of the base AB, and CD drawn.



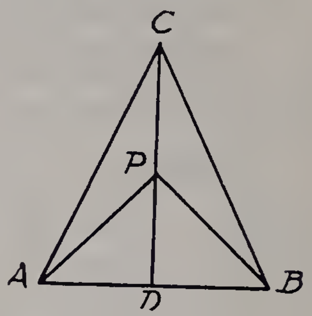
To prove that CD ⊥ AB.

Planning the Proof: 1. We can prove lines ⊥ by §14. 2. We shall use §14.

Proof: 1. AC = BC (Given). 2. D is the midpoint of AB (Given). 3. AD = DB (§22). 4. CD = CD (Iden.). 5. △ADC ≅ △BDC (§85).

6. $\angle ADC = \angle BDC$ (§78). 7. $CD \perp AB$ (§14). 8. From 3 and 7, CD is the $\perp$ bisector of AB and C is on the $\perp$ bisector of AB.

9. Given the isos. $\triangle ABC$ with $AC = BC$, CD bisecting $\angle ACB$, P any point in CD, and $PA = PB$ drawn.

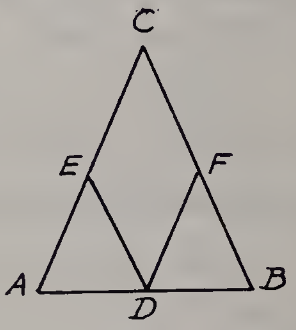


To prove that $PA = PB$.

Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§ 78, 79. 2. We shall use §78.

Proof: 1. $\triangle ABC$ is isos. having $AC = BC$ (Given). 2. Line segments PA and PB are drawn from P to A and B respectively (Given). 3. DC bisects $\angle ACB$ (Given). 4. $\angle PCA = \angle PCB$ (§22). 5. $CP = CP$ (Iden.). 6. $\triangle APC \cong \triangle BPC$ (§65). 7. $PA = PB$ (§78).

10. Given the isos. $\triangle ABC$ with $AC = BC$, DE connecting the midpoints of AB and AC, and DF connecting the midpoints of AB and BC.

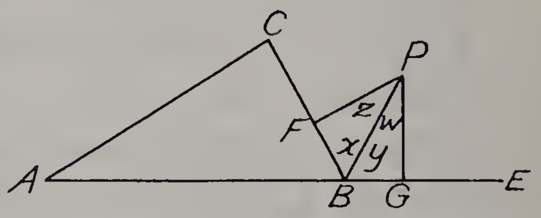


To prove that $DE = DF$.

Planning the Proof: (Same as in Ex. 9).

Proof: 1. $\triangle ABC$ is isos. having $AC = BC$, E the midpoint of AC, F the midpoint of BC, and D the midpoint of AB (Given). 2. $AE = EC$ and $BF = CF$ (§22). 3. $2AE = 2BF$ (Asmt. 6). 4. $AE = BF$ (Asmt. 4). 5. $\angle A = \angle B$ (§83). 6. $AD = DB$ (§ 22). 7. $\triangle ADE \cong \triangle DBF$ (§ 65). 8. $DE = DF$ (§78).

11. Given $\triangle ABC$ with $PF \perp BC$, $PG \perp AB$ produced through B, $\angle z = \angle w$, and $PF = PG$.

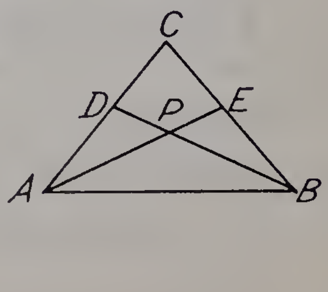


To prove that PB bisects $\angle CBE$.

Planning the Proof: 1. We can prove that a line bisects an angle by § 22. 2. We shall use §22.

Proof: 1. $PF = PG$ (Given). 2. $\angle z = \angle w$ (Given). 3. $BP = BP$ (Iden.). 4. $\triangle BPF \cong \triangle BPG$ (§ 65). 5. $\angle x = \angle y$ (§ 78). 6. BP bisects $\angle CBE$ (§22).

12. Given isos. $\triangle ABC$, base AB, AE bisecting $\angle BAC$, BD bisecting $\angle ABC$, and AE intersecting BD in P.



To prove that $\angle CDP = \angle CEP$.

Planning the Proof: 1. We can prove $\triangle$s = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. ABC is an isos. $\triangle$ having base AB

(Given). 2. AC = BC (§56). 3. $\angle CAB = \angle CBA$ (§83). 4. AE bisects $\angle BAC$ and BD bisects $\angle ABC$ (Given). 5. $\angle CAE = \angle EAB$ and $\angle CBD = \angle DBA$ (§22). 6. $2\angle CAE = 2\angle CBD$ (Asmt. 6). 7. $\angle CAE = \angle CBD$ (Asmt. 4). 8. $\angle C = \angle C$ (Iden.). 9. $\triangle CAE \cong \triangle CBD$ (§73). 10. $\angle CDP = \angle CEP$ (§78).

13. Planning the Proof: 1. We can prove $\measuredangle$ = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. $\angle x = \angle y$ and $\angle m = \angle n$ (Given). 2. $\angle x + \angle m = \angle y + \angle n$ (Asmt. 1). 3. $\angle x + \angle m = \angle BAC$ and $\angle y + \angle n = \angle ABD$ (Asmt. 9). 4. $\angle BAC = \angle ABD$ (Asmt. 7). 5. AB = AB (Iden.). 6. $\triangle BAC \cong \triangle ABD$ (§73). 7. $\angle C = \angle D$ (§78).

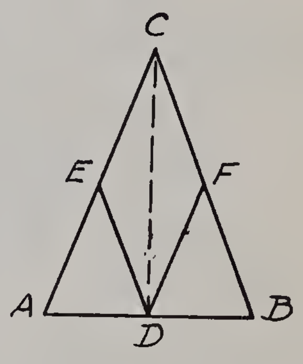
14. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

Proof: 1. BE = CE (Given). 2. $\angle x = \angle y$ (§83). 3. AC = BD (Given). 4. $\triangle ACE \cong \triangle BDE$ (§65). 5. AE = DE (§78).

15. Given the isos. $\triangle ABC$ with AC = BC; D, a point in AB; E and F midpoints of AC and BC, respectively; and DE = DF.

To prove that AD = DB.

Planning the Proof: (Same as in Ex. 14).



Proof: 1. Draw CD (Asmt. 11). 2. AC = BC (Given). 3. E is the midpoint of AC and F is the midpoint of BC (Given). 4. CE = EA and CF = FB (§22). 5. 2CE = 2CF (Asmt. 6). 6. CE = CF (Asmt. 4). 7. DE = DF (Given). 8. CD = CD (Iden.). 9. $\triangle CED \cong \triangle CFD$ (§85). 10. $\angle ECD = \angle BCD$ (§78). 11. From statements 2, 8, and 10, $\triangle ADC \cong \triangle BDC$ § 65). 12. AD = DB (§78).

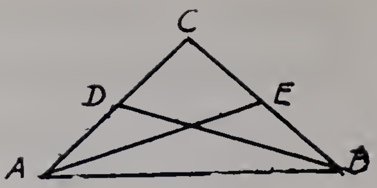
16. Given the isos. $\triangle ABC$ with AC = BC, D and E the midpoints respectively of AC and BC, GE $\perp$ CB and FD $\perp$ AC.

To prove that DF = EG.

Planning the Proof: (Same as in Ex. 14).

Proof: 1. In $\triangle ABC$, AC = BC (§56, Given). 2. D is the midpoint of AC and E the midpoint of BC (Given). 3. AD = DC and BE = EC (§22). 4. 2AD = 2BE (Asmt. 6). 5. AD = BE (Asmt. 4). 6. $\angle A = \angle B$ (§83). 7. GE $\perp$ BC and FD $\perp$ AC (Given). 8. $\angle BEG$ and $\angle ADF$ are rt. $\measuredangle$ (§14). 9. $\angle BEG = \angle ADF$ (§50a). 10. $\triangle BEG \cong \triangle ADF$ (§73). 11. DF = EG (§78).

17. Given the isos. $\triangle ABC$ with AC = BC, BD the median to AC, and AE the median to BC.

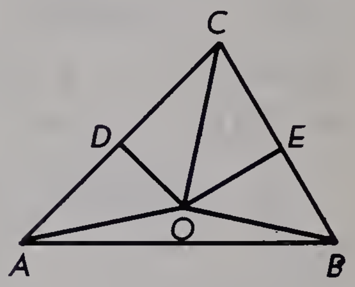


To prove that BD = AE.

Planning the Proof: (Same as in Ex. 14).

Proof: 1. AC = BC (Given). 2. AE and BD are medians (Given). 3. AD = DC and BE = EC (§§89, 22). 4. 2AD = 2BE (Asmt. 6). 5. AD = BE (Asmt. 4). 6. $\angle$CAB = $\angle$CBA (§83). 7. AB = AB (Iden.). 8. $\triangle$ABD $\cong$ $\triangle$BAE (§65). 9. BD = AE (§78).

18. Given $\triangle$ABC, OD the $\perp$ bisector of AC, OE the $\perp$ bisector of BC, and line segments AO, BO, and CO.

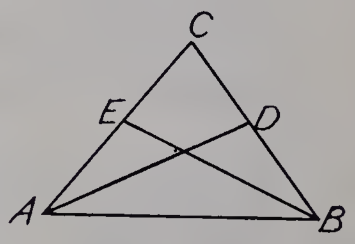


To prove that AO = BO = CO.

Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §79.

Proof: 1. Point O is on the $\perp$ bisector of AC (Given). 2. Then OA = OC (§79). 3. O is on the $\perp$ bisector of BC (Given). 4. Then OB = OC (§79). 5. Then OA = OC = OB (Asmt. 7).

19. Given isos. $\triangle$ABC, having AC = BC, AD bisecting $\angle$BAC and BE bisecting $\angle$ABC.



To prove that AD = BE.

Planning the Proof: (Same as in Ex. 18).

Proof: 1. AC = BC (Given). 2. $\angle$BAC = $\angle$ABC (§83). 3. AD bisects $\angle$BAC and BE bisects $\angle$ABC (Given). 4. $\angle$BAD = $\angle$DAC and $\angle$ABE = $\angle$EBC (§22). 5. 2$\angle$BAD = 2$\angle$ABE (Asmt. 6). 6. $\angle$BAD = $\angle$ABE (Asmt. 4). 7. AB = AB (Iden.). 8. $\triangle$ABE $\cong$ $\triangle$ABD (§73). 9. AD = BE (§78).

20. Planning the Proof: 1. We can prove: (a) lines $\perp$ by §14; (b) line seg. = by Asmt. 1-8, 26, §§78, 79. 2. We shall use: (a) §14; (b) §78.

Proof: 1. MR = PR and MN = PN (Given). 2. RN = RN (Iden.). 3. $\triangle$MRN $\cong$ $\triangle$PRN (§85). 4. $\angle$MRN = $\angle$PRN (§78). 5. RO = RO (Iden.). 6. From statements 1, 4, and 5, $\triangle$MRO $\cong$ $\triangle$PRO (§65). 7. MO = OP and $\angle$MOR = $\angle$POR (§78). 8. RO $\perp$ MP (§14).

PAGE 122

1. (1) Yes. (2) Yes. (3) §93. (4) 120°. (5) 155°.

2. (1) $\angle x > 60°$ and $\angle y = 120°$. Then $\angle x + \angle y > 180°$.

(2) The reason for $x > 60°$ is §93 and the reason for $\angle y = 120°$ is Asmt. 24.

3. $\angle x + \angle y > 180^\circ$ (Ex. 2) and $\angle z = 155^\circ$. Then $\angle x + \angle y + \angle z > 335^\circ$.

PAGES 122-123 (Review Questions)

1. Theorem. 2. Postulate.

3. Equilateral triangle. 4. Isosceles triangle.

5. Hypotenuse. 6. Legs. 7. §93.

8. (1) 3; (2) 3; (3) 3. 9. See § 42. 10. Yes.

11. Auxiliary lines are lines introduced in a figure to assist in the proof.

12. No. 13. Not necessarily. 14. (1) AC; (2) $\angle A$

15. No. 16. $\angle x > \angle B$ and $> \angle C$. It is supp. $\angle BAC$.

PAGE 124 (Test 4)

1. If the three sides of one triangle are equal respectively to the three sides of another, the triangles are congruent.

2. If the bisector of an angle of a triangle is perpendicular to the opposite side, the triangle is isosceles.

3. The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

4. If a point lies on the perpendicular bisector of a line segment, it is equidistant from the ends of the segment.

5. If the median of a triangle is also an altitude, the triangle is isosceles.

6. Two triangles are congruent if two sides and the median to one of these sides are equal respectively to two sides and the corresponding median of the other.

7. If two medians of a triangle are equal, the triangle is isosceles.

8. The bisectors of two corresponding angles of two congruent triangles are equal.

9. If two oblique line segments are drawn from a point on the perpendicular to a line so that they cut off equal line segments from the foot of the perpendicular, they are equal.

10. The base angles of an isosceles triangle are equal.

PAGES 124-125 (Test 5)

1. F. 2. T. 3. F. 4. F. 5. T. 6. T. 7. F.
8. T. 9. T. 10. F.

PAGE 125 (Test 6)

1. 52°. 2. 37°. 3. 10. 4. 120°. 5. 360°. 6. 95°. 7. 72°. 8. 66°. 9. 48°. 10. 10°.

PAGE 126 (Test 7)

1. §65 (Asmt. 32). 2. §78. 3. §73 (Asmt. 33). 4. §78. 5. §84. 6. §50d. 7. §83. 8. §93. 9. §65. 10. §78.

PAGE 127 (Test 8)

1. Planning the Proof: 1. We can prove a triangle isosceles by §56. 2. We shall use §56.

Proof: 1. In $\triangle ADE$, CE bisects $\angle AED$ and is $\perp AD$ (Given). 2. $\angle AEC = \angle CED$ (§22). 3. $\angle ACE = \angle ECD$ (§14). 4. $EC = EC$ (Iden.). 5. $\triangle ACE \cong \triangle CED$ (§73). 6. $AE = DE$ (§78). 7. $\triangle ADE$ is isos. (§56).

2. Planning the Proof: 1. We can prove $\measuredangle$ = by §§50, 78, 83, 84. 2. We shall use §78.

Proof: 1. $KH \perp RS$ (Given). 2. $\angle SHK = \angle KHR$ (§14). 3. $HR = SH$ (Given). 4. $HK = HK$ (Iden.). 5. $\triangle SHK \cong \triangle KHR$ (§65). 6. $\angle HKS = \angle RKH$ (§78).

3. Planning the Proof: 1. We can prove $\measuredangle$ = by §§50, 78, 83, 84. 2. We shall use §83 and Asmt. 2.

Proof: 1. In $\triangle ABC$, $AC = BC$ (Given). 2. $\angle BAC = \angle ABC$ (§83). 3. $AP = BP$ (Given). 4. $\angle BAP = \angle ABP$ (§83). 5. $\angle PAC = \angle PBC$ (Asmt. 2).

4. Planning the Proof: 1. We can prove line segments = by Asmt. 1-8, 26, §§78, 79. 2. We shall use §78.

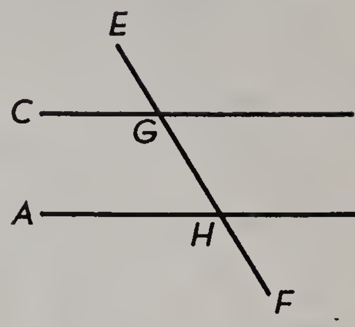
Proof: 1. $DF = EF$ (Given). 2. $\angle EDF = \angle DEF$ (§83). 3. $\angle DEH = \angle EDK$ (Given). 4. $DE = DE$ (Iden.). 5. $\triangle DEH \cong \triangle DEK$ (§73). 6. $DK = HE$ (§78).

Chapter 5. Parallel and Perpendicular Lines

PAGE 130

1. Line *ℓ* is not a transversal.

2. ∠ AHG and ∠ HGD are alt. int. ∠s ; ∠BHG and ∠HGC are alt. int. ∠s ; alternate interior angles seem to be equal when a transversal intersects parallel lines. Measurement of the angles indicates that alternate interior angles formed by a transversal of parallel lines are equal.



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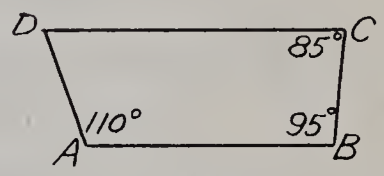


1. §§99, 104, 105, 106, 107.

2. AB ∥ DC (§106).

3. Planning the Proof: 1 We can prove lines ∥ by §§99, 104, 105, 106, 107. 2. We shall use §104.

Proof: 1. AE and BD bisect each other (Given). 2. AC = CE and BC = CD (§22). 3. ∠ACB = ∠DCE (§50d). 4. △ABC ≅ △DCE (§95a). 5. ∠A = ∠E (§98b). 6. AB ∥ DE (§104).

4. Planning the Proof: 1. We can prove lines ∥ by §§99, 104, 105, 106, 107. 2. We shall use §§105, 104.

Proof: 1. ∠x = ∠B (Given). 2. EC ∥ AB (§105). 3. ∠B = ∠E (Given). 4. ∠x = ∠E (Asmt. 7). 5. AE ∥ BD (§104).

5. Planning the Proof: (Same as in Ex. 3).

Proof: 1. △ABC ≅ △ADC (Given). 2. ∠DAC = ∠ACB and ∠BAC = ∠ACD (§98b). 3. AD ∥ BC and AB ∥ DC (§104).

6. Planning the Proof: 1. We can prove lines ∥ by §§99, 104, 105, 106, 107. 2. We shall use §106.

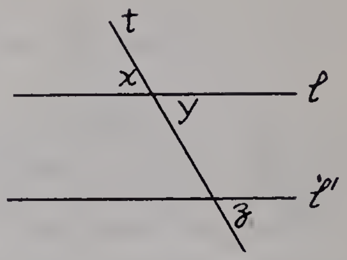
Proof: 1. ∠E is the supp. of ∠B (Given). 2. ∠B = ∠x (Given). 3. ∠E is the supp. of ∠x (Asmt. 6). 4. EF ∥ CB (§106).

7. Proof: 1. ∠A, ∠B, ∠C, and ∠D are rt. ∠s (Given). 2. ∠A = ∠B = ∠C = ∠D (§50a). 3. AD ⊥ AB and DC; and BC ⊥ AB and DC (§14). 4. AB ∥ DC and AD ∥ BC (§107). 5. ∠A + ∠C = 2 rt. ∠s and ∠B + ∠D = 2 rt. ∠s (Asmt. 1).

PAGES 137-138

1. Given the two ∥s ℓ and ℓ' cut by the transversal t, forming $\angle$s x, y, and z as shown.

To prove that $\angle x = \angle z$.



Planning the Proof: 1. We can prove $\angle$s = by §§50, 98, 111, 112. 2. We shall use §112, Asmt. 7.

Proof: 1. $\angle x = \angle y$ (§50d). 2. $\ell \parallel \ell'$ (Given). 3. $\angle y = \angle z$ (§112). 4. $\angle x = \angle z$ (Asmt. 7).

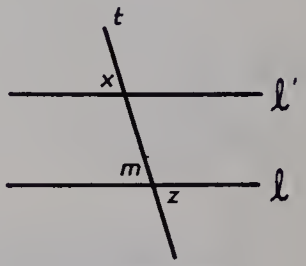
2. $\angle m = 51°$ (§50d, Ax. 6). $\angle x = 51°$ (111, Ax. 6). $\angle y = 51°$ (§50d, Ax. 6). $\angle r = 129°$ (§53b, Ax. 6). $\angle n = 129°$ (§50d, Ax. 6). $\angle w = 129°$ (§111, Ax. 6). $\angle z = 129°$ (§50d Ax. 6).

3. $\angle n = 118°$ (§50d, Ax. 6). $\angle w = 118°$ (§111, Ax. 6). $\angle z = 118°$ (§50d, Ax. 6). $\angle s = 62°$ (§53b, Ax. 6). $\angle m = 62°$ (§50d, Ax. 6). $\angle x = 62°$ (§111, Ax. 6). $\angle y = 62°$ (§50d, Ax. 6).

4. Let x = the number of degrees in $\angle m$. Then $2x + 15$ = the number of degrees in $\angle w$. $3x + 15 = 180$ (§113). Solving, $x = 55$ and $2x + 15 = 125$. Then $\angle m = \angle s = \angle x = \angle y = 55°$ and $\angle w = \angle z = \angle n = \angle r = 125°$.

5. Given lines ℓ and ℓ' cut by the transversal t, with $\angle x = \angle z$.

To prove that $\ell \parallel \ell'$.



Planning the Proof: 1. We can prove lines ∥ by §§99, 104, 105, 106, 107, 109. 2. We shall use §105.

Proof: 1. $\angle m = \angle z$ (§50d). 2. $\angle x = \angle z$ (Given). 3. $\angle x = \angle m$ (Asmt. 7). 4. $\ell \parallel \ell'$ (§105).

6. Planning the Proof: 1. We can prove line segments = by §97. 2. We shall use §97a.

Proof: 1. $AB \parallel DC$ and $AB = DC$ (Given). 2. $\angle x = \angle y$ (§111). 3. $AC = AC$ (Iden.). 4. $\therefore \triangle ADC \cong \triangle ABC$ (§95a). 5. $AD = BC$ (§97a).

7. Planning the Proof: (Same as in Ex. 6).

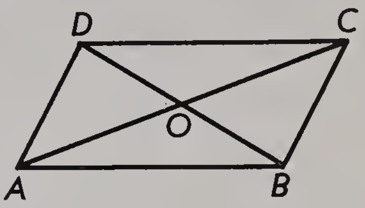
Proof: 1. $DC \parallel AB$ and $DC = AB$ (Given). 2. $\angle x = \angle n$ and $\angle y = \angle m$ (§111). 3. $\triangle DOC \cong \triangle AOB$ (§95b). 4. $AO = OC$ and $DO = OB$ (§97a).

8. Planning the Proof: 1. We can prove: (a, b) $\angle$s = by §§ 50, 98, 111, 112; (c, d) sum of $\angle$s is a st. $\angle$ by §§ 53, 113. 2. We shall use: (a, b) §111; (c) Asmt. 23; (d) Asmt. 6.

Proof: 1. $\ell \parallel \ell'$ (Given). 2. $\angle x = \angle m$ and $\angle y = \angle n$ (§111). 3. $\angle x + \angle y + \angle z$ = a st.$\angle$ (Asmt. 23). 4. $\angle m + \angle n + \angle z$ = a st. $\angle$ (Asmt. 6).

9. (1) The blade and head of the T-square are $\perp$. Lines drawn with a T-square are all $\perp$ to the edge of the drawing board and are $\therefore \parallel$ (§107). (2) By placing the head of the T-square flush with the horizontal edge of the drawing board. (3) 30°-60° and 45° right triangles made of celluloid or wood.

10. Given polygon ABCD, having diagonals AC and BD bisecting each other in O.



To prove that AD $\parallel$ BC and AB $\parallel$ DC.

Planning the Proof: 1. We can prove lines $\parallel$ by §§99, 104, 105, 106, 107, 109. 2. We shall use §104.

Proof: 1. BD and AC bisect each other in point O (Given). 2. BO = OD and AO = OC (§22). 3. $\angle AOB = \angle COD$ (§50d). 4. $\triangle AOB \cong \triangle DOC$ (§95a). 5. $\angle BAO = \angle OCD$ (§98b). 6. AB $\parallel$ DC (§104). 7. In like manner, we can prove $\triangle AOD \cong \triangle BOC$, $\angle DAO = \angle BCO$, and AD $\parallel$ BC (Statements 2-6).

11. Planning the Proof: 1. We can prove: (a) $\angle$s = by §§50, 98, 111, 112; (b) $\angle$s supp. by §§53, 113. 2. We shall use (a) §112; (b) §53b, Asmt. 6.

Proof: 1. AB $\parallel$ DE and BC $\parallel$ FG (Given). 2. $\angle B = \angle y$ and $\angle z = \angle y$ (§112). 3. $\angle B = \angle z$ (Asmt. 7). 4. $\angle w$ and $\angle z$ are supp. (§53b). 5. $\angle w$ and $\angle B$ are supp. (Asmt. 6).

12. (a) §113. (b) §113. (c) §50c.

13. Let x = the number of degrees in $\angle A$. Then x + 12 = the number of degrees in $\angle B$. 2x + 12 = 180. Solving, x = 84 and x + 12 = 96. Then $\angle A$ = 84° and $\angle B$ = 96°. Ans.

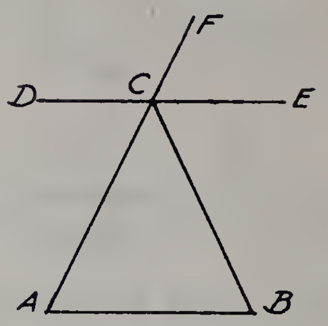
14. Planning the Proof: 1. We can prove line segments = by §97. 2. We shall use §97a.

Proof: 1. DO = OB and DC $\parallel$ AB (Given). 2. $\angle x = \angle y$ (§111). 3. $\angle COD = \angle AOB$ (§50d). 4. $\triangle COD \cong \triangle AOB$ (§95b). 5. AO = OC and DC = AB (§97a).

15. Planning the Proof: 1. We can prove lines $\parallel$ by §§99, 104, 105, 106, 107, 109. 2. We shall use §104.

Proof: 1. DC = AB and AD = BC (Given). 2. AC = AC (Iden.). 3. $\therefore \triangle ACD \cong \triangle ACB$ (§95c). 4. $\therefore \angle x = \angle y$ and $\angle m = \angle n$ (§98b). 5. $\therefore$ DC $\parallel$ AB and AD $\parallel$ BC (§104).

16. Given the isos. △ ABC with AC = BC, DCE ∥ AB, and BCF the ext. ∠ at the vertex.



To prove that ∠FCE = ∠BCE.

Planning the Proof: 1. We can prove △s = by §§ 50, 98, 111, 112. 2. We shall use Asmt. 7.

Proof: 1. DCE ∥ AB (Given). 2. ∴ ∠FCE = ∠A (§112). 3. ∠BCE = ∠B (§111). 4. AC = BC (Given). 5. ∴ ∠A = ∠B (§98a). 6. ∠FCE = ∠BCE (Asmt. 7).

17. $4x + (3x - 9) = 180$ (§113). Then $4x + 3x - 9 = 180$; whence $7x = 189$, and $x = 27$. Then $4x = 108$, and $3x - 9 = 72$.

108°, 72°. Ans.

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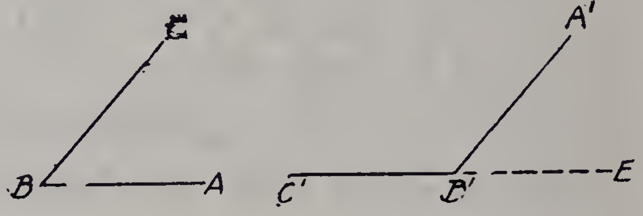
1. Given the △s ABC and A'B'C' with AB ∥ B'C' and BC ∥ B'A'.

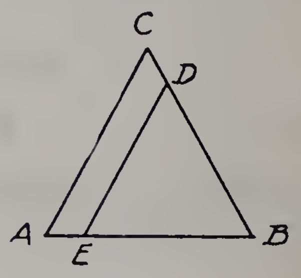


To prove that △s ABC and A'B'C' are supp.

Planning the Proof: 1. We can prove △s supp. by §§ 53, 113. 2. We shall use §53b, Asmt. 6.

Proof: 1. Extend C'B' to E (Asmt. 13). 2. AB ∥ EB'C' and BC ∥ B'A' (Given). 3. ∴ ∠ABC = ∠A'B'E (§117). 4. But △s A'B'C' and A'B'E are supp. (§53b). 5. ∴ △s ABC and A'B'C' are supp. (Asmt. 6).

2. Given the isos. △ ABC with AC = BC, DE ∥ CA and intersecting AB in E and BC in D.



To prove that ∠DEB = ∠B.

Planning the Proof: 1. We can prove △s = by §§50, 98, 111, 112. 2. We shall use Asmt. 7.

Proof: 1. DE ∥ CA (Given). 2. ∴ ∠DEB = ∠A (§112). 3. AC = BC (Given). 4. ∴ ∠A = ∠B (§98a). 5. ∴ ∠DEB = ∠B (Asmt. 7).

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1. Since no change was made in the T.V. set other than changing the station dial, it appears that the poor picture was due to the wave signals from Channel 8.

2. If the bulb did not glow when he placed it in the floor lamp, Dick would not have known what the difficulty was. Since

he found the trouble right away, the bulb glowed in the floor lamp. Therefore the bulb and the fuse for the floor lamp were good and Dick knew that the fuse for the kitchen lamp was bad.

3. The teacher was acquainted with both the machinist and dentist before the church dinner. Since Mr. Baker and Mr. Fuller had never met before, Mr. Smith was the teacher.

The reasoning can be illustrated as follows: The six possibilities are:

	Teacher	Machinist	Dentist
(1)	Mr. Baker	Mr. Smith	Mr. Fuller
(2)	Mr. Baker	Mr. Fuller	Mr. Smith
(3)	Mr. Fuller	Mr. Smith	Mr. Baker
(4)	Mr. Fuller	Mr. Baker	Mr. Smith
(5)	Mr. Smith	Mr. Baker	Mr. Fuller
(6)	Mr. Smith	Mr. Fuller	Mr. Baker

Only the combinations (5) and (6) are possible. In either case, the teacher is Mr. Smith.

4. Ted and Lawrence used right-hand gloves and Bill used a left-hand glove. Since Bill used a left-hand glove, he did not leave the bag. Since Ted had small feet, he did not leave the bag. Since neither Bill nor Ted left the bag, the coach knew that Lawrence left it.

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1. (1) §§ 50, 98, 111, 112, 117, 121. (2) § 97. (3) §§ 95, 96, 125. (4) §§ 53, 113. (5) §§ 52, 124. (6) §§ 99, 104, 105, 106, 107, 109 (7) §§ 51, 114. (8) §§ 96, 125.

2. (1) No. (§ 122). (2) The student should see that since the sum of the $\angle$s of a $\triangle$ = a st. $\angle$, if 2 of the $\angle$s were rt. $\angle$s there could be no third $\angle$ and hence no $\triangle$.

3. (1) Yes. (2) The = legs may form the rt. $\angle$.

4. 56° (§124).

5. 60° (§§120 and 84).

6. Let x = the number of degrees in each of the other two angles. Then $x + x + 70 = 180$ (§ 120). Then $2x = 110$, and $x = 55$.
55°. Ans.

7. Let x = the number of degrees in the vertex angle. Then $x + 62 + 62 = 180$ (§120). Solving, $x = 56$. 56°. Ans.

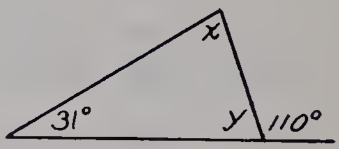
8. Let x = the number of degrees in the smaller of the two

other angles. Then $x + 12$ = the number of degrees in the larger of the two angles. $x + x + 12 + 46 = 180$ (§120). $2x = 122$, $x = 61$, and $x + 12 = 73$. 61° and 73°. Ans.

9. We have $2x + 3x + 4x = 180$; whence $9x = 180$, and $x = 20$. Then $2x = 40$, $3x = 60$, and $4x = 80$. 40°, 60°, 80°. Ans.

10. We have $3x + 4x + 5x = 180$; whence $12x = 180$, and $x = 15$. Then $3x = 45$, $4x = 60$, and $5x = 75$. 45°, 60°, 75°. Ans.

11. $\angle ACB = 180° - 48° - 34° = 98°$. $\angle DCB = \frac{1}{2}(98°) = 49°$. 98°, 49°. Ans.



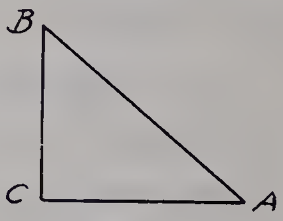
12. We have $x + 31° = 110°$ (§123); whence $x = 79°$. $y = 70°$ (§53b). 79°, 70°. Ans.

13. Let x, $3x$, and $6x$ = the no. of degrees in the first, second, and third $\angle$s, respectively. Then $10x = 180$ (§120); whence $x = 18$. Then $3x = 54$, and $6x = 108$. 18°, 54°, 108°. Ans.

14. Given the $\triangle ABC$ with $\angle$s A and B comp.

To prove that ABC is a rt. $\triangle$.

Planning the Proof: 1. We can prove that a $\triangle$ is a rt. $\triangle$ by §57. 2. We shall use §57.

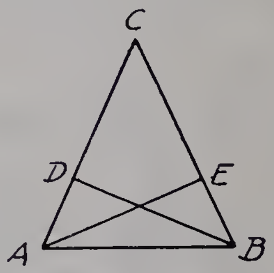


Proof: 1. $\angle A + \angle B$ = a rt. $\angle$ (Given). 2. $\angle A + \angle B + \angle C$ = a st. $\angle$ (§120). 3. $\therefore$ $\angle C$ = a rt. $\angle$ (Asmt. 2). 4. $\therefore$ ABC is a rt. $\triangle$ (§57).

15. Given the isos. $\triangle ABC$ with $AC = BC$, and altitudes AE and BD.

To prove that $AE = BD$.

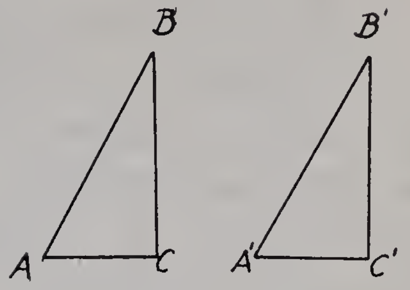
Planning the Proof: 1. We can prove line segments = by §97. 2. We shall use §97a.



Proof: 1. $AC = BC$ (Given). 2. $\therefore \angle A = \angle B$ (§98a). 3. $AB = AB$ (Iden.). 4. $AE \perp BC$ and $BD \perp AC$ (Given). 5. $\therefore$ $\triangle$s ABD and ABE are rt. $\triangle$s (§57, 14). 6. $\therefore \triangle ABD \cong \triangle ABE$ (§125). 7. $\therefore AE = BD$ (§97a).

16. Let x = the number of degrees in the first angle. Then $x - 26$ = the number of degrees in the smallest angle. And $x + 36$ = the number of degrees in the largest angle. $x + x - 26 + x + 36 = 180$. Solving, $x = 56\frac{2}{3}$, $x - 26 = 30\frac{2}{3}$, and $x + 36 = 92\frac{2}{3}$. $56\frac{2}{3}°$, $30\frac{2}{3}°$, $92\frac{2}{3}°$. Ans.

17. Given the rt. $\triangle$s ABC and A'B'C' with $\angle$s C and C' the rt. $\angle$s, $AC = A'C'$, and $\angle B = \angle B'$.



To prove that $\triangle ABC \cong \triangle A'B'C'$.

Planning the Proof: 1. We can prove $\triangle$s $\cong$ by §§ 95, 96, 125. 2. We shall use §95b.

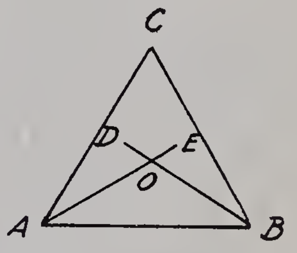
Proof: 1. $\angle C = \angle C'$ (§ 50a). 2. $\angle B = \angle B'$ and $AC = A'C'$ (Given). 3. $\therefore \angle A = \angle A'$ (§121). 4. $\therefore \triangle ABC \cong \triangle A'B'C'$ (§ 95b).

18. (1) No. (2) The sum of the angles of any triangle is a straight angle (§120). Since the size of an $\angle$ is independent of the length of its sides (§11) one triangle may be larger than another even though the three angles of one are equal respectively to the three angles of the other.

19. Planning the Proof: 1. We can prove values of sums of $\angle$s by Asmt. 22, §§120, 123. 2. We shall use § 120 and assumptions.

Proof: 1. Draw AC (Asmt. 11). 2. $\angle x + \angle D + \angle m =$ a st.$\angle$ (§120). 3. $\angle y + \angle B + \angle n =$ a st. $\angle$ (§120). 4. $\therefore \angle A + \angle D + \angle B + \angle C = 2$ st. $\angle$s (Asmts. 1, 6, and 9).

20. Given the equilateral $\triangle ABC$ with AE bisecting $\angle A$ and BD bisecting $\angle B$.



To find the no. of degrees in $\angle$s AOB, AOD, DOE, and BOE.

Solution: 1. $\angle A + \angle B + \angle C = 180°$ (§120). 2. $\angle A = \angle B = \angle C$ (§84). 3. $3\angle A = 180°$ (Asmt. 6). 4. $\therefore \angle A = 60°$ (Asmt. 4) and $\angle B = 60°$ (Asmt. 6). 5. $\angle EAB$ and $\angle DBA = 30°$ (Given, Asmt. 4 and 6). 6. $\angle OAB + \angle OBA + \angle AOB = 180°$ (§ 120). 7. $\therefore \angle AOB = 120°$ (Asmt. 6 and 2). 8. $\angle DOE = \angle AOB$ (§ 50d) $= 120°$ (Asmt. 6). 9. $\angle AOD = 60°$ (§ 53b). 10. $\angle BOE = 60°$ (§ 50d and Asmt. 6). 120°, 60°, 120°, 60°. Ans.

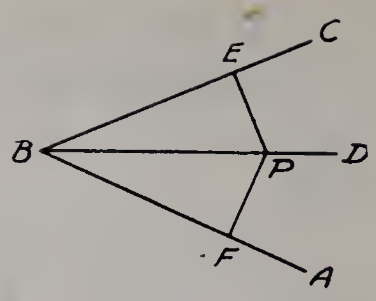
21. Given the isos. $\triangle ABC$ with $AC = BC$ and CD bisecting the ext. $\angle$ at C.

To prove that $CD \parallel AB$.

Planning the Proof: 1. We can prove lines $\parallel$ by §§99, 104, 105, 106, 107, 109. 2. We shall use §105.

Proof: 1. $\angle x = \angle y$ and $AC = BC$ (Given). 2. $\angle A = \angle B$ (§ 98a). 3. $\angle x + \angle y = \angle A + \angle B$ (§123). 4. $2\angle x = 2\angle A$ (Asmt. 6). 5. $\angle x = \angle A$ (Asmt. 4). 6. $CD \parallel AB$ (§ 105).

22. Given the $\angle ABC$ with its bisector BD, P any point in BD, $PE \perp BC$, and $PF \perp BA$.

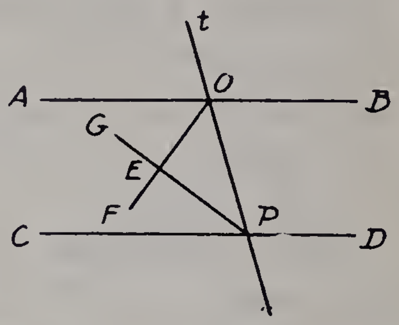


To prove that PE = PF.

Planning the Proof: (Same as in Ex. 15).

Proof: 1. BD bisects ∠ABC (Given). 2. ∠ABP = ∠PBE (§22). 3. PB = PB (Iden.). 4. PE ⊥ AC and PF ⊥ AB (Given). 5. ∠s BFP and BEP are rt. ∠s (§14). 6. △BFP and △BEP are rt. △s (§57). 7. △PBE ≅ △PBF (§125). 8. PE = PF (§97a).

23. Given the two ∥s AB and CD cut by the transversal t in O and P, OF bisecting ∠AOP, and PG bisecting ∠OPC.

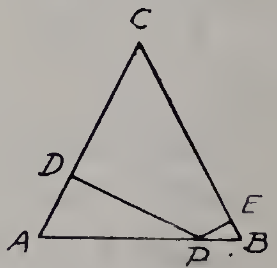


To prove that OF ⊥ PG.

Planning the Proof: 1. We can prove lines ⊥ by §§51, 114. 2. We shall use §51a.

Proof: 1. ∠AOP + ∠CPO = a st. ∠ (§§113, 21). 2. ∠AOF = ∠FOP and ∠OPG = ∠CPG (Given). 3. 2∠FOP + 2∠OPG = a st. ∠ (Asmt. 9, 6). 4. ∠FOP + ∠OPG = a rt. ∠ (Asmt. 4). 5. If OF and PG do not intersect, OF ∥ GP (§100). 6. Then ∠FOP + ∠OPG = a st. ∠ (§113). 7. This is impossible (Statement 4). Hence OF and PG intersect at some point E. 8. ∴ ∠FOP + ∠OPG + ∠OEP = a st. ∠ (§120). 9. ∴ ∠OEP = a rt. ∠ (Asmt. 2). 10. ∴ OF ⊥ PG (§51a).

24. Given the isos. △ABC with AC = BC, P any point in AB, PD ⊥ AC, and PE ⊥ BC.

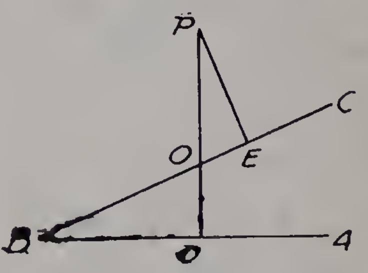


To prove that ∠DPA = ∠EPB.

Planning the Proof: 1. We can prove ∠s = by §§50, 98, 111, 112, 117, 121. 2. We shall use §121.

Proof: 1. AC = BC, PD ⊥ AC, and PE ⊥ BC (Given). 2. ∠A = ∠B (§98a). 3. ∠PDA = ∠PEB (§§14, 50a). 4. ∠DPA = ∠EPB (§121).

25. Given the acute ∠ABC, P any point outside ∠ABC, PE ⊥ BC, PD ⊥ BA and intersecting BC at O.



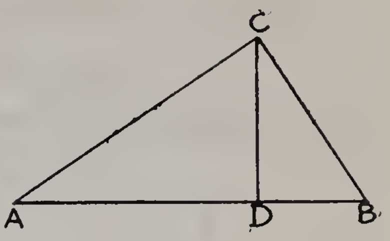
To prove that ∠EPD = ∠ABC.

Planning the Proof: (Same as in Ex. 24).

Proof: 1. PD ⊥ BA and PE ⊥ BC (Given). 2. ∴ ∠PEO = ∠ODB (§§14, 50a). 3. ∠POE = ∠BOD (§50d). 4. ∴ ∠EPO = ∠ABC, or ∠EPD = ∠ABC (§121).

26. Given the rt. △ABC with ∠C the rt. ∠ and CD ⊥ AB.

To prove that ∠ADC = ∠BDC, ∠ACD = ∠B, and ∠A = ∠BCD.

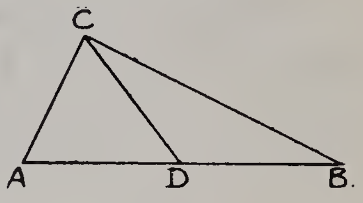


Planning the Proof: 1. We can prove $\angle$s = by §§50, 98, 111, 112, 117, 121. 2. We shall use §§ 50a, 121.

Proof: 1. CD $\perp$ AB (Given). 2. $\angle$CDA and $\angle$CDB are rt. $\angle$s (§14). 3. In $\triangle$s ACB and ADC: (a) $\angle$ACB = $\angle$ADC (§50a). (b) $\angle$ A = $\angle$A (Iden.). (c) $\therefore$ $\angle$ B = $\angle$ ACD (§121). 4. In $\triangle$s ADC and BDC: (a) $\angle$ACD = $\angle$B (Statement 3c), (b) $\angle$ADC = $\angle$BDC (§50a), (c) $\therefore$ $\angle$A = $\angle$BCD (§121).

27. Given the $\triangle$ABC with CD the median to AB and CD = $\frac{1}{2}$AB.

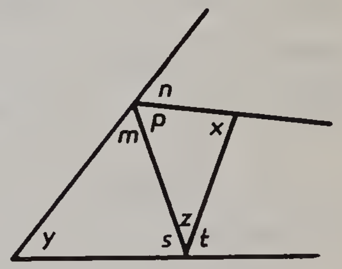
To prove that ABC is a rt. $\triangle$.



Planning the Proof: 1. We can prove that a $\triangle$ is a rt. $\triangle$ by §57. 2. We shall use §57.

Proof: 1. CD = $\frac{1}{2}$AB and AD = DB (Given). 2. AD = $\frac{1}{2}$AB (Asmt. 9, 6, 4). 3. $\therefore$ CD = AD = DB (Asmt. 7). 4. $\angle$ACD = $\angle$A and $\angle$DCB = $\angle$B (§98a). 5. $\angle$ ACD + $\angle$DCB = $\angle$A + $\angle$B (Asmt. 1). 6. But $\angle$ACD + $\angle$DCB + $\angle$A + $\angle$B = a st. $\angle$ (§120). 7. 2$\angle$ACD + 2$\angle$DCB = a st. $\angle$ (Asmt. 6). 8. $\therefore$ $\angle$ ACD + $\angle$ DCB = a rt. $\angle$ (Asmt. 4). 9. $\therefore$ $\triangle$ ACB is a rt. $\triangle$ (§57).

28. Planning the Proof: 1. We can prove that an angle equals the sum of $\angle$s by §§120, 123 and Asmt. 2. We shall use §120 and Asmt.



Proof: 1. $\angle$x + $\angle$p + $\angle$z = 180° (§120). 2. $\angle$x = 180° - $\angle$p - $\angle$z (Asmt. 2). 3. $\angle$ m + $\angle$ p + $\angle$n = 180° and $\angle$s + $\angle$z + $\angle$t = 180° (Asmt. 23). 4. $\angle$p = 180° - $\angle$m - $\angle$n and $\angle$z = 180° - $\angle$s - $\angle$t (Asmt. 2). 5. From (2) and (4), $\angle$ x = 180° - (180° - $\angle$ m - $\angle$n) - (180° - $\angle$s - $\angle$t) (Asmt. 6). 6. $\angle$ x = $\angle$m + $\angle$n + $\angle$s + $\angle$t - 180° (Asmt. 6). 7. $\angle$ m = $\angle$n and $\angle$s = $\angle$t (Given). 8. $\angle$x = $\angle$m + $\angle$m + $\angle$s + $\angle$s - 180° (Asmt. 6). 9. $\angle$x = $\angle$m + $\angle$s + $\angle$m + $\angle$s - 180° (Asmt. 6). 10. $\angle$y + $\angle$m + $\angle$s = 180° (§120). 11. $\angle$ m + $\angle$s = 180° - $\angle$y. 12. From (9) and (11), $\angle$x = 180° - $\angle$y + $\angle$m + $\angle$s - 180° (Asmt. 6). 13. $\angle$x = $\angle$m + $\angle$s - $\angle$y (Asmt. 6).

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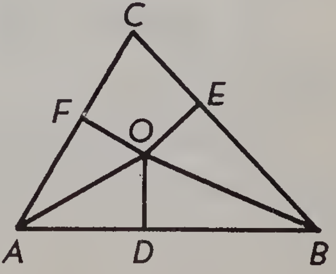

Given $\triangle$ABC, OA the bisector of $\angle$ BAC, and OB the bisector of $\angle$ABC.

To prove that O is equidistant from AC and BC.

Planning the Proof: 1. We can prove line segments = by §§97, 126. 2. We shall use §126.

Proof: 1. Draw $OD \perp AB$, $OE \perp BC$, and $OF \perp AC$ (Asmt. 20). 2. $OF = OD$ and $OE = OD$ (§126). 3. $OF = OE$ (Asmt. 7). 4. $\therefore$ O is equidistant from AC and BC (§15).

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Given $\angle$s ABC and LEF with $BA \perp LE$ and $BC \perp EF$.

To prove that $\angle B$ and $\angle x$ are supp.

Planning the Proof: 1. We can prove $\angle$s supp. by §§ 53, 113. 2. We shall use § 53b.

Proof: 1. Extend LE to H (Asmt. 13). 2. $BA \perp LH$ and $BC \perp EF$ (Given). 3. $\therefore \angle ABC = \angle y$ (§127). 4. $\angle x$ and $\angle y$ are supp. (§53b). 5. $\therefore \angle ABC$ and $\angle x$ are supp. (Asmt. 6).

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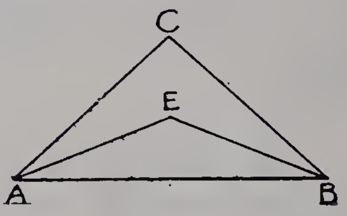
1. §§97, 126, 128, 129.

2. Given the isos. $\triangle ABC$ with $\angle BAC = \angle ABC$, AE bisecting $\angle BAC$ and BE bisecting $\angle ABC$.

To prove that $\triangle ABE$ is isos.

Planning the Proof: 1. We can prove a $\triangle$ isos. by § 56. 2. We shall use §56.

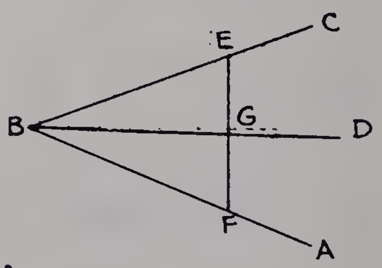


Proof: 1. $\angle BAC = \angle ABC$, $\angle CAE = \angle EAB$, and $\angle CBE = \angle EBA$ (Given). 2. $2\angle EAB = \angle BAC$ and $2\angle EBA = \angle ABC$ (Asmt. 9, 6). 3. $2\angle EAB = 2\angle EBA$ (Asmt. 7). 4. $\angle EAB = \angle EBA$ (Asmt. 4). 5. $AE = EB$ (§128). 6. $\therefore \triangle AEB$ is isos. (§56).

3. Given the $\angle ABC$, BD bisecting $\angle ABC$, $EF \perp BD$ and intersecting BD at G.

To prove that $\triangle BEF$ is isos.

Planning the Proof: (Same as in Ex. 2).

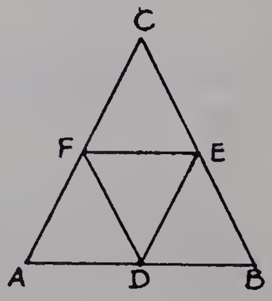


Proof: 1. BD bisects $\angle ABC$ (Given). 2. $\angle EBG = \angle FBG$ (§22). 3. $EF \perp BD$ (Given). 4. $\angle BGE = \angle BGF$ (§14). 5. $\angle BEG = \angle BFG$ (§121). 6. $\therefore BF = BE$ (§128). 7. $\therefore \triangle BEF$ is isos. (§56).

4. Given the equilateral $\triangle ABC$ with DE, EF, and FD joining the midpoints of AB, BC, and CA.

To prove that $\triangle$s ADF, DBE, EFD, and FEC are equilateral.

Planning the Proof: 1. We can prove a $\triangle$ equilateral by §§56, 129. 2. We shall use § 129.



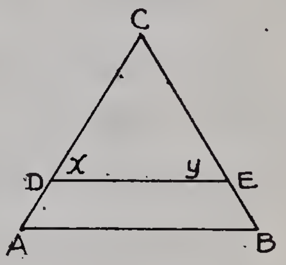
Proof: 1. $AB = BC = CA$ and $AD = DB$, $BE = EC$, and $CF = FA$ (Given). 2. $AD = AF = BD = BE = CE = CF$ (Asmt. 6, 4, 7). 3. $\angle A = \angle B = \angle C = 60°$ (§§84, 120). 4. $\therefore \triangle ADF \cong \triangle DBE \cong \triangle EFC$ (§95a). 5. $FD = DE = EF$ (§97a). 6. $\triangle DEF$ is equilateral (§56). 7. $\angle AFD = \angle ADF = 60°$ (§§98a, 120). 8. $\therefore \triangle ADF$ is equiangular (Statements 3 and 7). 9. $\therefore \triangle ADF$ is equilateral (§129). 10. Similarly, $\triangle$s DBE and EFC are equilateral.

5. *Planning the Proof*: 1. We can prove line segments = by §§97, 126, 128. 2. We shall use §126.

Proof: 1. $AE \parallel CD$ and $\angle x = \angle y$ (Given). 2. $\angle x = \angle m$ (§111). 3. $\angle y = \angle n$ (§112). 4. $\angle n = \angle m$ (Asmt. 7). 5. $\therefore AC = EC$ (§128).

6. *Given* the isos. $\triangle ABC$ with $AC = BC$ and $DE \parallel AB$.

To prove that $\triangle DEC$ is isos.



Planning the Proof: (Same as in Ex. 2).

Proof: 1. $AC = BC$ and $DE \parallel AB$ (Given). 2. $\angle x = \angle A$ and $\angle y = \angle B$ (§112). 3. $\angle A = \angle B$ (§98a). 4. $\therefore \angle x = \angle y$ (Asmt. 7). 5. $\therefore CD = CE$ (§128). 6. $\therefore \triangle DEC$ is isos. (§56).

7. Let x = the no. of degrees in the vertex angle, and $2x$ = the no. of degrees in each base angle. Then $x + 2x + 2x = 180$ (§120); whence $5x = 180$, and $x = 36$. Then $2x = 72$.

36°, 72°, 72°. *Ans*.

8. *Given* the isos. $\triangle ABC$ with $AC = BC$ and $\angle CBE$ an ext. $\angle$ at the base.

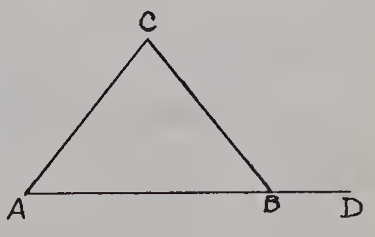
To prove that $\angle CBE$ = a rt. $\angle + \frac{1}{2}\angle ACB$.

Planning the Proof: 1. We can prove values of sums of $\angle$s by Asmt. 22, §§120, 123. 2. We shall use §123.

Proof: 1. Draw CD the bisector of $\angle ACB$ (Asmt. 18). 2. $\angle ACD = \angle y$ (Const. §22). 3. $\angle y = \frac{1}{2}\angle ACB$ (Asmt. 9, 6, 4). 4. $AC = BC$ (Given). 5. $CD = CD$ (Iden.). 6. $\therefore \triangle ADC \cong \triangle BDC$ (§95a). 7. $\therefore \angle ADC = \angle z$ (§98b). 8. $\therefore \angle z$ is a rt. $\angle$ (§14). 9. $\angle x = \angle z + \angle y$ (§123). 10. $\therefore \angle CBE$ = a rt. $\angle + \frac{1}{2}\angle ACB$ (Asmt. 6).

9. *Given* the $\triangle ABC$ with $\angle DBC$ supp. to $\angle A$.

To prove that $\triangle ABC$ is isos.



Planning the Proof: 1. We can prove a $\triangle$ isos. by §56. 2. We shall use §56.

Proof: 1. $\angle DBC$ is supp. to $\angle A$ (Given). 2. $\angle DBC$ is supp. to $\angle ABC$ (§53b). 3. $\therefore \angle A = \angle ABC$ (§50c). 4. $\therefore AC = BC$ (§128). 5. $\therefore \triangle ABC$ is isos. (§56).

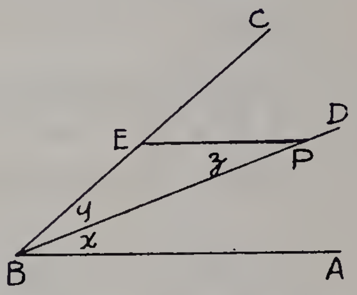
10. Given the isos. △ABC with AC = BC, ∠ACB = 4∠A or 4∠ABC, BD ⊥ AB, and BD meeting AC produced at D.

To prove that △BCD is equilateral.

Planning the Proof: 1. We can prove a △ equilateral by §§ 56, 129. 2. We shall use § 129.

Proof: 1. AC = BC (Given). 2. ∠A = ∠ABC (§ 98a). 3. ∠DCB = ∠A + ∠ABC (§ 123). 4. ∴ ∠DCB = 2∠A (Asmt. 6). 5. ∠A + ∠ACB + ∠ABC = 180° (§ 120). 6. ∠ACB = 4∠A (Given). 7. ∴ 6∠A = 180° (Asmt. 6). 8. ∴ ∠A = 30° (Asmt. 4). 9. ∠DCB = 60° (Asmt. 6). 10. ∠ABC = 30° (Asmt. 6). 11. ∠DBA = a rt. ∠ (Given). 12. ∴ ∠DBC = 60° (Asmt. 2). 13. ∠D = 60° (§ 124). 14. ∴ △BCD is equiangular (Statements 9, 12, and 13). 15. ∴ △BCD is equilateral (§ 129).

11. Given the ∠ABC with BD the bisector of ∠ABC, P any point in BD, and PE ∥ AB.

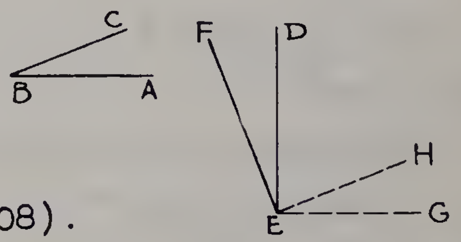


To prove that △BEP is isos.

Planning the Proof: (Same as in Ex. 9).

Proof: 1. PE ∥ AB and ∠y = ∠x (Given, § 22). 2. ∠z = ∠x (§ 111). 3. ∴ ∠y = ∠z (Asmt. 7). 4. ∴ BE = PE (§ 128). 5. ∴ △BEP is isos. (§ 56).

12. Given the ∠s ABC and DEF with BA ⊥ ED and BC ⊥ EF.



To prove that ∠ABC = ∠DEF.

Planning the Proof: 1. We can prove ∠s = by §§ 50, 98, 111, 112, 117, 121, 127. 2. We shall use §§ 117, 50b, and Asmt. 7.

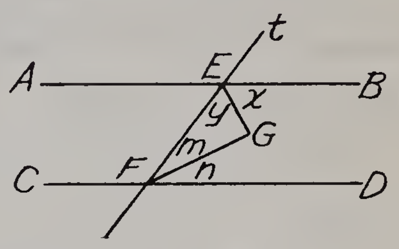
Proof: 1. Draw EG ∥ BA and EH ∥ BC (§ 108). 2. ∴ ∠GEH = ∠ABC (§ 117). 3. BA ⊥ ED and BC ⊥ EF (Given). 4. ∴ EG ⊥ ED and EH ⊥ EF (§ 114). 5. ∴ ∠s DEG and FEH are rt. ∠s (§ 14). 6. ∴ ∠GEH is comp. to ∠DEH and ∠DEF is comp. to ∠DEH (§ 52a). 7. ∴ ∠GEH = ∠DEF (§ 50b). 8. ∴ ∠ABC = ∠DEF (Asmt. 7).

13. Planning the Proof: 1. We can prove line segments = by §§ 97, 126, 128, 129. 2. We shall use § 97a, Asmt. 7.

Proof: 1. △ABC is equilateral (Given). 2. AB = AC (§ 56). 3. AE bisects ∠BAC (Given). 4. AE = AE (Iden.). 5. △ABE ≅ △ACE (§ 95a). 6. ∠AEB = ∠AEC (§ 98b). 7. AD ⊥ CB (§ 51a). 8. OF ⊥ AC (Given). 9. ∠CEO and ∠CFO are rt. ∠s (§ 14). 10. △s CEO and CFO are rt. △s (§ 57). 11. CO = CO (Iden.). 12. CO bisects ∠ECF (Given). 13. ∠FCO = ∠ECO (§ 22).

14. △CEO ≅ △CFO (§125). 15. OF = OE (§97a). 16. ∠FCE = ∠CAB + ∠B (§123). 17. 2∠ECO = 2∠B (Asmt. 9, 6). 18. ∠ECO = ∠B (Asmt. 4). 19. ∠CEO = ∠AEB (§50d). 20. CE = EB (§97a). 21. △CEO ≅ △AEB (§95b). 22. OE = AE (§97a). 23. AE = OF (Asmt. 7).

14. Given AB and CD cut by the transversal, t. GE bisecting ∠FEB, EG ⊥ FG, and FG bisecting ∠DFE.



To prove that AB ∥ CD.

Planning the Proof: 1. We can prove lines ∥ by §§99, 104, 105, 106, 107, 109. 2. We shall use §106.

Proof: 1. EG ⊥ FG (Given). 2. ∠FGE is a rt. ∠ (§14). 3. △FGE is a rt. △ (§56). 4. ∠m + ∠y = a rt. ∠ (§124). 5. ∠m = ∠n and ∠x = ∠y (Given, §22). 6. ∠n + ∠x = a rt. ∠ (Asmt. 6). 7. ∠m + ∠y + ∠n + ∠x = 2 rt. ∠s (Asmt. 1). 8. ∠DFE + ∠FEB = a st. ∠ (Asmt. 6, 9). 9. ∠DFE is the supp. of ∠FEB (§21). 10. AB ∥ CD (§106).

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1. If a man lives in Indiana, he lives in Indianapolis. False.

2. If two lines form equal alternate interior angles with a transversal, the lines are parallel. True.

3. If two triangles have the three angles of one equal respectively to the three angles of the other, the triangles are congruent. False.

4. If a point is equidistant from the sides of an angle, it is on the bisector of the angle. True.

Converse

5. All animals are horses. False.

6. Equal angles are right angles. False.

7. If a point is equidistant from the end points of a line segment, it lies on the perpendicular bisector of the line segment. True.

8. If two angles are complementary, they are the acute angles of a right triangle. False.

9. A boy lives in Los Angeles if he lives in California. False.

10. (1) If the median on the base of a triangle is perpendicular to the base, it bisects the vertex angle and the triangle is isosceles. True.

(2) If the median on the base of a triangle bisects the vertex angle, it is perpendicular to the base and the triangle is isosceles. True.

(3) If a line through the vertex of an isosceles triangle is perpendicular to the base, it bisects the base and the vertex angle. True.

(4). If a line through the vertex of an isosceles triangle bisects the vertex angle, it bisects the base and is perpendicular to the base. True.

(5) If a line is perpendicular to the base of a triangle and bisects the vertex angle, it bisects the base and the triangle is isosceles. True.

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Inverse

1. If two angles are not right angles, they are unequal. False.

2. If two sides of a triangle are unequal, the angles opposite these sides are unequal. True.

3. When there is no ice on the road, the road is not slippery. False.

4. If a man does not live in Indianapolis, he does not live in Indiana. False.

5. If two triangles are not congruent, the corresponding angles are unequal. False.

6. If a point does not lie on the perpendicular bisector of a line segment, it is not equidistant from the end points of the line segment. True.

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ontrapos

1. If the complements of two angles are not equal, the angles are not equal. True.

2. If a man is not at least 35 years old, he is not president of the United States. True.

3. If two angles are not equal, they (both) are not right angles. True.

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1. (1) If two lines are not parallel, they do not form equal alternate interior angles with a transversal.

(2) Asmt. 38.

2. Same as Ex. 1.

3. 12.

4. If a point is equidistant from the sides of an angle, it is on the bisector of the angle.

5. If two angles are equal and their right sides are parallel, their left sides are parallel.

If two angles are equal and their left sides are parallel, their right sides are parallel.

Both are true.

6. If the right sides of two angles are not parallel and the left sides are parallel, the angles are unequal. If the left sides of two angles are not parallel and the right sides are parallel, the angles are unequal. Both are true.

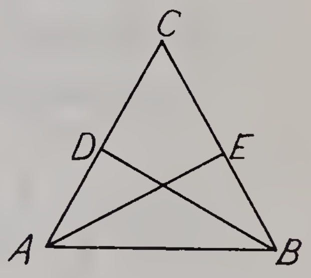
7. If two angles are unequal and their right sides are parallel, their left sides are not parallel. If two angles are unequal and their left sides are parallel, their right sides are not parallel. Both are true.

8. If two angles are not the acute angles of a right triangle, they are not complementary. No.

9. If two angles are not complementary, they are not the acute angles of a right triangle. Yes.

PAGE 154

1. Given the △ABC with AE the altitude on BC, BD the altitude on AC, and AE = BD.



To prove that △ABC is isos.

Planning the Proof: 1. We can prove a △ isos. by § 56. 2. We shall use § 56.

Proof: 1. AE and BD are altitudes of △ABC (Given). 2. AE ⊥ BC and BD ⊥ AC (§ 90). 3. ∠BDA and ∠AEB are rt. ∠s (§ 14). 4. △BDA and △AEB are rt. △s (§ 57). 5. AE = BD (Given). 6. AB = AB (Iden.). 7. △BDA ≅ △AEB (§ 135). 8. ∠BAC = ∠ABC (§ 98b). 9. AC = BC (§ 128). 10. △ABC is isos. (§ 56).

2. Figure same as in Ex. 1.

Given △ABC, AE the altitude on BC, BD the altitude on AC, and BE = AD.

To prove that AD = BE.

Planning the Proof: 1. We can prove line segments = by §§ 97, 126, 128, 129. 2. We shall use § 97a.

Proof: 1. AE ⊥ BC and BD ⊥ AC (Given). 2. ∠BDA and ∠AEB are rt. ∠s (§ 14). 3. △BDA and △AEB are rt. △s (§ 57). 4. AE = BD (Given). 5. AB = AB (Iden.). 6. △BDA ≅ △AEB (§ 135). 7. AD = BE (§ 97a).

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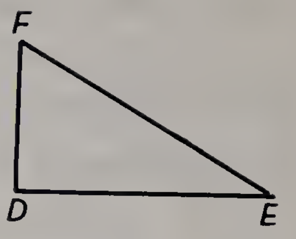
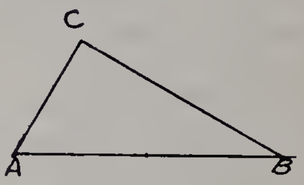


Let x = the number of degrees in $\angle E$. Then $2x$ = the number of degrees in $\angle F$. And $3x$ = the number of degrees in $\angle D$. $x + 2x + 3x = 180$ (§120). $\angle E = \angle x = 30°$, $\angle F = 2x = 60°$, and $\angle D = 3x = 90°$. $FE = 2FD = 2 \times 6 = 12$. FE = 12 in. Ans.

PAGE 156

1. (1) $a \parallel b \parallel c$ (§107). (2) $d \parallel e \parallel f$ (§105).

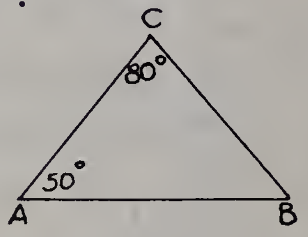


2. Given the rt. $\triangle ABC$ with $\angle C$ the rt. $\angle$, and $\angle A = 2\angle B$.

To prove that $AC = \frac{1}{2}AB$.

Planning the Proof: 1. We can prove that one leg of a rt. $\triangle = \frac{1}{2}$ the hypotenuse (§136). 2. We shall use (§136).

Proof: 1. $\angle A = 2\angle B$ (Given). 2. $\angle A + \angle B = 90°$ (§124). 3. $\therefore 3\angle B = 90°$ (Asmt. 6). 4. $\therefore \angle B = 30°$ (Asmt. 4). 5. $\therefore AC = \frac{1}{2}AB$ (§136).



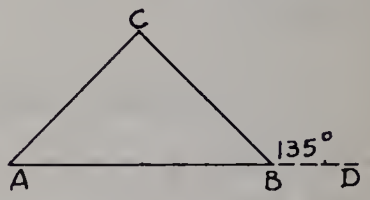
3. Given the $\triangle ABC$ with $\angle A = 50°$ and $\angle C = 80°$.

To prove that $\triangle ABC$ is isos.

Planning the Proof: 1. We can prove a $\triangle$ isos. by §56. 2. We shall use §56.

Proof: 1. $\angle A + \angle B + \angle C = 180°$ (§120). 2. But $\angle A = 50°$ and $\angle C = 80°$ (Given). 3. $\therefore \angle B = 50°$ (Asmt. 6 and 2). 4. $\therefore \angle A = \angle B$ (Asmt. 7). 5. $\therefore AC = BC$ (§128). 6. $\therefore \angle ABC$ is isos. (§56).

4. Given the isos. $\triangle ABC$ with $AC = BC$ and ext. $\angle CBD = 135°$.



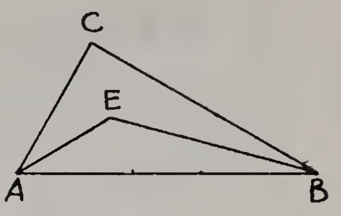
To prove that ABC is a rt. $\triangle$.

Planning the Proof: 1. We can prove that a $\triangle$ is a rt. $\triangle$ by § 57. 2. We shall use § 57.

Proof: 1. $\angle CBD + \angle ABC = 180°$ (§ 53b). 2. But $\angle CBD = 135°$ (Given). 3. $\therefore \angle ABC = 45°$ (Asmt. 2). 4. $AC = BC$ (Given). 5. $\therefore \angle A = \angle ABC$ (§ 98a). 6. $\therefore \angle A = 45°$ (Asmt. 7). 7. $\angle A + \angle C + \angle ABC = 180°$ (§ 120). 8. $\therefore \angle C = 90°$ (Asmt. 2). 9. ABC is a rt. $\triangle$ (§ 57).

5. Given the rt. $\triangle ABC$ with C the rt. $\angle$, $\angle ABC = 30°$, AE bisecting $\angle BAC$, and BE bisecting $\angle ABC$.

To find the size of $\angle AEB$.



__Solution__: 1. $\angle ABC = 30^\circ$ (Given). 2. $\angle BAC + \angle ABC = 90^\circ$ (§124). 3. ∴ $\angle BAC = 60^\circ$ (Asmt. 2). 4. $\angle EAB = \frac{1}{2}\angle BAC$ and $\angle EBA = \frac{1}{2}\angle ABC$ (Given, §22). 5. ∴ $\angle EAB = 30^\circ$ and $\angle EBA = 15^\circ$ (Asmt. 6). 6. $\angle EAB + \angle EBA + \angle AEB = 180^\circ$ (§120). 7. ∴ $\angle AEB = 135^\circ$ (Asmt. 2).

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__1__. 1. $CB = AD$ if $\triangle COB \cong \triangle AOD$ (§97a). 2. $\triangle COB \cong \triangle AOD$ if $AO = OB$, $CO = OD$, and $\angle x = \angle y$ (§95a). 3. But $AO = OB$ (Given) and $\angle x = \angle y$ (§50d). 4. $CO = OD$ if $\triangle CAO \cong \triangle BOD$ (§97a). 5. $\triangle CAO \cong \triangle BOD$ if $\angle m = \angle n$, $\angle AOC = \angle BOD$, and $AO = OB$ (§95b). 6. But $\angle m = \angle n$ (§50a), $\angle AOC = \angle BOD$ (§50d), and $AO = OB$ (Given). 7. ∴ $CB = AD$ (Statements 1-6).

__2__. __Planning the Proof__: 1. We can prove line segments = by §§97, 126, 128, 129. 2. We shall use §97a.

__Proof__: 1. $PM = PN$ and $MR = SN$ (Given). 2. $\angle M = \angle N$ (§98a). 3. $\triangle MRP \cong \triangle NSP$ (§95a). 4. $PR = PS$ (§97a).

__3__. __Planning the Proof__: 1. We can prove $\angle$s = by §§50, 98, 111, 112, 117, 121, 127. 2. We shall use §121.

__Proof__: 1. $\angle a = \angle b$ and $\angle x = \angle y$ (Given). 2. $\angle MRP = \angle NSP$ (§50c). 3. $\angle M = \angle N$ (§121).

__4__. __Planning the Proof__: (Same as in Ex. 2).

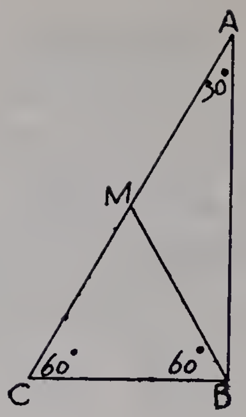
__Proof__: 1. $MR = SN$ and $\angle x = \angle y$ (Given). 2. $PR = PS$ (§128). 3. $\angle MRP = \angle NSP$ (§50c). 4. ∴ $\triangle MRP \cong \triangle NSP$ (§95a). 5. ∴ $MP = NP$ (§97a).

__5__. __Planning the Proof__: 1. (Same as in Ex. 3). 2. We shall use §98b.

__Proof__: 1. $\angle DAB = \angle DBA$ (Given). 2. $AD = BD$ (§128). 3. $\angle CDA = \angle CDB$ (Given). 4. $CD = CD$ (Iden.). 5. ∴ $\triangle ADC \cong \triangle BDC$ (§95a). 6. ∴ $\angle m = \angle n$ (§98b).

__6__. __Planning the Proof__: 1. (Same as in Ex. 3). 2. We shall use §98a.

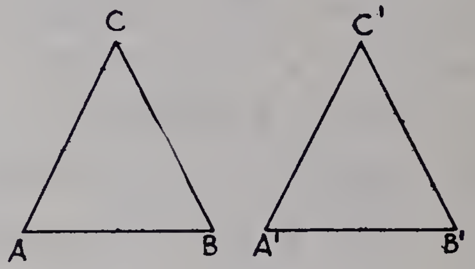
__Proof__: 1. $CA = CB$ and $\angle m = \angle n$ (Given). 2. $CD = CD$ (Iden.). 3. ∴ $\triangle ADC \cong \triangle BDC$ (§95a). 4. ∴ $AD = DB$ (§97a). 5. ∴ $\angle DAB = \angle DBA$ (§98a).



7. Planning the Proof: 1. (Same as in Ex. 2). 2. We shall use §128, Asmt. 7.

Proof: 1. ∠C = 60° and ∠MBC = 60° (Given). 2. ∴ BM = CM (§128). 3. ∠A + ∠ABC + ∠C = 180° (§120). 4. ∠A = 30° (Given). 5. ∴ ∠ABC = 90° (Asmt. 2). 6. ∴ ∠ABM = 30° (Asmt. 2). 7. ∴ AM = BM (§128). 8. ∴ AM = BM = CM (Asmt. 7).

8. Given the isos. △s ABC and A'B'C' with AC = CB, A'C' = C'B', AB = A'B', and ∠C = ∠C'.

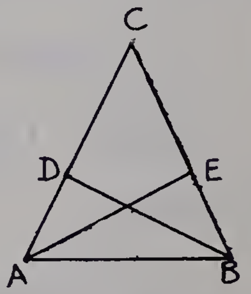


To prove that △ABC ≅ △A'B'C'.

Planning the Proof: 1. We can prove △s = by §§95, 96, 125, 135. 2. We shall use §95b.

Proof: 1. ∠C = ∠C' (Given). 2. ∠A + ∠B + ∠C = a st. ∠ and ∠A' + ∠B' + ∠C' = a st. ∠ (§120). 3. ∠A + ∠B + ∠C = ∠A' + ∠B' + ∠C' (Asmt. 7). 4. ∠A + ∠B = ∠A' + ∠B' (Asmt. 2). 5. AC = CB and A'C' = C'B' (Given). 6. ∴ ∠A = ∠B and ∠A' = ∠B' (§98a). 7. 2∠A = 2∠A' and 2∠B = 2∠B' (Asmt. 6). 8. ∠A = ∠A' and ∠B = ∠B' (Asmt. 4). 9. AB = A'B' (Given). 10. ∴ △ABC ≅ △A'B'C' (§95b).

9. Given the △ABC with AE the altitude on BC, BD the altitude on AC, and BD = AE.

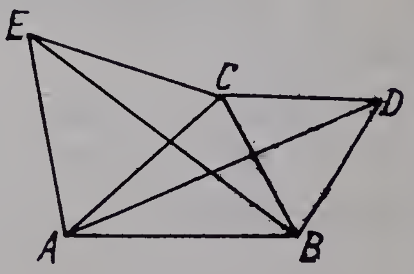


To prove that △ABC is isos.

Planning the Proof: 1. We can prove a △ isos. by §56. 2. We shall use §56.

Proof: 1. △s BDC and AEC are rt. △s (Given and §14). 2. ∴ ∠BDC = ∠AEC (§50a). 3. ∠C = ∠C (Iden.). 4. ∠DBC = ∠EAC (§121). 5. BD = AE (Given). 6. ∴ △BDC ≅ △AEC (§95b). 7. ∴ AC = BC (§97a). 8. ∴ △ABC is isos. (§56).

10. Given △ABC, equil. △s ACE and BCD, and line segments BE and AD.



To prove that AD = BE.

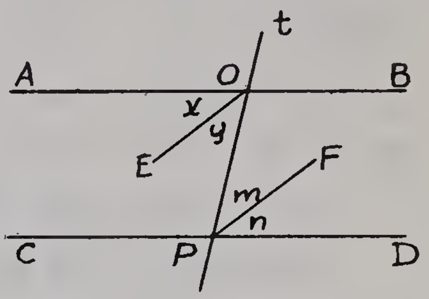
Planning the Proof: 1. We can prove line segments = by §§97, 126, 128, 129. 2. We shall use §97a.

Proof: 1. △s BCD and ACE are equilateral (Given). 2. ∠ACE + ∠CAE + ∠AEC = 180° (§120). 3. △ACE is equiangular (§84). 4. ∴ 3∠ACE = 180° (Asmt. 6). 5. ∴ ∠ACE

$= 60°$ (Asmt. 4). 6. In like manner, $\angle BCD = 60°$ (Statements 2-5). 7. $\angle ACE = \angle BCD$ (Asmt. 7). 8. $\angle ACB = \angle ACB$ (Iden.). 9. $\therefore \angle ECB = \angle ACD$ (Asmt. 1, 9, 7). 10. $AC = CE$ and $CD = CB$ (§ 56). 11. $\therefore \triangle ACD \cong \triangle BCE$ (§ 95a). 12. $\therefore AD = BE$ (§ 97a).

11. Given the lines AB and CD cut by the transversal t in O and P, OE bisecting $\angle POA$, PF bisecting $\angle OPD$, and $EO \parallel PF$.

To prove that $AB \parallel CD$.



Planning the Proof: 1. We can prove lines $\parallel$ by §§99, 104, 105, 106, 107, 109. 2. We shall use § 104.

Proof: 1. $EO \parallel PF$ (Given). 2. OE bisects $\angle POA$ and PF bisects $\angle OPD$ (Given). 3. $\angle x = \angle y$ and $\angle m = \angle n$ (§ 22). 4. $\angle y = \angle m$ (§ 111). 5. $\angle x = \angle n$ (Asmt. 7). 6. $\angle x + \angle y = \angle m + \angle n$ (Asmt. 1). 7. $\angle AOP = \angle OPD$ (Asmt. 9, 6). 8. $AB \parallel CD$ (§ 104).

12. Planning the Proof: 1. We can prove $\angle$s = by §§ 50, 98, 111, 112, 117, 121, 127. 2. We shall use §§ 127, 50a, 121.

Proof: 1. $AC \perp AB$ and $CD \perp DB$ (Given). 2. $\angle C = \angle B$ (§ 127). 3. $DE \perp AB$ (Given). 4. $\angle A$ and $\angle DEB$ are rt. $\angle$s (§ 14). 5. $\angle A = \angle DEB$ (§ 50a). 6. $\angle AOC = \angle BDE$ (§ 121).

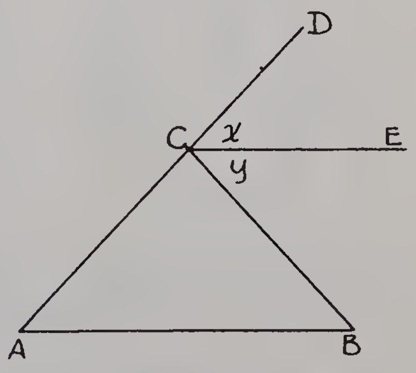
13. Planning the Proof: 1. We can prove line segments = by §§ 97, 126, 128, 129. 2. We shall use § 128.

Proof: 1. $FH = GH$ and $KL \parallel FH$ (Given). 2. $\therefore \angle F = \angle G$ (§ 98a). 3. $\angle F = \angle LKG$ (§ 112). 4. $\therefore \angle LKG = \angle G$ (Asmt. 7). 5. $\therefore KL = LG$ (§ 128).

14. Proof: 1. $MN = NO$ and $OP = PS$ (Given). 2. $\angle M = \angle MON$ and $\angle POS = \angle S$ (§ 98a). 3. $\angle MON = \angle POS$ (§ 50d). 4. $\angle M = \angle S$ (Asmt. 7). 5. $MN \parallel PS$ (§ 104). 6. $\angle N = \angle P$ (§ 111).

15. Given the $\triangle ABC$, CE bisecting ext. $\angle BCD$, and $CE \parallel AB$.

To prove that $\triangle ABC$ is isos.



Planning the Proof: 1. We can prove a $\triangle$ isos. by § 56. 2. We shall use § 56.

Proof: 1. $CE \parallel AB$ (Given). 2. $\therefore \angle y = \angle B$ (§ 111) and $\angle x = \angle A$ (§ 112). 3. But $\angle x = \angle y$ (Given, § 22). 4. $\therefore \angle A = \angle B$ (Asmt. 7). 5. $\therefore AC = CB$ (§ 128). 6. $\therefore \triangle ABC$ is isos. (§ 56).

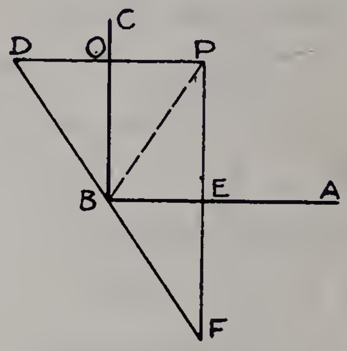
16. Given the quad. ABCD, $AB = DC$, $BC = AD$, AC a diagonal, $DF \perp AC$, and $BE \perp AC$.

To prove that DF = BE.

Planning the Proof: 1. We can prove line segments = by §§ 97, 126, 128, 129. 2. We shall use §97a.

Proof: 1. AB = DC and BC = AD (Given). 2. AC = AC (Iden.). 3. ∴ △ABC ≅ △ADC (§95c). 4. ∴ ∠DAF = ∠ECB (§98b). 5. ∠s AFD and BEC are rt. ∠s (Given and §14). 6. ∴ △s AFD and BEC are rt. △s (§57). 7. ∴ △AFD ≅ △BEC (§125). 8. ∴ DF = BE (§97a).

17. Given the rt. ∠ABC, P any point within ∠ABC, PD ⊥ BC, PF ⊥ AB, PO = OD, and PE = EF.



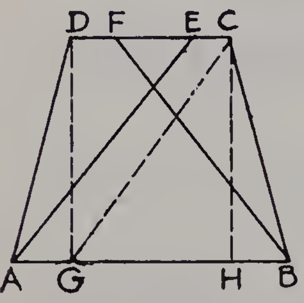
To prove that DBF is a st. line.

Planning the Proof: 1. We can prove that a line is straight by §16. 2. We shall use §16.

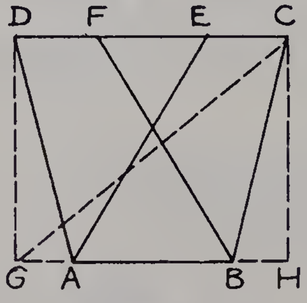
Proof: 1. Draw BP (Asmt. 11). 2. ∠BEP = ∠BEF (Given, §14). 3. PE = EF (Given). 4. BE = BE (Iden.). 5. ∴ △BEP ≅ △BEF (§95a). 6. ∴ ∠FBE = ∠PBE (§98b). 7. Similarly, ∠DBO = ∠PBO (Reasons 2-6). 8. ∠FBE + ∠DBO = ∠PBE + ∠PBO (Asmt. 1). 9. But ∠ABC = a rt. ∠ (Given). 10. ∴ ∠PBE + ∠PBO = a rt. ∠ (Asmt. 9 and 7). 11. ∴ ∠FBE + ∠DBO = a rt. ∠ (Asmt. 7). 12. ∴ ∠PBE + ∠PBO + ∠FBE + ∠DBO = a st. ∠ (Asmt. 1). 13. ∴ ∠DBF = a st. ∠ (Asmt. 9 and 6). 14. ∴ DBF is a st. line (§16).

18. Planning the Proof: 1. We can prove line segments = by §§ 97, 126, 128, 129. 2. We shall use § 97a.

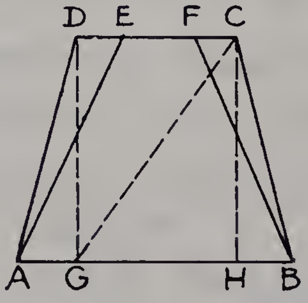
Proof: Cases I and II. 1. From D and C draw DG ⊥ AB and CH ⊥ AB or AB extended (§94). 2. Draw GC (Asmt. 11). 3. DG ∥ CH (§107) and AB ∥ DC (Given). 4. ∴ ∠DGC = ∠GCH and ∠DCG = ∠HGC



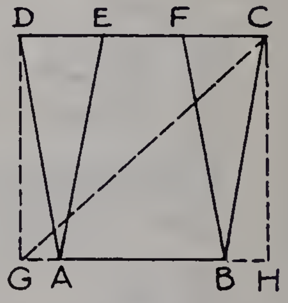
Case I



Case II



Case III



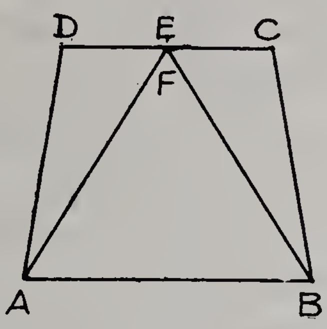
Case IV

(§111). 5. GC = GC (Iden.). 6. ∴ △GDC ≅ △GHC (§95b). 7. ∴ DG = CH (§97a). 8. ∠AGD = ∠BHC (§§14, 50a). 9. DG ⊥ DC and CH ⊥ DC (§114). 10. ∴ ∠GDC = ∠HCD (§§ 14, 50a). 11. ∠ADC = ∠BCD (Given). 12. ∴ ∠ADG = ∠HCB (Asmt. 2). 13. ∴ △ADG ≅ △HCB (§95b). 14. ∴ AD = BC (§97a). 15. CE = DF (Given).

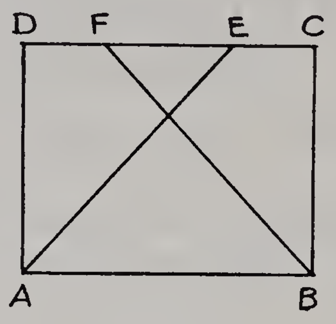
16. FE = FE (Iden.). 17. CF = DE (Asmt. 1). 18. ∴ $\triangle ADE \cong \triangle BCF$ (§95a). 19. ∴ AE = BF (§97a).

Cases III and IV. The steps in the proof are the same as those for Cases I and II with the exception of Statement 17, in which the reason is Asmt. 2 instead of Asmt. 1.

Discussion: The teacher might lead the students to consider also the special cases (1) in which E and F are identical with the midpoint of CD and (2) in which ABCD is a rect. [See Figs. (1) and (2) below.]



(1)



(2)

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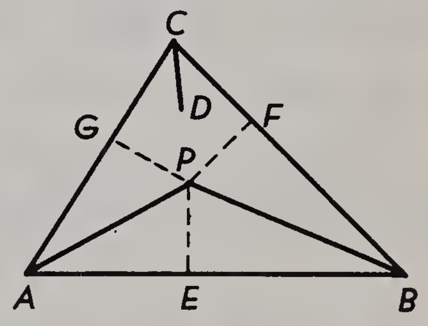
1. Given $\triangle ABC$, PA bisecting $\angle BAC$, PB bisecting $\angle ABC$, and CD bisecting $\angle ACB$.

To prove that point P is on line CD.

Planning the Proof: 1. We can prove that a point is on a line by §§139, 141. 2. We shall use §141.

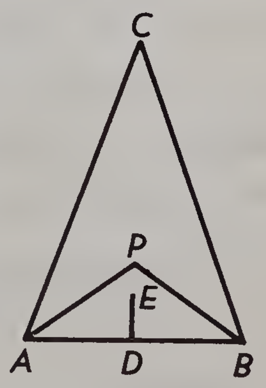


Proof: 1. Draw PE $\perp$ AB, PF $\perp$ BC, and PG $\perp$ AC (Asmt. 20). 2. PA bisects $\angle BAC$, PB bisects $\angle ABC$, and DC bisects $\angle ACB$ (Given). 3. PG = PE and PF = PE (§126). 4. PG = PF (Asmt. 7). 5. P is on DC (§141).

2. Given $\triangle ABC$, PA bisecting $\angle BAC$, PB bisecting $\angle ABC$, AC = BC, and DE the $\perp$ bisector of AB.

To prove that P is on DE.

Planning the Proof: 1. We can prove that a point lies on a line by §§139, 141. 2. We shall use §139.



Proof: 1. In $\triangle ABC$, AC = BC, PA bisects $\angle BAC$ and PB bisects $\angle ABC$ (Given). 2. $\angle BAC = \angle ABC$ (§98a). 3. $\angle BAC = \angle BAP + \angle PAC$ and $\angle ABC = \angle ABP + \angle PBC$ (Asmt. 9). 4. $2\angle BAP = 2\angle ABP$ (Asmt. 6). 5. $\angle BAP = \angle ABP$ (Asmt. 4). 6. PA = PB (§128). 7. DE $\perp$ bisector of AB (Given). 8. P is on DE (§139).

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1. Solution: 1. CD = 433, and BC = 500 (§136). 2. AC $\perp$ CB and AB $\perp$ CD (Given). 4. $\therefore \angle$ A = $\angle$DCB (§127). 5. $\therefore \angle$ DCB = 30° (Asmt. 6). 6. $\therefore$ DB = 250 (Statement 1, §136). 7. AD = 750 (Asmt. 2). 433, 500, 250, 750. Ans.

2. Planning the Proof: 1. We can prove lines $\perp$ by §§14, 51, 114, 140. 2. We shall use §140.

Proof: 1. DF = EF and DG = GE (Given). 2. $\therefore$ GF $\perp$ DE (§140).

3. Planning the Proof: 1. We can prove that a line is the $\perp$ bisector of a line segment by § 140. 2. We shall use §140.

Proof: 1. Draw OC, OD, O'C, and O'D (Asmt. 11). 2. OC = OD and O'C = O'D (Asmt. 26). 3. $\therefore$ OO' is the $\perp$ bisector of CD (§140).

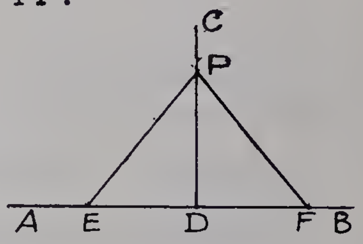
4. Planning the Proof: 1. We can prove line segments = by §§97, 126, 128, 129, 140. 2. We shall use §140.

Proof: 1. $\angle$s = $\angle$t and $\angle$x = $\angle$y (Given). 2. $\therefore$ AD = AB and CD = CB (§128). 3. $\therefore$ OB = OD (§§140, 22).

5. Given CD $\perp$ AB, P a point in CD, and PE = PF.

To prove that DE = DF.

Planning the Proof: 1. We can prove line segments = by §§97, 126, 128, 129, 140. 2. We shall use §97a.

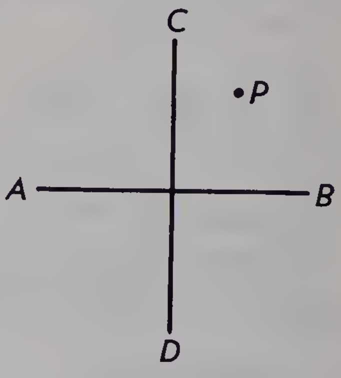


Proof: 1. PE = PF and PD $\perp$ AB (Given). 2. $\therefore$ $\triangle$ PDE and PDF are rt. $\triangle$ (§57). 3. PD = PD (Iden.). 4. $\therefore \triangle$PDE $\cong \triangle$PDF (§135). 5. $\therefore$ DE = DF (§97a).

6. Given CD the $\perp$ bisector of AB and point P not on CD.

To prove that P is not equidistant from A and B.

Proof: 1. If P is equidistant from A and B, it is on CD (§139). 2. The contrapositive of statement 1 is "If P is not on CD, it is not equidistant from A and B" (§132). 3. Then P is not equidistant from A and B. (Asmt. 38, § 134).



7. Planning the Proof: 1. We can prove line segments = by §§97, 126, 128, 129, 140. 2. We shall use §128.

Proof: 1. $\angle$ y = $\angle$ x + $\angle$C (§123). 2. But $\angle$y = 2$\angle$x (Given). 3. $\therefore$ 2$\angle$x = $\angle$x + $\angle$C (Asmt. 6). 4. $\therefore$ $\angle$x = $\angle$C (Asmt. 2). 5. $\therefore$ AB = BC (§128).

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1. The measurement needed is BC because $AB = \frac{1}{2}BC$ (§136).

2. *Planning the Proof*: 1. We can prove $\angle$s = by §§ 50, 98, 111, 112, 117, 121, 127. 2. We shall use §98a and Asmt. 1.

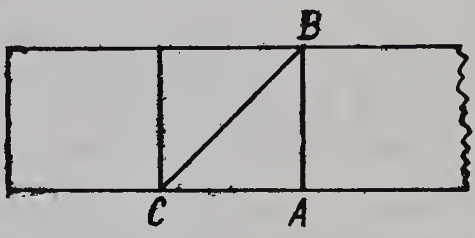
Proof: 1. $BE = DE$ (Given). 2. $\angle EBD = \angle EDB$ (§98a). 3. $\angle$s ABE and CDE are rt. $\angle$s (Given). 4. $\therefore \angle ABE = \angle CDE$ (§50a). 5. $\angle ABD = \angle CDB$ (Asmt. 1, 9, 6).

3. *Planning the Proof*: 1. We can prove line segments = by §§ 97, 126, 128, 129, 140. 2. We shall use §128.

Proof: 1. $\angle CDB = 2\angle A$ (Given). 2. $\angle CDB = \angle A + \angle B$ (§123). 3. $\therefore 2\angle A = \angle A + \angle B$ (Asmt. 7). 4. $\angle A = \angle B$ (Asmt. 2). 5. $\therefore DB = DA$ (§128).

4. (1) 70° (§113). (2) 70° (§113).

5. He can use his square to mark off $AB \perp$ to the side of the board. He can then lay off $AC = AB$ and draw CB. He saws along line CB.



Proof: 1. $\angle CAB =$ rt. $\angle$ (Const.). 2. $AC = AB$ (Const.). 3. $\therefore \angle ABC = \angle ACB$ (§98a). 4. $\angle ACB + \angle ABC = 90°$ (§124). 5. $2\angle ACB = 90°$ (Asmt. 6). 6. $\therefore \angle ACB = 45°$ (Asmt. 4).

6. 45° (see Ex. 5).

7. 1. $\angle ABD = 150°$ (Given). 2. If $CB \perp AB$, $\angle CBD = 60°$ (Asmt. 2). 3. $\angle C = 90°$ (§114). 4. $\therefore \angle CDB = 30°$ (§124, Asmt. 2). 5. $\therefore BC = \frac{1}{2}BD$ (§136).

8. (1) *Proof*: 1. $AO = OD$ and $CO = OB$ (Given). 2. $\angle AOB = \angle COD$ (§50d). 3. $\therefore \triangle AOB \cong \triangle COD$ (§95a). 4. $\therefore \angle A = \angle D$ (§98b). 5. $\therefore AB \parallel CD$ (§104).

(2) $AB = CD$ (§97a). (3) Lowered.

9. (1) $180 - 35 - 35 = 110$. 110°. *Ans*. (2) §128.

10. (1) 1. $AO = OC$ (Given). 2. $\angle OAC = \angle OCA$ (§98a). 3. $\angle OAC + \angle OCA + \angle AOC = 180°$ (§120). 4. $\angle AOC = 50°$ (Given). 5. $\angle OAC + \angle OCA = 130°$ (Asmt. 2). 6. $2\angle OAC = 130°$ (Asmt. 6). 7. $\therefore \angle OAC = 65°$ (Asmt. 4). 8. The exterior angles at A and C = $180° - 65° = 115°$.

(2) 65° (§111).

(3) Nail a board to AO and CO, making it $\parallel AC$.

11. Proof: 1. $\angle BDE = \angle A + \angle E$ (§123). 2. $AD = DE$ (Given). 3. $\therefore \angle A = \angle E$ (§98a). 4. $\therefore \angle BDE = 2\angle A$ (Asmt. 6). 5. $\therefore \frac{1}{2}\angle BDE = \angle A$ (Asmt. 4).

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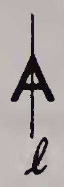
1. Circles, vase, and hexagon.

2. (1)



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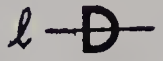

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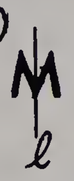

In each figure, l is the axis of symmetry.

3. O, H, and X. 4. Any two of the following: H, O, S, and X. 5. Equal.

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1. (1) An infinite number. (2) Yes. 2. A st. line. 3. (1) An infinite number. (2) On a st. line. 4. The bisector of the $\angle$ formed by the 2 roads. 5. A st. line 2 in. from AB, 1 inch from CD, and $\parallel$ AB and CD. 6. (1) Yes. (2) It is the $\perp$ bisector of the line segment joining M and N. 7. A rectangle whose dimensions are 2 in. less than those of the page. 8. Find the point where the $\perp$ bisector of the line segment joining T and T' intersects the river bank. 9. (1) Curve line. (2) Circle. 10. A st. line $\parallel$ the road. 11. The line $\parallel$ the given lines and midway between them. 12. The bisector of the $\angle$. 13. A ⊙ with the given point as the center and a radius = 1 in. 14. The $\perp$ bisector of the line segment joining the two points. 15. A ⊙ having the same center as the larger ⊙ and a radius = the sum of the radii of the 2 given ⓢ.

16. A circle having the same center as the larger circle and a radius = the difference of the radii of the two given circles.

17. A circle.

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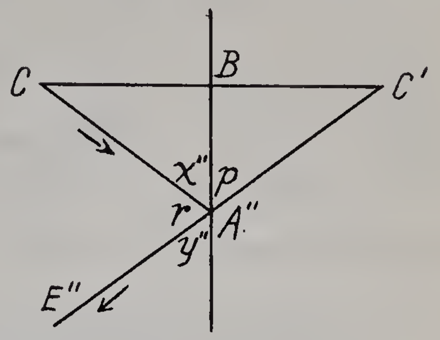
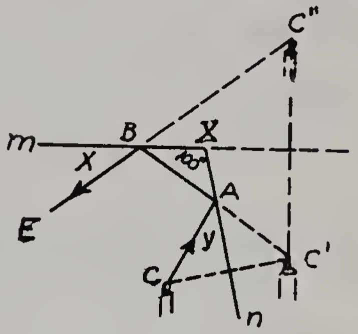
1. 1. $\angle x = \angle y$ and $\angle x' = \angle y'$ (Given). 2. $\angle y = \angle C'AB$ and $\angle y' = \angle C'A'B$ (§50d). 3. $\angle x = \angle C'AB$ and $\angle x' = \angle C'A'B$ (Asmt. 7). 4. $AA' = AA'$ (Iden.). 5. $\therefore \triangle CAA' \cong \triangle C'AA'$ (§95b). 6. $\angle CA = C'A$ and $CA' = C'A'$ (§97a). 7. $\therefore AA'$ is the $\perp$ bisector of CC' (§140); that is, $CC' \perp AA'$ and $CB = BC'$ (§22).



2. 1. $CC' \perp BA''$ and $CB = BC'$ (Ex. 1). 2. $A''B = A''B$ (Iden.)
3. $\therefore \triangle CBA'' \cong \triangle C'BA''$ (§96d). 4. $\therefore \angle x'' = \angle p$ (§98b). 5. $\angle x'' = \angle y''$ (Given in Ex. 1). 6. $\angle y'' = \angle p$ (Asmt. 7).
7. $\angle x'' + \angle r + \angle y'' = 180°$ (Asmt. 24).
8. $\therefore \angle x'' + \angle r + \angle p = 180°$ (Asmt. 6).
9. $\therefore E''A''C'$ is a st. line (Asmt. 25).



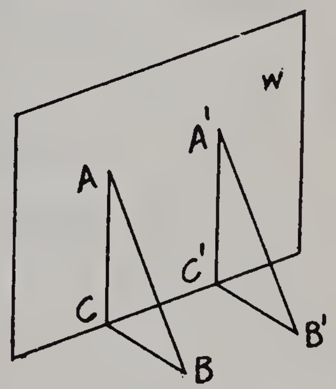
3. (1) $CA + AB + BE$. (2) From Ex. 1, $\angle x = \angle XBA$ and $\angle y = \angle BAX$. But $\angle X + \angle XBA + \angle BAX = 180°$ (§120) and $\angle X = 100°$ (Given). Then $\angle x + \angle y = 80°$ (Asmt. 6, 2). If $\angle y = 50°$, $\angle x = 30°$. (3) If $\angle y = 40°$ $\angle x = 40°$.

4. $y = 56°$.

5. 1. From Ex. 1, $\angle y = \angle x$ (Asmt. 7) and $\angle r = \angle s$. 2. But $\angle s = \angle x$ (§111). 3. $\therefore \angle r = \angle y$ (Asmt. 7). 4. $\angle CAB$ = a st. $\angle - \angle x - \angle y$ and $\angle ABE$ = a st. $\angle - \angle s - \angle r$ (§16).
5. $\therefore \angle CAB = \angle ABE$ (Asmt. 6, 7). 6. $\therefore CA \parallel BE$ (§104).

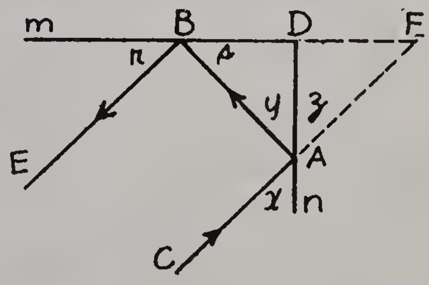
6. Given the wall w, ladders AB and A'B' equal in length, $BC = B'C'$, $BC \perp AC$ and $B'C' \perp A'C'$.

To prove that $AC = A'C'$.



Planning the Proof: 1. We can prove line segments = by §§ 97, 126, 128, 129, 140. 2. We shall use §97a.

Proof: 1. $\angle$s ACB and A'C'B' are rt. $\angle$s (Given). 2. $\therefore \triangle$s ACB and A'C'B' are rt. $\triangle$s (§57). 3. $AB = A'B'$ and $BC = B'C'$ (Given).
4. $\therefore \triangle ABC \cong \triangle A'B'C'$ (§135). 5. $\therefore AC = A'C'$ (§97a).



7. 1. $\angle x = \angle y$ (Ex. 1). 2. $\angle x = \angle z$ (§50d). 3. $\therefore \angle y = \angle z$ (Asmt. 7).
4. $AD \perp BD$ (Given). 5. $\therefore \angle ADB = \angle ADF$ (§§14, 50a). 6. $AD = AD$ (Iden.).
7. $\therefore \triangle ABD \cong \triangle ADF$ (§95b). 8. $\therefore \angle F = \angle s$ (§98b). 9. $\angle r = \angle s$ (Ex. 1).
10. $\therefore \angle F = \angle r$ (Asmt. 7). 11. $\therefore AC \parallel BE$ (§105).

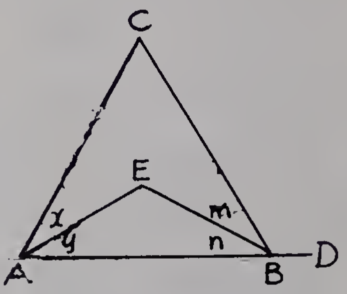
8. 1. $\triangle ABC$ is equilateral (Given). 2. $\therefore \triangle ABC$ is equiangular (§84). 3. $\therefore \angle A = \angle B = \angle C = 60°$ (§120, Asmt. 6, 4).

9. (1) $\angle ABC$ is made = 120°, $\angle BCD$ = 60°, and $\angle CDE$ = 120°. If BC and CD are made =, $\triangle BCD$ is equiangular and equilateral (§§98a, 120, 129). Since $\angle CBD$ is 60°, ABD is a st. line (Asmt. 25). Similarly, BDE is a st. line. If ABD and BDE are st. lines, AE is a st. line (Asmt. 1). (2) Since $\triangle BCD$ is equilateral, BD = BC, which is known.

10. Planning the Proof: 1. We can prove: (a, b) $\angle$s = by §§ 50, 98, 111, 112, 117, 121, 127; (c) line segments = by §§ 97, 126, 128, 129, 140. 2. We shall use: (a) § 121; (b) § 50c; (c) § 128.
Proof: 1. $\angle x = \angle y$ and AE = DE (Given). 2. $\angle r = \angle s$ (§ 98a). 3. $\angle h = \angle k$ (§ 121). 4. $\angle m = \angle n$ (§ 50c). 5. $\therefore$ BE = CE (§ 128).

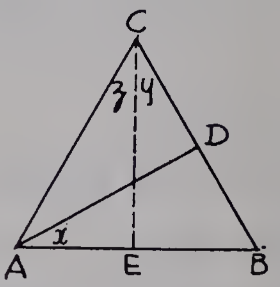
11. Planning the Proof: 1. We can prove line segments = by §§97, 126, 128, 129, 140. 2. We shall use § 128.
Proof: 1. $\angle r = \angle s$ and $\angle x = \angle y$ (Given). 2. $\therefore \angle h = \angle k$ (§ 121). 3. $\therefore \angle m = \angle n$ (§ 50c). 4. $\therefore$ BE = CE (§ 128).

12. Given the isos. $\triangle ABC$ with AC = BC, the ext. $\angle CBD$, AE bisecting $\angle BAC$, and BE bisecting $\angle ABC$.



To prove that $\angle CBD = \angle AEB$.
Planning the Proof: 1. (Same as in Ex. 10). 2. We shall use Asmt. 7, 6, 2.
Proof: 1. $\angle CBD + \angle CBA$ = a st. $\angle$ (§ 53b). 2. AC = BC, $\angle x = \angle y$, and $\angle m = \angle n$ (Given). 3. $\therefore \angle BAC = \angle CBA$ (§98a). 4. $\therefore 2\angle y = 2\angle n$ (Asmt. 9, 6). 5. $\therefore \angle n = \angle y$ (Asmt. 4) 6. $\angle m = \angle y$ (Asmt. 7). 7. $\angle CBD + \angle y + \angle n$ = a st. $\angle$ (§ 16, Asmt. 6 8. $\angle AEB + \angle y + \angle n$ = a st. $\angle$ (§ 120). 9. $\therefore \angle CBD + \angle n + \angle y$ = $\angle AEB + \angle y + \angle n$ (Asmt. 7). 10. $\therefore \angle CBD = \angle AEB$ (Asmt. 2).

13. Given the isos. $\triangle ABC$ with $\angle A = \angle B$ and AD $\perp$ BC.



To prove that $\angle x = \frac{1}{2}\angle ACB$.
Planning the Proof: 1. (Same as in Ex. 10). 2. We shall use § 121, Asmt.
Proof: 1. Draw CE, the bisector of $\angle ACB$ (Asmt. 18). 2. CE $\perp$ AB (Ex. 17, p. 108). 3. AD $\perp$ BC (Given). 4. $\angle$s ADB and CEB are rt. $\angle$s (§ 14). 5. $\angle ADB = \angle CEB$ (§ 50a). 6. $\angle B = \angle B$ (Iden.). 7. $\angle x = \angle y$ (§ 121). 8. $\angle z = \angle y$ (§ 22). 9. $\angle y = \frac{1}{2}\angle ACB$ (Asmt. 9, 6, 2). 10. $\angle x = \frac{1}{2}\angle ACB$ (Asmt. 7).

14. Planning the Proof: 1. We can prove a $\triangle$ isos. by § 56. 2. We shall use § 56.

Proof: 1. $AC = CB$ (Given). 2. $\therefore \angle A = \angle B$ (§98a). 3. $\angle$s AEF and DEB are rt. $\angle$s (Given and §14). 4. $\therefore$ $\triangle$s AEF and DEB are rt. $\triangle$s (§57). 5. $\therefore \angle A$ is comp. to $\angle F$, and $\angle B$ is comp. to $\angle BDE$ (§124). 7. $\therefore \angle BDE = \angle F$ (§50b). 8. $\angle BDE = \angle CDF$ (§50d). 9. $\therefore \angle F = \angle CDF$ (Asmt. 7). 10. $\therefore CF = CD$ (§128). 11. $\therefore \triangle CFD$ is isos. (§56).

15. Planning the Proof: 1. We can prove an angle = to the sum of $\angle$s by §§120, 123, 124, Asmt. 2. We shall use §123, Asmt. 6, 7.

Proof: 1. $AC = BC$ and $BE = BD$ (Given). 2. $\therefore \angle A = \angle ABC$ and $\angle BDE = \angle E$ (§98a). 3. $\angle ABC = \angle BDE + \angle E$ and $\angle CFE = \angle A + \angle FDA$ (§123). 4. But $\angle FDA = \angle BDE$ (§50d). 5. $\therefore \angle CFE = 3\angle CEF$ (Asmt. 6, 7).

16. Given the $\triangle ABC$ with CD the altitude on AB and CE bisecting $\angle ACB$.

To prove that $\angle x = \frac{1}{2}(\angle A - \angle B)$.

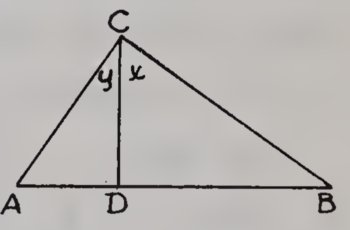
Planning the Proof: 1. (Same as in Ex. 15). 2. We shall use § 124, Asmt.

Proof: 1. $CD \perp AB$ (Given). 2. $\therefore$ $\triangle$s ADC and CDB are rt. $\triangle$s (§14). 3. $\angle A + \angle z - \angle x =$ a rt. $\angle$ and $\angle B + \angle w + \angle x =$ a rt. $\angle$ (§124). 4. $\therefore \angle A + \angle z - \angle x = \angle B + \angle w + \angle x$ (Asmt. 7). 5. But $\angle w = \angle z$ (Given). 6. $\therefore \angle A - \angle x = \angle B + \angle x$ (Asmt. 2). 7. $\therefore 2\angle x = \angle A - \angle B$ (Asmt. 1, 2). 8. $\therefore \angle x = \frac{1}{2}(\angle A - \angle B)$ (Asmt. 4).

17. Given the $\triangle ABC$ with CD the altitude on AB.

To prove that $\angle x - \angle y = \angle A - \angle B$.

Planning the Proof: (Same as in Ex. 16).



Proof: 1. $CD \perp AB$ (Given). 2. $\therefore$ $\triangle$s ADC and CDB are rt. $\triangle$s (§14). 3. $\angle A + \angle y =$ a rt. $\angle$ and $\angle B + \angle x =$ a rt. $\angle$ (§124). 4. $\therefore \angle B + \angle x = \angle A + \angle y$ (Asmt. 7), 5. $\therefore \angle x - \angle y = \angle A - \angle B$ (Asmt. 2).

PAGES 172-173 (Space Geometry)

1. A point. 2. Straight line. 4. No. 5. (1) An infinite number. (2) One. (3) None. 6. (1) An infinite number. (2) An infinite number. (3) One. 7. (1) An infinite number. (2) One. (3) One. 8. (1) One. (2) Two.

PAGES 174-175

1. Yes. 2. § 145c. 3. §145c. 4. Yes; §145c. 5. No. 6. Two. 7. See § 145c. 8. Three (§ 145e). 9. Three (§ 145f).

10. (a) Yes. (b) $\overline{PB}^2 = 150^2 + 80^2 = 28{,}900$; whence PB = 170.
170 ft. Ans.
(c) 1. AB = AC (Given). 2. PA = PA (Iden.). 3. ∠PAB = ∠PAC (§50a). 4. ∴ △PAB ≅ △PAC (§95a). 5. ∴ PB = PC (§97a).

PAGE 176 (Space Geometry)

1. Planning the Proof: 1. We can prove: (a) line segments = by §§ 97, 126, 128, 129, 140; (b) ∠s = by §§ 50, 98, 111, 112, 117, 121, 127. 2. We shall use: (a) § 97a; (b) § 98b.
Proof: 1. PO ⊥ m (Given). 2. PO ⊥ OA and OB (§147). 3. ∠AOP and ∠BOP are rt. ∠s (§14). 4. ∠AOP = ∠BOP (§50a). 5. OA = OB (Asmt. 26). 6. PO = PO (Iden.). 7. △POB ≅ △POA (§95a). 8. PA = PB (§97a). 9. ∠PAO = ∠PBO (§98b).

2. One. 3. An infinite number. 4. Yes. 5. No. 6. No. Vertical lines pass through the center of the earth. 7. (1) Yes. (2) No. 8. (1) One. (2) One. 9. (1) One. (2) One.

PAGE 177 (Space Geometry)

1. No, they meet at the center of the earth. 2. (1) No. (2) No. 3. (1) No. (2) Yes. 4. Yes.

PAGES 178-179 (Review Questions)

1. (1) In plane geometry they are parallel (§107), and in space geometry they may be skew (§151). (2) In plane geometry they form the same line (Asmt. 20), and in space geometry they lie in a plane (§145e).

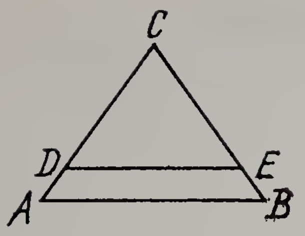
2. One. 3. Two. 4. (1) Right. (2) Obtuse. (3) Acute. 5. 180°. 6. Converse. 7. Inverse. 8. (1) When the angle adjacent to the exterior angle is obtuse. (2) When the angle adjacent to the exterior angle is right. 9. On the perpendicular bisector of the line segment joining the two given points. 10. Complementary (§124). 11. 45°, 45°, 90°. 12. Alternate interior ∠s; corresponding ∠s; alternate exterior ∠s. 13. 30°-60° right triangle. 14. 60°. 15. 135°. 16. 20°, 80°, 80°. 17. See § 142. 18. See §§ 96, 125, 135.

PAGE 180 (Test 9)

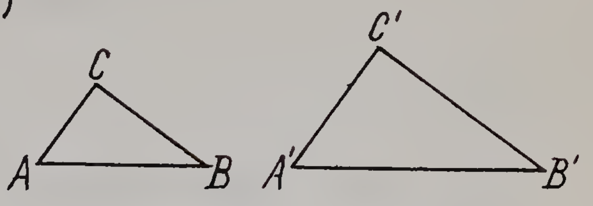
1. §112. 2. §111. 3. §113. 4. §50d. 5. §112. 6. §98a. 7. §112. 8. Asmt. 7. 9. §128.

PAGE 181 (Test 10)

(1)



(2)



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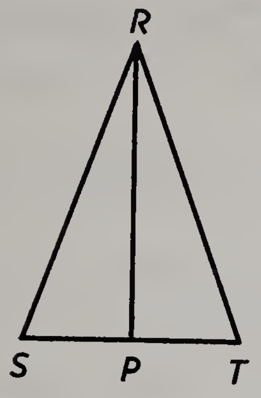

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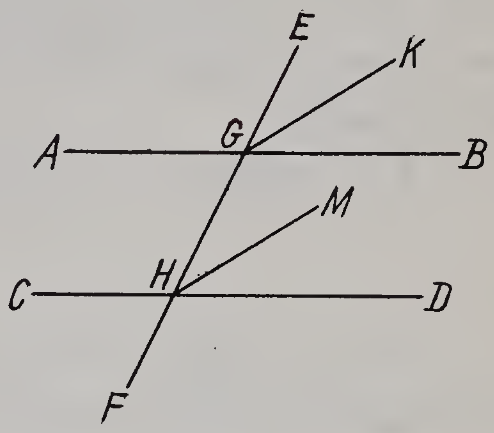


1. Given isos. △ABC with AC = BC and DE ∥ AB.

To prove that △CDE is isos.

2. Given rt. △ABC and A'B'C' with ∠C and C' rt. ∠ , and ∠A = ∠A'.

To prove that ∠B = ∠B'.

3. Given △RST, having altitude RP bisecting ST.

To prove that △RST is isos.

4. Given AB and CD cut by transversal EF, GK bisecting ∠EGB, HM bisecting ∠GHD, and GK ∥ HM.

To prove that AB ∥ CD.

PAGES 181-182 (Test 11)

1. (1) 70°. (2) 40°. 2. 8". 3. 180°. 4. 45°.
5. (1) 40°. (2) 100°. (3) 140°. 6. 5". 7. 40°.
8. (90 - x)°. 9. 90°. 10. 110°. 11. 30°, 60°.

PAGES 182-183 (Test 12)

1. (c) 2. (a). 3. (d). 4. (c) 5. (d). 6. (c).
7. (a). 8. (d). 9. (c). 10. (c). 11. (a).

Chapter 6. Constructions

PAGE 186

1. Direct application of §165.

2. Direct application of §165.

3. By §85 we know that so long as the lengths of the three sides of a triangle remain constant, the shape of the triangle cannot change.

4. No, because the sum of 3" and 4" is not greater than 7".

PAGE 187

1. Direct applications of §166.

2. Direct applications of §166.

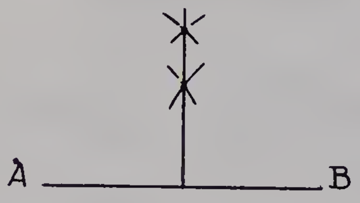
PAGE 188

Direct application of §167.

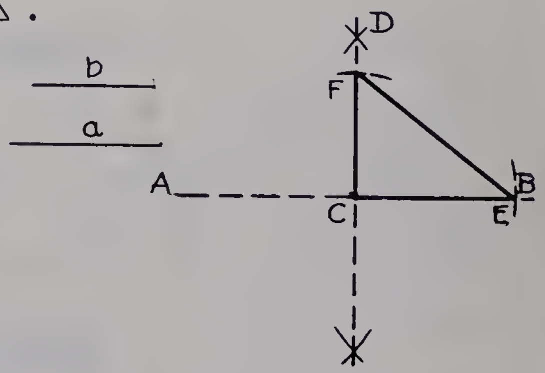
PAGE 190

1. (1) Yes. (2) Yes.

2. (1)



(2) Proof same as in §171.



3. Given a and b the legs of a rt. △.

To construct the △.

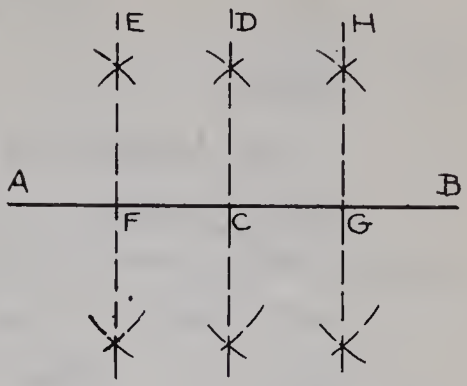
Construction: 1. Construct CD, the ⊥ bisector of a line segment AB (§171). 2. On AB lay off CE = a (§9). 3. On CD lay off CF = b (§9). 4. Draw EF (Asmt. 11).

Then △ECF is the required △.

Proof: 1. ∠ECF is a rt. ∠ (Const.). 2. △ECF is a rt.△ (§57). 3. Leg CE = a and leg CF = b (Const.). 4. ∴ △ECF is the required △.

4. Given line segment AB.

To divide AB into four equal parts.



Construction: 1. Construct CD the ⊥ bisector of AB (§171). 2. Construct FE the ⊥ bisector of AC and GH the ⊥ bisector of CB (§171).

Then AF, FC, CG, and GB are the required parts.

Proof: 1. AC = CB (Const.). 2. AF = FC and CG = GB (Const.). 3. 2FC = 2CG (Asmt. 9, 6). 4. ∴ FC = CG (Asmt. 4). 5. ∴ AF, FC, CG, and GB are = (Asmt. 6).

PAGE 191

1. Direct application of §172.

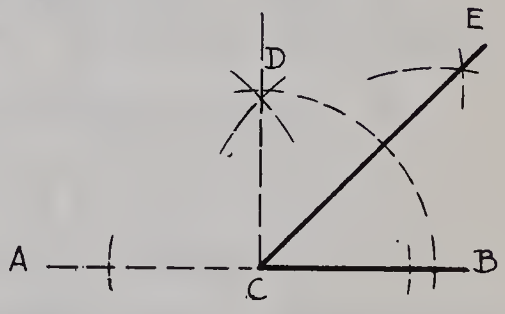
2. Direct application of §172.

3. Direct application of §172.

4. Use Prob. 6, §171, to construct a rt. ∠ , then make direct application of §172.

5. The altitudes of a △ seem to meet in a point.

PAGE 192

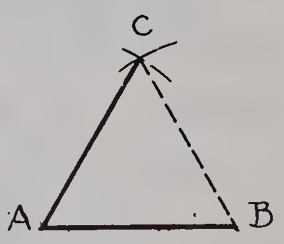


1. To construct a 45° ∠.

Construction: 1. At a point C on line AB, construct CD ⊥ AB (§173). 2. Construct CE bisecting ∠BCD (§166).

Then ∠BCE = 45°.

Proof: 1. ∠BCD = 90° (Const.). 2. Then 2∠BCE = 90° (Asmt. 9, 6). 3. Then ∠BCE = 45° (Asmt. 4).

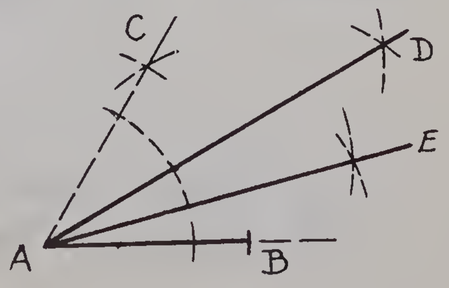


2. To construct a 60° ∠ .

Construction: 1. Draw AB any line segment (Asmt. 11). 2. With A and B as centers and a radius = AB, draw arcs intersecting in C (Asmt. 15). 3. Draw AC (Asmt. 11).

Then ∠A is the required ∠ .

Proof: 1. Draw BC (Asmt. 11). 2. AB = BC = AC (Const.). 3. ∠A + ∠B + ∠C = 180° (§120). 4. Then ∠A = ∠B = ∠C (§84). 5. 3∠A = 180° (Asmt. 6). 6. ∴ ∠A = 60° (Asmt. 4).



3. (1) To construct a 30° ∠ .

Construction: 1. Construct ∠BAC = 60° (Ex. 2, above). 2. Construct AD the bisector of ∠BAC (§166).

Then ∠BAD is the required ∠ .

Proof: 1. ∠ BAC = 60° (Const.). 2 ∠BAD = $\frac{1}{2}$(60°), or 30° (Const.).

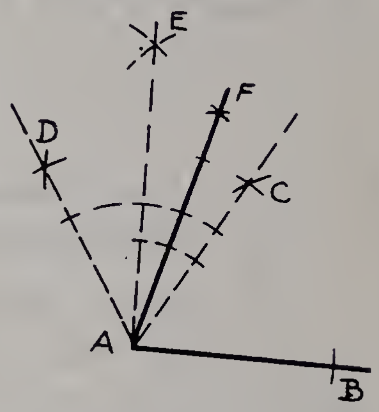
(2) To construct a 15° ∠.

Construction: 1. Construct ∠BAD = 30° [(1), above]. 2. Construct AE the bisector of ∠BAD (§166).

Then ∠BAE is the required ∠.

Proof: 1. ∠BAD = 30° (Const.). 2. ∠BAE = $\frac{1}{2}$(30°), or 15° (Const.).

4. To construct a 75° ∠.



Construction: 1. Construct ∠BAC = 60° (Ex. 2, above). 2. Construct ∠CAD = 60° (Ex. 2, above). 3. Bisect ∠CAD, constructing ∠CAE = 30° (§166). 4. Bisect ∠CAE, constructing ∠CAF = 15° (§166).

Then ∠BAF is the required ∠.

Proof: 1. ∠BAC = 60° (Const.). 2. ∠CAF = 15° (Const.). 3. Then ∠BAF = 60° + 15° = 75° (Asmt. 1, 9).

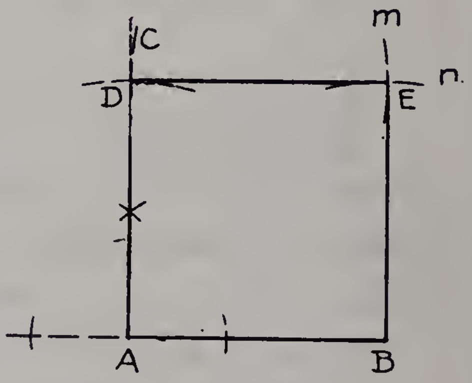
5. (1), (2), and (3) are direct applications of §166. (4) The bisectors of the ∠s seem to meet in a point.

6. Direct applications of §172.

7. Construct a rt. △ by applying §173, and then apply §171.

8. (1) and (2) are direct applications of §171. 3. The ⊥ bisectors of the sides of a △ seem to meet in a point.

9. To construct a square.



Construction: 1. Using AB, any line segment, as a side, construct AC ⊥ AB (§173). 2. On AC lay off AD = AB (§9). 3. With D and B as centers and radii = AB, draw arcs m and n intersecting at E (Asmt. 15). 4. Draw DE and BE (Asmt. 11).

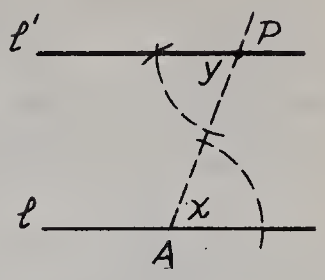
Then ABED is the required square.

Proof: 1. ∠A is a rt. ∠ (Const.). 2. AD = AB = BE = DE (Const.). 3. ∴ ABED is a square (§185).

PAGE 193

1. Given (same as in §174).

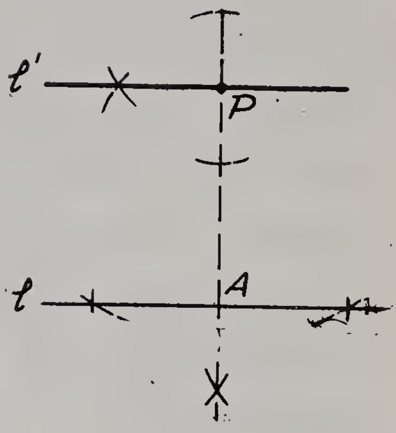
To construct (same as in §174).



Construction: 1. Through P and A, a point in ℓ, draw AP (Asmt. 11). 2. With P as a vertex and PA as a side, construct $\angle y = \angle x$ (§167).

Then ℓ' is the required line.

Proof: 1. $\angle y = \angle x$ (Const.). 2. Then $\ell' \parallel \ell$ (§160b).



2. Given the point P and the line ℓ.

To construct a line through P $\parallel \ell$.

Construction: 1. Through P construct PA $\perp \ell$ (§172). 2. At P construct $\ell' \perp$ PA (§173).

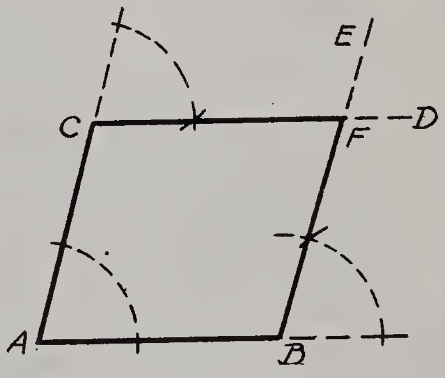
Then ℓ' is the required line.

Proof: 1. ℓ' passes through P and is $\perp$ PA (Const.). 2. $\ell \perp$ PA (Const.). 3. $\therefore \ell' \parallel \ell$ (§160d).

3. Direct application of §174.

4. Direct application of §174.

5. To construct a polygon of four sides having the opposite sides $\parallel$.



Construction: 1. Draw AB and AC, two intersecting sides of the polygon (Asmt. 35). 2. Through C construct CD $\parallel$ AB (§174). 3. Through B construct BE $\parallel$ AC and intersecting CD in F (§174).

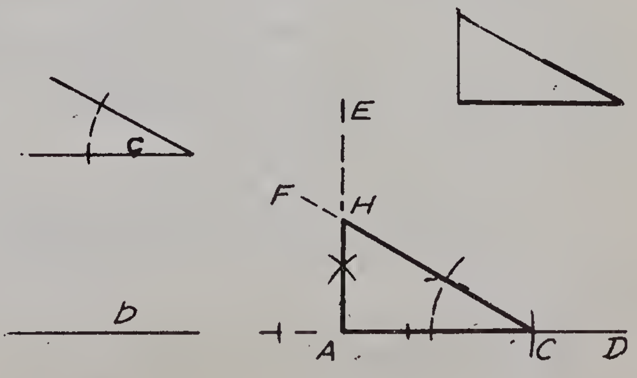
Then ABFC is the required polygon.

Proof: 1. CF $\parallel$ AB (Const.). 2. BF $\parallel$ AC (Const.).

PAGE 195

1. Given a leg b and the adjacent acute $\angle c$ of a rt. $\triangle$.

To construct the rt. $\triangle$.



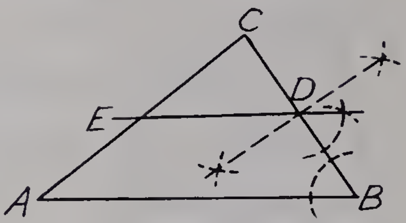
Construction: 1. On line AD lay off AC = b (§9). 2. At A construct AE $\perp$ AC (§173). 3. At C construct CF making $\angle ACF = \angle c$ (§167) and intersecting AE in H (Asmt. 12).

Then △ACH is the required △.

Proof: 1. △ACH has a leg AC = b (Const.). 2. ∠C = ∠c and is adjacent to AC (Const.). 3. ∠A is a rt. ∠ (Const.). 4. ∴ △AHC is a rt. △ (§57).

2. Given the △ABC.

To construct a line through the midpoint of BC and parallel to AB.



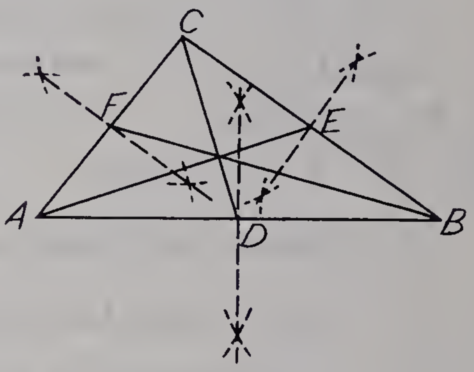
Construction: 1. Bisect BC (§171). and let D be the bisection point. 2. Through D construct DE ∥ AB (§174).

Then DE is the required line.

Proof: ED bisects BC and is parallel to AB (Const.).

3. (1) Given △ABC.

To construct the medians of △ABC.



Construction: 1. Bisect AB in D, BC in E, and AC in F (§171). 2. Draw AE, CD, and BF (Asmt. 11).

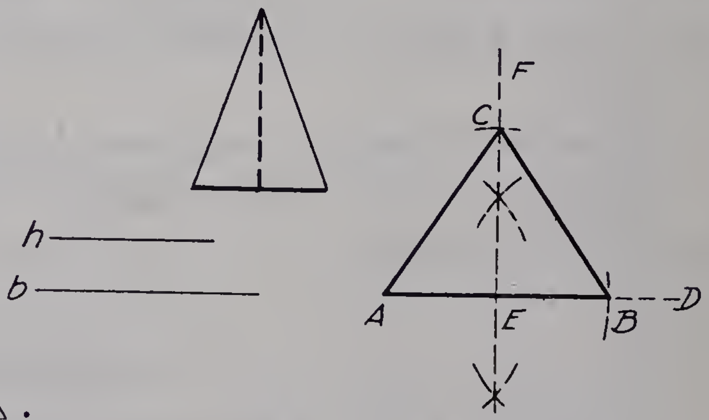
Then AE, CD, and BF are the required medians.

Proof: 1. D, E, and F are midpoints of AB, BC, and AC respectively (Const.). 2. AE, BF, and CD are the medians of △ABC (§89).

(2) Yes.

4. Given b the base and h the altitude of an isos. △.

To construct the △.



Construction: 1. On line AD lay off AB = b (§9). 2. Construct EF the ⊥ bisector of AB (§171). 3. On EF lay off EC = h (§9). 4. Draw AC and BC (Asmt. 11).

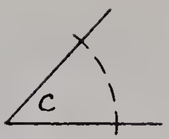
Then △ABC is the required △.

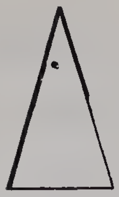
Proof: 1. EF is the ⊥ bisector of AB (Const.). 2. CA = CB (§97b). 3. △ABC is isosceles (§56). 4. AB is the base of △ABC (§55) and = b (Const.). 5. CE is the altitude of △ABC (§90) and = h (Const.).

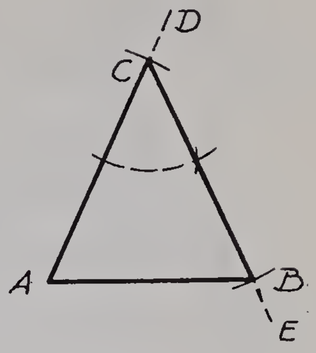
5. Given b one of the equal sides and c the vertex ∠ of an isos. △.

To construct the △.

Construction: 1. On line AD lay off AC = b (§9). 2. Construct $\angle$ACE = $\angle$c (§167). 3. On CE lay off CB = b (§9). 4. Draw AB (Asmt. 11).







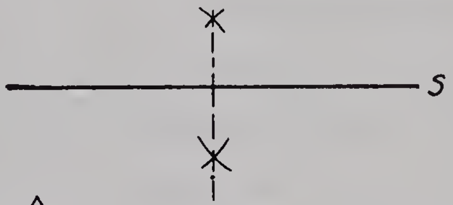
Then $\triangle$ ABC is the required $\triangle$.

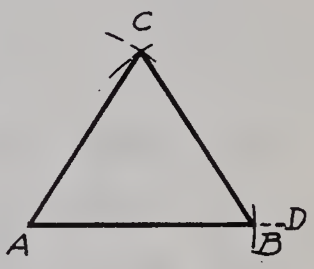
Proof: 1. $\triangle$ ABC has $\angle$C = $\angle$c (Const.). 2. AC = b = BC (Const.). 3. $\triangle$ ABC is isos. (§56).

6. Given b the base and s the sum of the equal sides of an isos. $\triangle$.

To construct the $\triangle$.

Construction: 1. On line AD lay off AB = b (§9). 2. Bisect s (§171). 3. With A and B as centers and radii = $\frac{1}{2}$s, draw arcs intersecting in C (Asmt. 15). 4. Draw AC and BC (Asmt. 11).





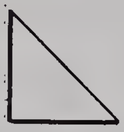
Then $\triangle$ABC is the required $\triangle$.

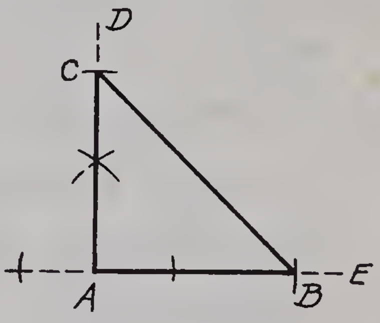
Proof: 1. AC = BC = $\frac{1}{2}$s (Const.). 2. AB = b (Const.). 3. $\triangle$ ABC is isos. (§56).

7. Given s a leg of an isos. rt. $\triangle$.

To construct the $\triangle$.

Construction: 1. On line AE lay off AB = s (§9). 2. At A construct $AD \perp AE$ (§173). 3. On AD lay off AC = s (§9). 4. Draw BC (Asmt. 11).





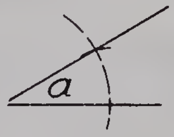
Then $\triangle$ABC is the required $\triangle$.

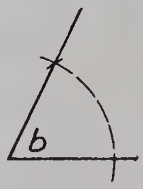
Proof: 1. AC = s = AB (Const.). 2. $\triangle$ABC is isos. (§56). 3. $\angle$A = a rt. $\angle$ (Const.). 4. $\triangle$ABC is a rt. $\triangle$ (§57).

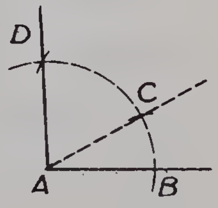
8. Given $\angle$a and $\angle$b.

To construct an $\angle$ = $\angle$a + $\angle$b.

Construction: 1. On any line AB as a side and with A as a vertex, construct $\angle$BAC = $\angle$a (§167). 2. On AC as a side and with A as a vertex construct $\angle$CAD = $\angle$b (§167).





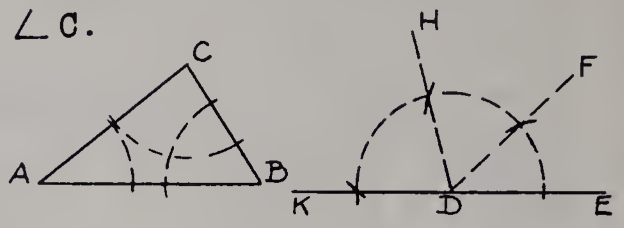


Then $\angle$BAD is the required $\angle$.

Proof: 1. $\angle BAC = \angle a$ and $\angle CAD = \angle b$ (Const.). 2. $\angle BAD = \angle BAC + \angle CAD$ (Asmt. 9). 3. Then $\angle BAD = \angle a + \angle b$ (Asmt. 6).

9. Given $\triangle ABC$.

To construct an $\angle = \angle A + \angle B + \angle C$.



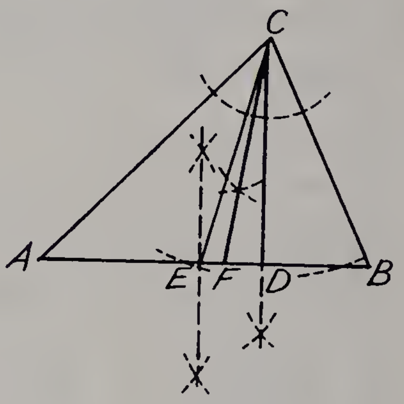
Construction: 1. On line DE as a side and with D as a vertex, construct $\angle EDF = \angle A$ (§167). 2. On DF as a side and with D as a vertex, construct $\angle FDH = \angle C$ (§167). 3. On DH as a side and with D as a vertex, construct $\angle HDK = \angle B$ (§167).

Then $\angle EDK$ is the required $\angle$.

Proof: 1. $\angle EDF = \angle A$, $\angle FDH = \angle C$, and $\angle HDK = \angle B$ (Const.). 2. $\angle EDK = \angle EDF + \angle FDH + \angle HDK$ (Asmt. 9). 3. Then $\angle EDK = \angle A + \angle C + \angle B$ (Asmt. 6).

10. Given scalene $\triangle ABC$.

To construct the altitude on AB, the median to AB, and the bisector of $\angle C$.



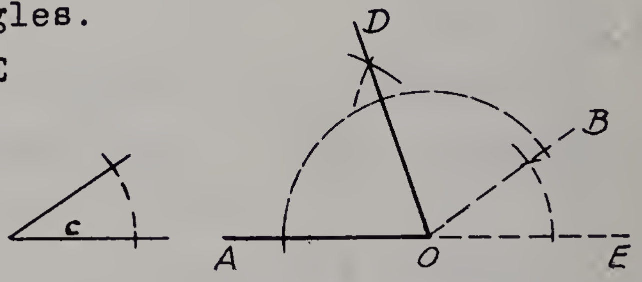
Construction: 1. Construct $CD \perp AB$ (§172). 2. Bisect AB (§171). 3. Draw CE (Asmt. 11). 4. Bisect $\angle C$ (§166).

Then CD is the required altitude, CE is the required median, and CF is the required bisector of $\angle C$.

Proof: 1. CD is the altitude from C to AB (§90). 2. CE is the median to AB (§89). 3. CF is the bisector of $\angle C$ (§91).

11. Given c the vertex angle of an isos. $\triangle$.

To construct one of the base angles.



Construction: 1. Draw st. $\angle AOE$ (§16, Asmt. 11). 2. Construct $\angle EOB = \angle c$ (§167). 3. Construct OD bisecting $\angle AOB$ (§166).

Then $\angle AOD$ is the required $\angle$.

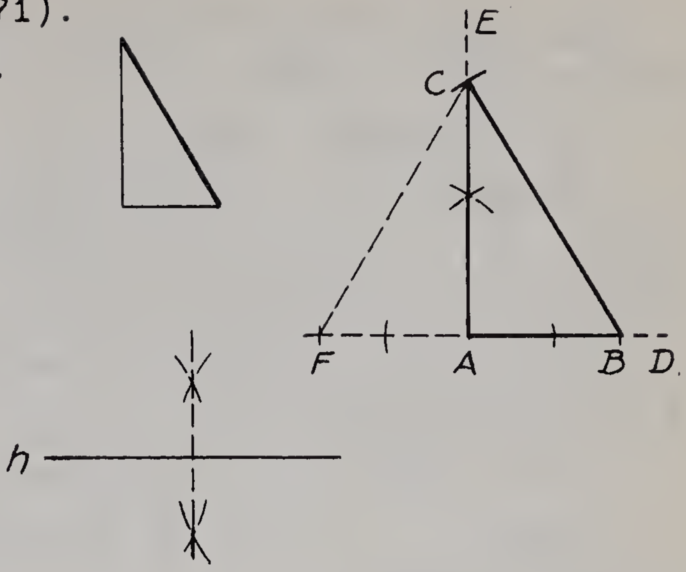
Proof: 1. $\angle AOD + \angle DOB + \angle BOE =$ a st. $\angle$ (Asmt. 9). 2. Let $x =$ one base $\angle$ of the isos. $\triangle$. Then $x =$ the other base $\angle$ (§98a). 3. Then $\angle C + 2\angle x =$ a st. $\angle$ (§120). 4. Then $\angle C + 2\angle x = \angle AOD + \angle DOB + \angle BOE$ (Asmt. 7). 5. $\angle C = \angle BOE$ and $\angle BOD = \angle DOA$ (Const.). 6. Then $2\angle x = \angle AOD + \angle DOB$ (Asmt. 2). 7. $2\angle x = 2\angle AOD$ (Asmt. 6). 8. $\angle x = \angle AOD$ (Asmt. 4).

12. Given h, the hypotenuse of a 30°-60° rt. $\triangle$.

To construct the $\triangle$.

Construction: 1. Bisect h (§ 171). 2. On line AD lay off AB = $\frac{1}{2}$h (§ 9). 3. At A construct AE ⊥ AB (§ 173). 4. With B as a center and radius = h, draw an arc intersecting AE in C (Asmt. 15). 5. Draw BC (Asmt. 11).

Then △ABC is the required △.



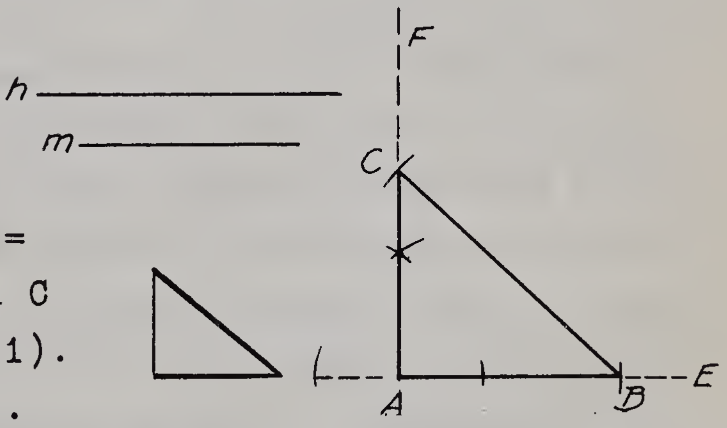
Proof: 1. Extend BA to F so that AF = AB (Asmt. 13). 2. Draw CF (Asmt. 11). 3. AC = AC (Iden.). 4. ∠FAC = ∠BAC (§ 14). 5. △FAC ≅ △ABC (§ 95a). 6. ∴ CF = CB (§ 97a). 7. But FB = h and BC = h (Const.). 8. ∴ FB = BC = CF (Asmt. 6). 9. ∠ABC + ∠BCF + ∠CFB = 180° (§ 120). 10. ∴ ∠ABC = ∠BCF = ∠CFB (§ 84). 11. ∴ ∠ ABC = 60° (Asmt. 6, 5). 12. ∴ ∠ACB = 30° (§ 161).

13. Given h the hypotenuse and m one leg of a rt. △.

To construct the △.

Construction: 1. On line AE lay off AB = m (§ 9). 2. At A construct AF ⊥ AB (§ 173). 3. With B as a center and radius = h, draw an arc intersecting AF in C (Asmt. 15). 4. Draw BC (Asmt. 11).

Then △ ABC is the required △.

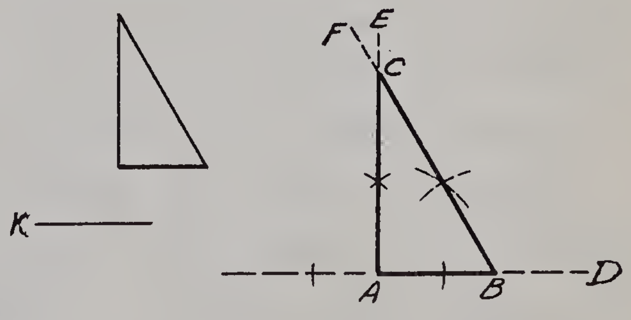


Proof: 1. Hypotenuse BC = h and leg AB = m (Const.). 2. ∠BAC = a rt. ∠ (Const.). 3. △ ABC is a rt. △ (§ 57).

14. Given k, the shorter leg of a 30°-60° rt. △.

To construct the △.

Construction: 1. On line AD lay off AB = k (§ 9). 2. At B with AB as one side construct ∠ABE = 60° (Ex. 2, page 192). 3. At A construct a ⊥ to AB, intersecting BF at C (§ 173). If these lines did not intersect, CB would be ⊥ AB (Asmt. 35, § 159a).



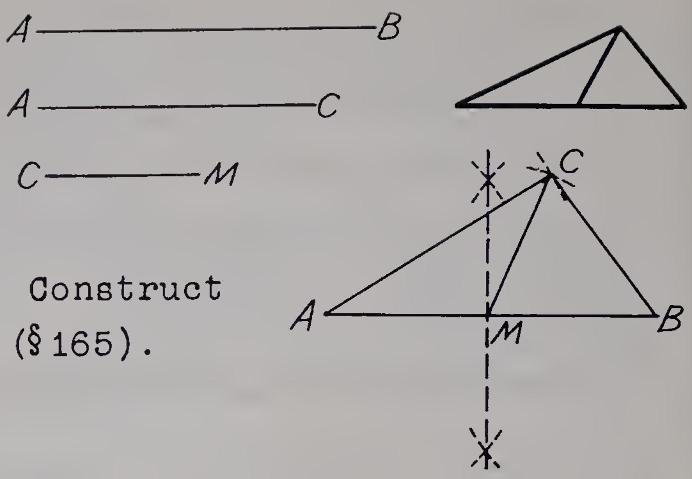
Then △ABC is the required △.

Proof: 1. ∠ABC = 60° (Const.). 2. AB = k (Const.). 3. ∠ C = 30° and lies opp. AB (§ 161).

15. Given sides AB and AC and median CM of △ABC.

To construct △ABC.

Construction: 1. Draw AB (Asmt. 11). 2. Bisect AB (§171) and denote its midpoint by M. 3. Construct △ AMC having sides AM, CM, and AC (§165). 4. Draw BC (Asmt. 11).



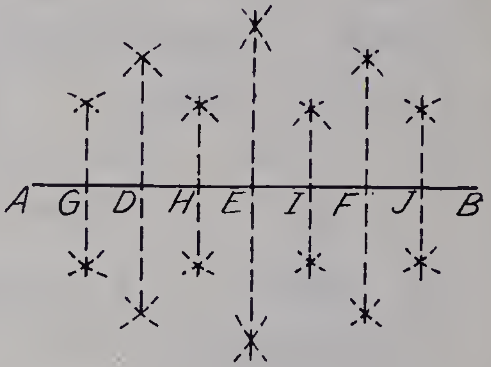
Then △ABC is the required △.

Proof: 1. AB and AC are sides of △ABC (Const.). 2. CM is a median (§89).

16. Given line segment AB.

To divide AB into 8 equal parts.

Construction: 1. Bisect AB in E (§171). 2. Bisect AE and EB in D and F respectively (§171). 3. Bisect AD, DE, EF, and FB in G, H, I, and J respectively (§171).



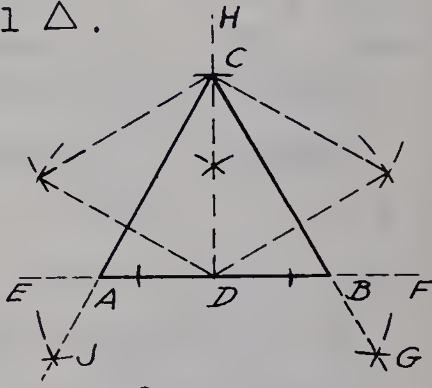
Then AB is divided into 8 equal parts.

Proof: 1. AE = EB (Const.). 2. AD = DE and EF = FB (Const.). 3. AD = DE = EF = FB (Asmt. 4, 6). In like manner, AG = GD = DH = HE = EI = IF = FJ = JB (Asmt. 4, 6).

17. Given a, the altitude of an equilateral △.

To construct the △.

Construction: 1. At D, a point on EF, construct DH ⊥ EF (§173). 2. On DH lay off DC = a (§9). 3. At C construct CJ intersecting EF in A and making ∠ACD = 30° (Ex. 3, p.192). 4. Likewise construct CB making ∠DCB = 30° (Ex.3, p.192).



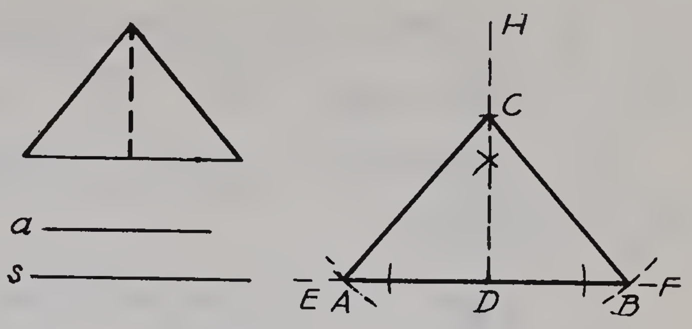
Then △ABC is the required △.

Proof: 1. CD = a (Const.). 2. CD is the altitude of △ABC (§90). 3. ∠ACD = 30° (Const.). 4. △CAD is a rt. △ (§57). 5. ∴ ∠A = 60° (§161). 6. Similarly, ∠B = 60° (Reasons 2-5). 7. ∠ACB = 60° (Asmt. 1). 8. ∠A = ∠B = ∠C (Asmt. 7). 9. ∴ △ ABC is equilateral (§129).

18. Given a the altitude and s one of the equal sides of an isos. △.

To construct the △.

Construction: 1. At D, a point on a line EF, construct DH ⊥ EF



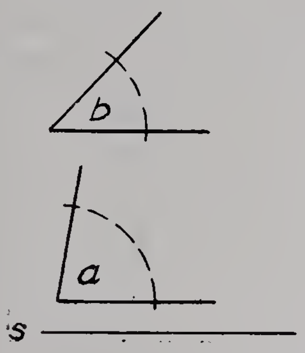
(§173). 2. On DH lay off DC = a (§9). 3. With C as a center and a radius = s, draw arcs intersecting EF in A and B (Asmt. 15). 4. Draw AC and BC (Asmt. 11).

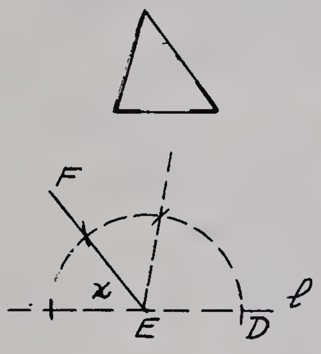
Then △ABC is the required △.

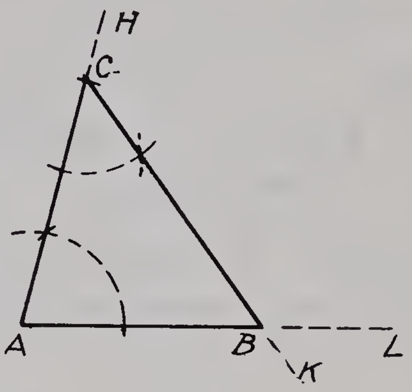
Proof: 1. AC = BC = s (Const.). 2. △ABC is isos. (§56). 3. CD ⊥ EF (Const.). 4. CD is the altitude of △ABC (§90). CD = a (Const.).

19. Given ∠a and ∠b, two ∠s of a △, and s a side opp. ∠b.

To construct the △.







Construction: 1. On line ℓ, using ED as a side and E as the vertex, construct ∠DEF = ∠a + ∠b (Ex. 8, page 195). 2. On line AL as a side and with A as the vertex construct ∠A = ∠a (§167). 3. On AH, a side of ∠A, lay off AC = s (§9). 4. Using AC as a side and C as the vertex, construct ∠ACK = ∠x, the supplement of ∠DEF (§167). 5. CK will intersect AL, since if it did not, ∠A + ∠C would = 180° (Asmt. 35, §162). 6. Let B be the point of intersection (Asmt. 12).

Then △ABC is the required △.

Proof: 1. ∠A = ∠a (Const.). 2. AC = s (Const.). 3. ∠A + ∠B + ∠C = 180° (§120). 4. ∠a + ∠b + ∠x = 180° (Const.). 5. ∴ ∠A + ∠B + ∠C = ∠a + ∠b + ∠x (Asmt. 7). 6. But ∠A = ∠a and ∠C = ∠x (Const.). 7. Then ∠B = ∠b (Asmt. 2).

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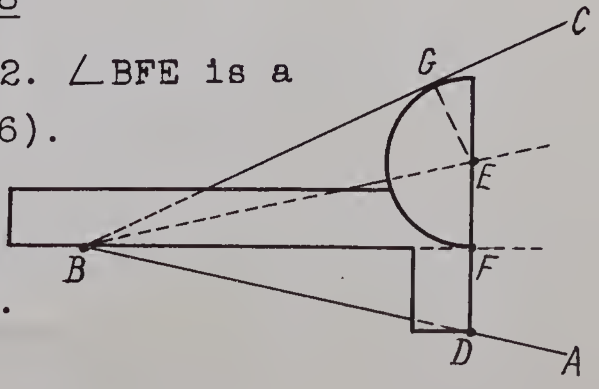


1. 1. Construct EG ⊥ BC (§172). 2. ∠BFE is a rt. ∠ (Const.). 3. EG = EF (Asmt. 26). 4. BE = BE (Iden.). 5. ∴ Rt. △BGE ≅ rt. △BFE (§156b). 6. ∠BFD is a rt. ∠ (Const.). 7. EF = FD (Const.). 8. BF = BF (Iden.).

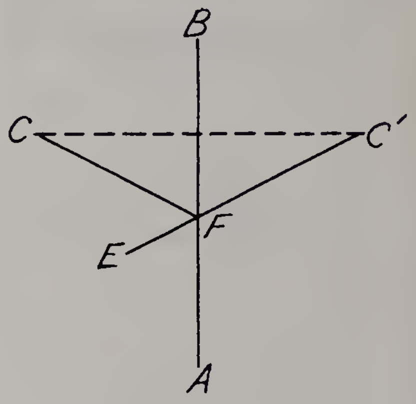
9. ∴ Rt. △BFE ≅ rt. △BFD (§96d). 10. ∴ Rt. △BGE ≅ rt. △BFE ≅ rt. △BFD (Asmt. 7). 11. ∴ ∠GBE = ∠EBF = ∠FBD (§98b).

2. Construct a right ∠ (§173) and then follow the method of Ex. 1, above.

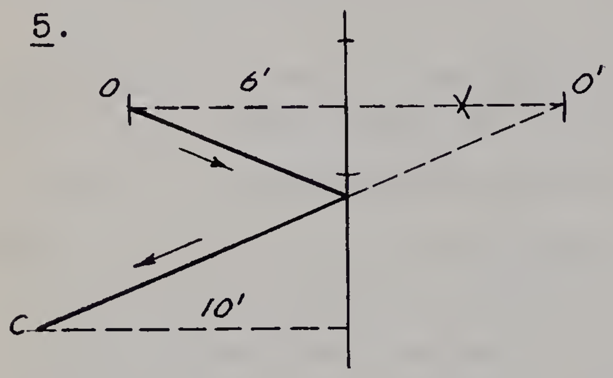
3. Proof: 1. AB = BE (Given). 2. ∠a = ∠b (§83). 3. ∠a + ∠b = ∠c (§93). 4. 2∠a = ∠c (Asmt. 6). 5. BE = CE (Given). 6. ∠c = ∠d (§83). 7. 2∠a = ∠d (Asmt. 6). 8. ∠a + ∠d = ∠x (§93). 9. ∠a + 2∠a = ∠x (Asmt. 6). 10. 3∠a = ∠x (Asmt. 6). 11. ∠a = 1/3∠x (Asmt. 4).

4. Draw EC′ intersecting AB in F (Asmt. 11). 2. Draw CF (Asmt. 11). 3. Then CFE is the path of a ray of light.

Proof: 1. The two △s are ≅ (§95a). 2. Then ∠CFB = ∠C′FB (§98b). 3. But ∠AFE = ∠C′FB (§50d). 4. Then ∠AFE = ∠CFB (Asmt. 7) and the incident and reflected rays make equal ∠s with the mirror.

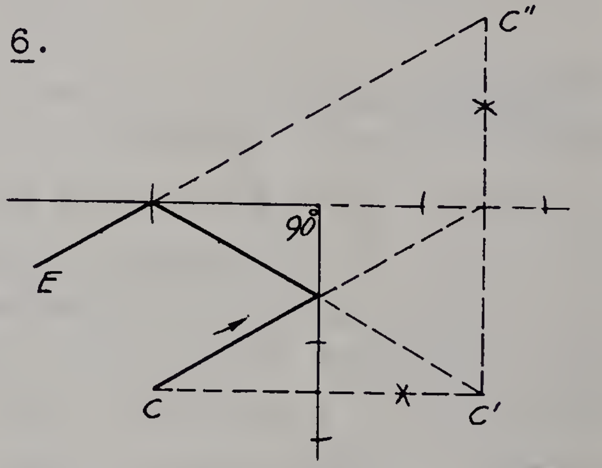


5.



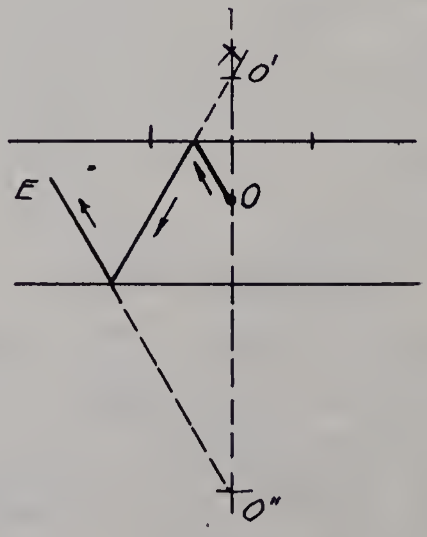
(This is not to the scale 1 inch = 4 feet.)

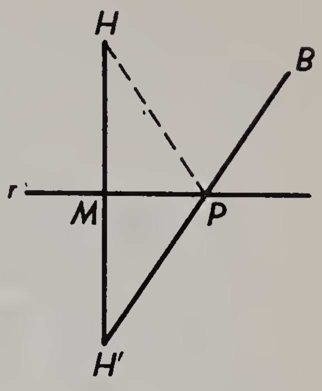
6.



See Ex. 7, p. 169.

7.



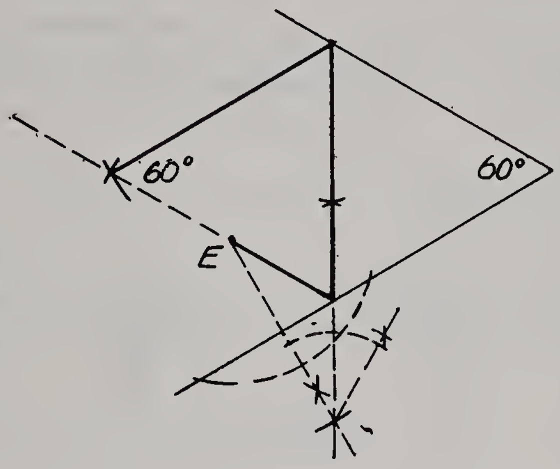


8. Construction: 1. Construct HM $\perp$ r (§172). 2. Extend HM to H' (Asmt. 13) so that HM = MH' (§9). 3. Draw H'B (Asmt. 11).

The pump should be placed at P, the point where H'B intersects r.

Proof: 1. Draw PH (Asmt. 11). 2. PM is the $\perp$ bisector of HH' (Const.). 3. $\therefore$ H and H' are equally distant from P, that is, PH = PH' (§79). 4. BP = BP (Iden.). 5. PH + BP = PH' + BP (Asmt. 1). 6. PH' + BP is the shortest distance from B to H' (Asmt. 14). 7. PH + BP is the shortest distance from B to H via P (Asmt. 6).

9.



PAGE 198 (Review Questions)

1. (1) By 2 of its points. (2) By the intersection of 2 lines, of 2 arcs, or of a line and an arc.

2. A theorem is a statement to be proved. A problem may be a computation to be performed or a construction to be made and a proof that the construction is correctly made.

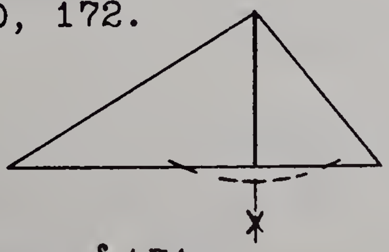
3. See §163.

4. a. See §165. b. See §166. c. See §167. d. See §169. e. See §170. f. See §171. g. See §172. h. See §173. i. See §174.

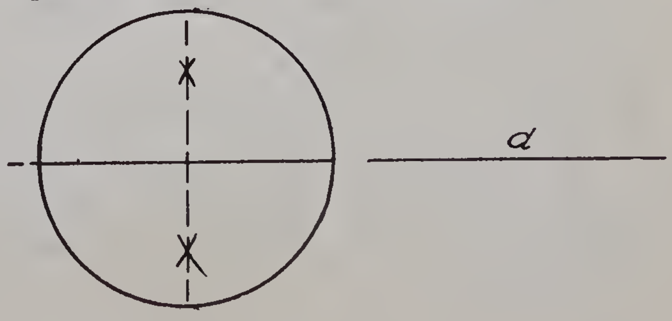
PAGE 199 (Test 13)

1. §165, §169, or §170.

2. §§90, 172.



3. §§23, 171, Asmt. 15.



4. Same as §174.

5. Same as Ex. 5, page 195.

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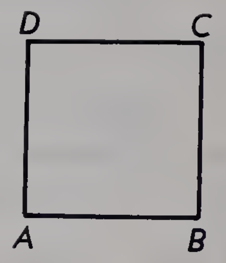


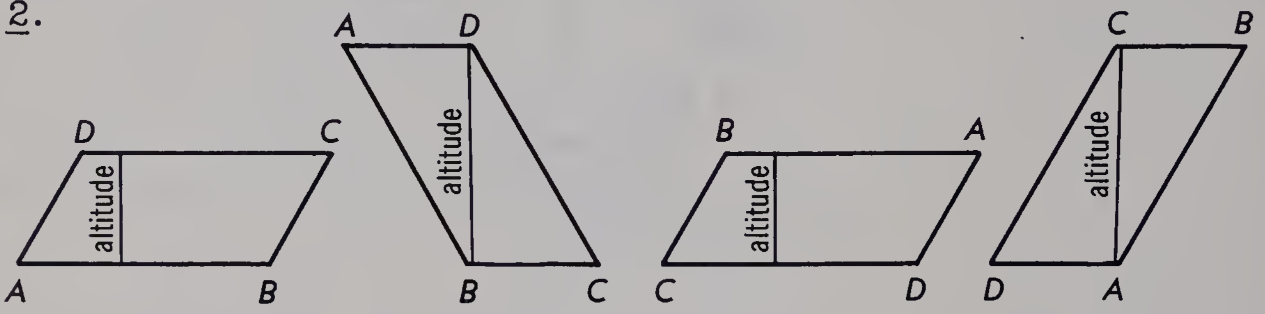
1. Given square ABCD.

To prove ABCD is a rhombus.

Planning the Proof: 1. We can prove a quadrilateral is a rhombus by § 185—Definition of rhombus. 2. We shall use § 185—Definition of rhombus.

Proof: 1. ABCD is a square (Given). 2. ABCD is a rectangle having two equal adjacent sides (§ 185—Def. of square). 3. ABCD is a parallelogram having two equal adjacent sides (§ 185—Def. of a rectangle). 4. ABCD is a rhombus (§ 185—Def. of rhombus).

2.



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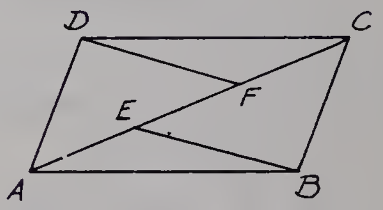
1. § 194 (See Ex. 1, p. 202).

2. Given the ▱ABCD with AC the diagonal and AE = EF = FC.

To prove that BE = DF.

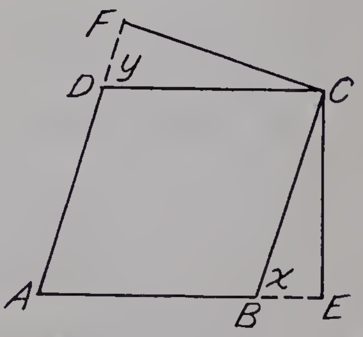


Planning the Proof: 1. We can prove line segments equal by §§ 97, 157, 188, 192, 193, 194. 2. We shall use § 97a.

Proof: 1. AE = FC (Given). 2. ∠EAB = ∠FCD (§ 158a). 3. AB = DC (§ 188). 4. ∴ △ AEB ≅ △ CFD (§ 95a). 5. BE = DF (§ 97a).

3. Given rhombus ABCD with altitude CE and altitude CF.

To prove that CE = CF.



Planning the Proof: (Same as in Ex. 2).

Proof: 1. DC = BC (§ 194). 2. AD ∥ BC and AB ∥ DC (§ 185). 3. ∠A = ∠x = ∠y (§ 158a). 4. CF and CE are altitudes (Given). 5. CF ⊥ AD and CE ⊥ AB (§ 186). 6. △s CFD and CEB are rt. △s (§ 57). 7. Rt. △ CFD ≅ rt. △ CEB (§ 156a). 8. CF = CE (§ 97a).

4. $8x - 3 = 5x + 9$. $3x = 12$ (Asmt. 1, 2). $x = 4$ (Asmt. 4). 29°. Ans.

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1. a. §§188, 192, 193, 194, 195. b. §189. c. §187. d. §§191, 196. e. §190.

2. Opp. $\angle = 65°$ (§189). Adj. $\angle$s $= 115°$ (§190).

3. Let x = the no. of degrees in one $\angle$ and $3x$ = the no. of degrees in the adj. $\angle$.

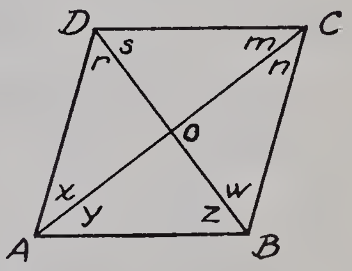
Then $x + 3x = 180°$ (§190); whence $x = 45°$, and $3x = 135°$. The $\angle$s are 45°, 135°, 45°, 135° (§189).

4. (1) Yes (§185). (2) Yes (§185). (3) Yes (Ex. 1, p. 202).

5. (1) No (§185). (2) Yes (§185). (3) No (§185).

6. (1) 4 sides. (2) All sides =. (3) 4 rt. $\angle$s. (4) All $\angle$s =. (5) Diagonals $\perp$.

7. Given the rhombus ABCD with diagonals AC and BD intersecting at O.

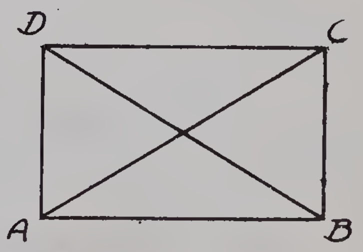


To prove that $\angle x = \angle y$, $\angle m = \angle n$, $\angle r = \angle s$, and $\angle z = \angle w$.

Planning the Proof: 1. We can prove $\angle$s = by §§50, 98, 158, 189. 2. We shall use §§98a, 98b.

Proof: 1. $\triangle ADC \cong \triangle ABC$ (§187). 2. $\angle x = \angle n$ and $\angle m = \angle y$ (§98b). 3. $AD = DC$ (§185). 4. $\angle x = \angle m$ (§98a). 5. $\therefore \angle x = \angle y = \angle m = \angle n$ (Asmt. 7). 6. In like manner, $\angle r = \angle s = \angle z = \angle w$ (Statements 1-5).

8. Given the rect. ABCD with diagonals AC and BD.



To prove that $AC = BD$.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 188, 192, 193, 194, 195. 2. We shall use §97a.

Proof: 1. $\angle$s DAB and ABC are rt. $\angle$s (§191). 2. $\therefore$ $\triangle$s DAB and ABC are rt. $\triangle$s (§57). 3. $AD = BC$ (§188). 4. $AB = AB$ (Iden.). 5. $\therefore \triangle DAB \cong \triangle ABC$ (§96d). 6. $\therefore AC = BD$ (§97a).

9. Given the ▱ ABCD with the diagonal AC, and DE and BF $\perp$ AC.

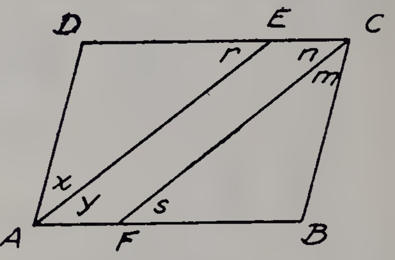
To prove that $DE \parallel BF$ and $DE = BF$.

Planning the Proof: 1. We can prove (a) lines $\parallel$ by §§160, 185; (b) line segments = by §§97, 157, 188, 192, 193, 194, 195. 2. We shall use (a) §160d; (b) §97a.

Proof: 1. DE ⊥ AC and BF ⊥ AC (Given). 2. ∴ DE ∥ BF (§160d). 3. △ AED and BFC are rt. △ (§57). 4. AD = BC (§188). 5. ∠ EAD = ∠ BCF (§158a). 6. ∴ △ AED ≅ △ BFC (§156a). 7. ∴ DE = BF (§97a).

10. 1. DB ⊥ AC (§196). 2. DB bisects ∠ADC (Ex. 7 above). 3. ∠ADO = ∠CDO = 60° (§ 22). 4. ∠DAO = 30° (§124). 5. DO = OB = 6 (§ 195 and Asmt. 6). 6. DO = $\frac{1}{2}$AD (§136). 7. $\frac{1}{2}$AD = 6 (Asmt. 6). 8. AD = 12 (Asmt. 3). 9. DC = AD = 12 (§194 and Asmt. 6). DO = 6 in., AD = DC = 12 in. Ans.

11. Given the ▱ ABCD with AE bisecting ∠ BAD and CF bisecting ∠ BCD.



To prove that AE ∥ FC and AE = FC.

Planning the Proof: 1. (Same as in Ex. 9). 2. We shall use (a) § 160b; (b) § 97a.

Proof: 1. ∠ x = ∠ y and ∠ m = ∠ n (Given). 2. ∠ x + ∠ y = ∠ m + ∠ n (§189 and Asmt. 9). 3. ∴ 2∠ x = 2∠ m (Asmt. 6). 4. ∴ ∠ x = ∠ m (Asmt. 4). 5. ∠ B = ∠ D (§189). 6. AD = BC (§188). 7. ∴ △ ADE ≅ △ FBC (§ 95b). 8. ∴ ∠ r = ∠ s (§ 98b). 9. ∠ n = ∠ s (§ 158a). 10. ∴ ∠ r = ∠ n (Asmt. 7). 11. ∴ AE ∥ FC (§ 160b). 12. AE = FC (§97a).

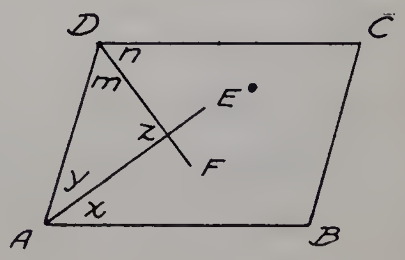
12. Given the isos. △ ABC with AC = BC, D any point in the base AB, DF ∥ AC, and DE ∥ BC.

To prove that DFCE is a ▱ and DF + FC + CE + ED = AC + BC.

Planning the Proof: 1. We can prove (a) that a quad. is a ▱ by § 185; (b) line segments = by §§ 97, 157, 188, 192, 193, 194, 195. 2. We shall use (a) § 185; (b) Asmt. 1 and 6.

Proof: 1. DF ∥ AC and DE ∥ BC (Given). 2. ∴ DFCE is a ▱ (§185). 3. ∠ A = ∠ B (§ 98a). 4. ∠ B = ∠ ADE (§ 158a). 5. ∴ ∠ A = ∠ ADE (Asmt. 7). 6. ∴ AE = ED (§157a). 7. In like manner, BF = DF. 8. CE = CE and FC = FC (Iden.). 9. ∴ ED + DF + CE + FC = AE + BF + CE + FC (Asmt. 1). 10. ∴ ED + DF + CE + FC = AC + BC (Asmt. 6).

13. Given the ▱ ABCD with AE bisecting ∠BAD and DF bisecting ∠ADC.



To prove that AE ⊥ DF.

Planning the Proof: 1. We can prove lines ⊥ by §§ 51, 159, 191, 196. 2. We shall use §51a.

Proof: 1. $\angle x + \angle y + \angle m + \angle n =$ a st. $\angle$ (§190). 2. $\angle x = \angle y$ and $\angle n = \angle m$ (Given). 3. $\therefore 2\angle y + 2\angle m =$ a st. $\angle$ (Asmt. 6). 4. $\therefore \angle y + \angle m =$ a rt. $\angle$ (Asmt. 4). 5. $\angle y + \angle m + \angle z =$ a st. $\angle$ (§120). 6. $\therefore \angle z =$ a rt. $\angle$ (Asmt. 2). 7. $\therefore AE \perp DF$ (§51a).

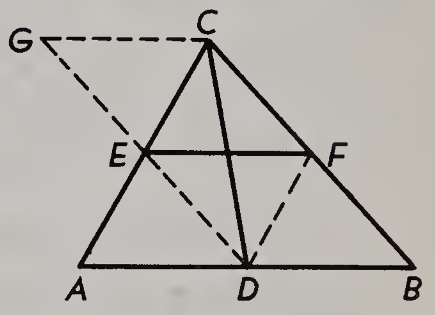
14. Planning the Proof: 1. We can prove a quadrilateral a square by §185. 2. We shall use §185.

Proof: 1. $\angle C$ is a rt. $\angle$ (Given). 2. $CR \perp CA$ (§51a). 3. $ST \perp CA$ (Given). 4. $ST \parallel CR$ (§160d). 5. Similarly, $TC \parallel SR$ (Reasons 1-4). 6. $\therefore$ CRST is a ▱ (§185—Def. of a ▱). 7. CRST is a rectangle (§185—Def. of a rectangle). 8. SA bisects $\angle TAB$ and SB bisects $\angle ABR$ (Given). 9. Draw $SE \perp AB$ (§178a). 10. $ST = SE$ and $SR = SE$ (§157b). 11. $ST = SR$ (Asmt. 7). 12. $\therefore$ CRST is a square (§185—Def. of a square).

15. Given $\triangle ABC$ with median CD and line segment EF joining E and F the midpoints of AC and BC, respectively.

To prove that EF and CD bisect each other.

Planning the Proof: 1. We can prove that two line segments bisect each other by §195. 2. We shall use §195.



Proof: 1. CD is a median of $\triangle ABC$ (Given). 2. D is the midpoint of AB (§89). 3. E is the midpoint of AC (Given). 4. $AE = EC$ (§22). 5. Draw ED (Asmt. 11) and extend it to G so that $DE = EG$ (Asmt. 13). 6. Draw GC (Asmt. 11). 7. $\angle GEC = \angle AED$ (§50d). 8. $\triangle AED = \triangle GEC$ (§95a). 9. $\angle CGD = \angle GDA$ (§98b). 10. $\therefore GC \parallel AB$ (§160b). 11. $GC = AD$ (§97a). 12. $AD = DB$ (§22). 13. $\therefore GC = DB$ (Asmt. 7). 14. $\angle GCD = \angle CDB$ (§158a). 15. $CD = CD$ (Iden.). 16. $\triangle GCD \cong \triangle BCD$ (§95a). 17. $\angle GDC = \angle DCB$ (§98b). 18. $GD \parallel BC$ (§160b). 19. In similar manner DF may be proved $\parallel AC$ (Reasons 1-17). 20. $\therefore$ DFCE is a ▱ (§185—Def. of a ▱). 21. Then CD and EF bisect each other (§195).

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1. §§ 185, 197.

2. The opposite sides of the quad. were made =, $\therefore$ by Th. 20 the quad. is a ▱.

3. Since the quad. always has its opposite sides =, it is a ▱ (§197). $\therefore$ the edges of the ruler are always $\parallel$ (§185—Def. of ▱).

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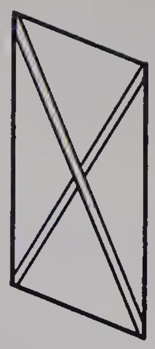
(1)



(2) a ▱.

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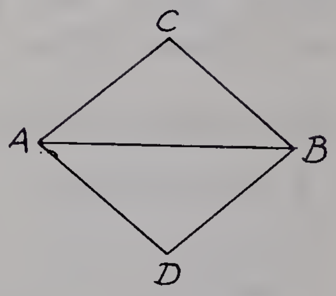
1. §§185, 197, 198, 199.

2. Given the ≅ isos. △s ABC and ABD with AC = BC, AD = BD, the common base AB, and vertices C and D on opp. sides of AB.

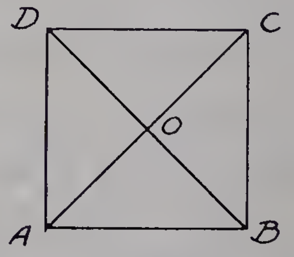


To prove that ADBC is a rhombus.

Planning the Proof: 1. We can prove that a quad. is a rhombus by §185. 2. We shall use §185.

Proof: 1. △ ABC ≅ △ ABD (Given). 2. ∴ BC = AD and AC = BD (§ 97a). 3. ∴ ADBC is a ▱ (§197). 4. AC = BC (Given). 5. ∴ ADBC is a rhombus (§185).

3. Given the rect. ABCD with AC ⊥ BD.

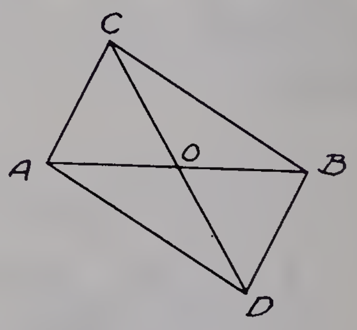


To prove that ABCD is a square.

Planning the Proof: 1. We can prove that a rect. is a square by §185. 2. We shall use §185.

Proof: 1. AC ⊥ BD (Given). 2. ∴ ∠AOD = ∠AOB (§14). 3. OD = OB (§195). 4. AO = AO (Iden.). 5. ∴ △ AOD ≅ △ AOB (§95a). 6. ∴ AD = AB (§ 97a). 7. ABCD is a rect. (Given). 8. ∴ ABCD is a square (§185).

4. Given the △ ABC with median CO, COD a st. line, CO = OD, and AD and BD.



To prove that ADBC is a ▱.

Planning the Proof: 1. We can prove that a quad. is a ▱ by §§185, 197, 198, 199. 2. We shall use §199.

Proof: 1. AO = OB and CO = OD (Given). 2. ∴ ADBC is a ▱ (§199).

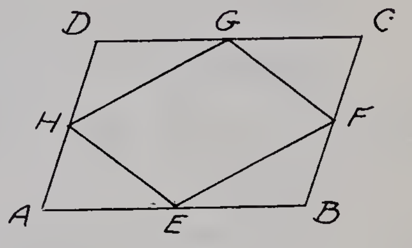
5. Three statements that can be proved are (a) AGFE and EFHD

are ▱s, (b) GF = FH, and (c) △GBF ≅ △FHC. The students will probably find others.

Planning the Proof: 1. We can prove (a) that a quad. is a ▱ by §§ 185, 197, 198, 199; (b) line segments = by §§ 97, 157, 188, 192, 193, 194, 195; (c) △s ≅ by §§ 95, 187. 2. We shall use (a) § 185; (b) § 97c; (c) § 95b.

Proof: 1. AB ∥ DH, EF ∥ AB, and GH ∥ AD (Given). 2. ∴ EF ∥ DH (§160e). 3. ∴ AGFE and EFHD are ▱s (§185). 4. AE = ED (Given). 5. AE = GF and ED = FH (§188). 6. ∴ GF = FH (Asmt. 7). 7. ∠CFH = ∠BFG (§50d). 8. ∠FHC = ∠FGB (§158a). 9. ∴ △GBF ≅ △FHC (§95b).

6. Given the ▱ ABCD with EF, FG, GH, and HE joining the midpoints of AB, BC, CD, and DA in order.

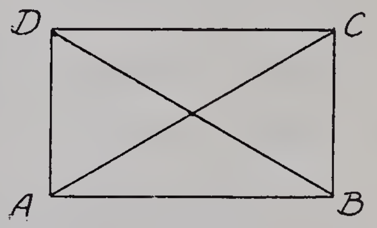


To prove that EFGH is a ▱.

Planning the Proof: 1. We can prove that a quad. is a ▱ by §§ 185, 197, 198, 199. 2. We shall use § 197.

Proof: 1. AD = BC (§188). 2. AH = HD and CF = FB (Given). 3. ∴ 2AH = 2CF (Asmt. 6). 4. ∴ AH = CF (Asmt. 4). 5. Similarly, AE = GC. 6. ∠A = ∠C (§189). 7. ∴ △AEH ≅ △FCG (§95a). 8. ∴ HE = GF (§97a). 9. Similarly, HG = EF. 10. ∴ EFGH is a ▱ (§197).

7. Given the ▱ ABCD with AC = BD.

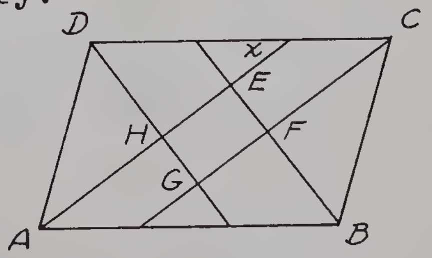


To prove that ▱ ABCD is a rect.

Planning the Proof: 1. We can prove that a ▱ is a rect. by §185. 2. We shall use §185.

Proof: 1. AC = BD (Given). 2. AD = BC (§188). 3. AB = AB (Iden.). 4. ∴ △ABC ≅ △BAD (§95c). 5. ∴ ∠DAB = ∠ABC (§98b). 6. ∠DAB + ∠ABC = a st. ∠ (§190). 7. ∴ 2∠DAB = a st. ∠ (Asmt. 6). 8. ∴ ∠DAB = a rt. ∠ (Asmt. 4). 9. ∴ ▱ ABCD is a rect. (§185).

8. (1) Given the ▱ ABCD not a rhombus with AE, BE, CG, and DG bisecting ∠s BAD, CBA, BCD, and CDA respectively.



To prove that HGFE is a rect.

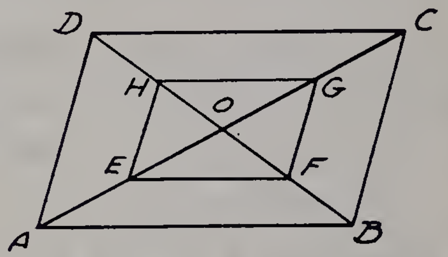
Planning the Proof: 1. We can prove that a quad. is a rect. by §185. 2. We shall use §185.

Proof: 1. ∠DAB = ∠BCD (§189). 2. ∠DAH = ∠BAH and ∠DCF = ∠BCF (Given). 3. ∴ ∠BAH = ∠DCF

(Asmt. 9, 6, and 4). 4. ∠BAH = ∠x (§158a). 5. ∴ ∠DCF = ∠x (Asmt. 7). 6. ∴ HE ∥ GF (§160b). 7. Similarly, EF ∥ HG. 8. ∴ HGFE is a ▱ (§185). 9. ∠BAD and ∠ADC are supp. (§190). 10. ∠DAH and ∠ADH are comp. (Asmt. 6 and 4). 11. ∠DAH + ∠ADH + ∠DHA = a st. ∠ (§120). 12. ∴ ∠DHA = a rt. ∠ (Asmt. 2). 13. ∠EHG = ∠DHA (§50d). 14. ∴ ∠EHG = a rt. ∠ (Asmt. 6). 15. ∴ ▱HGFE is a rect. (§185).

(2) A rhombus has only two angle bisectors since the bisector of one angle also bisects the opposite angle.

9. Given the ▱ABCD with E, F, G, and H, the midpoints of the half diagonals AO, BO, CO, and DO respectively.



To prove that EFGH is a ▱.

Planning the Proof: 1. (Same as in Ex. 6). 2. We shall use §199.

Proof: 1. AO = CO, DO = BO, AE = EO, BF = FO, GC = GO, and DH = HO (Given). 2. ∴ 2EO = 2GO and 2FO = 2HO (Asmt. 9, 6). 3. ∴ EO = GO and FO = HO (Asmt. 4). 4. ∴ EFGH is a ▱ (§199).

10. Planning the Proof: (Same as in Ex. 6).

Proof: 1. ABCD is a ▱ (Given). 2. AB = DC (§188). 3. EA = CG (Given). 3. EA + AB = CG + DC (Asmt. 1). 4. EB = DG (Asmt. 9, 6). 5. BF = HD (Given). 6. ∠EBF = ∠HDG (§158b). 7. △EBF ≅ △GDH (§95a). 8. EF = HG (§97a). 9. In like manner, EH = FG (Statements 2-8). 10. EFGH is a ▱ (§197).

11. Planning the Proof: 1. We can prove line segments = by §§97, 157, 188, 192-195. 2. We shall use §§97a, 157a; Asmt. 7.

Proof: 1. ABCD is a square (Given). 2. ∠s ADC and BCD are rt. ∠s (§191). 3. Draw SB (Asmt. 11). 4. SR ⊥ DB (Given). 5. ∠s DRS and SRB are rt. ∠s (§14). 6. △s BRS and BCS are rt. △s (§57). 7. BR = BC (Given). 8. BS = BS (Iden.). 9. △BRS ≅ △BCS (§156b). 10. RS = SC (§97a). 11. DB bisects ∠ADC (See Ex. 1, p. 202 and Ex. 7, p. 206). 12. ∠BDC = 1/2 a rt. ∠ (Asmt. 4). 13. ∠DRS + ∠RDS + ∠DSR = a st. ∠ (§120). 14. ∠DSR = 1/2 a rt. ∠ (Asmt. 2). 15. DR = RS (§157a). 16. DR = RS = SC (Asmt. 7).

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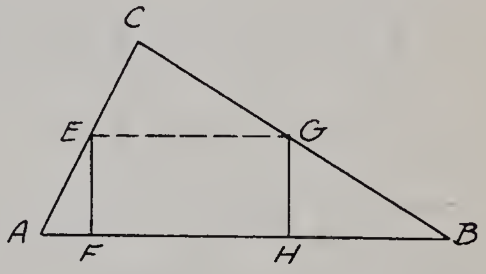
1. DE = 13 (§204). 2. AB = 26.5 (§204). 3. 4, 5, 6.5 (§204).

4. *Planning the Proof*: 1. We can prove line segments = by §§97, 157, 188, 192-195, 202-204. 2. We shall use (a) § 97a, (b) § 204, (c) Asmt. 6.

Proof: 1. $AB \parallel EF$, D is the midpoint of BF, and C is the midpoint of AE (Given). 2. BD = DF (§ 22). 3. $\angle ADB = \angle FDG$ (§ 50d). 4. $\angle ABD = \angle GFD$ (§ 158a). 5. $\triangle ABD \cong \triangle FDG$ (§ 95b). 6. AD = DG (§ 97a). 7. D is the midpoint of AG (§ 22). 8. CD = 1/2 EG (§ 204). 9. CD = 1/2(EF + FG) (Asmt. 6). 10. AB = FG (§ 97a). 11. AB = 1/2(EF + AB) (Asmt. 6).

5. *Given* the $\triangle ABC$ with E the midpoint of AC, G the midpoint of BC, $EF \perp AB$, and $GH \perp AB$.

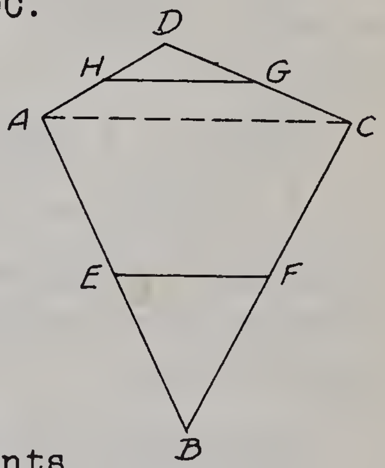


To prove that EF = GH.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 188, 192-195, 202-204. 2. We shall use § 193.

Proof: 1. E and G are the midpoints of AC and BC, respectively (Given). 2. Draw EG (Asmt. 11). 3. $EG \parallel AB$ (§ 204). 4. $EF \perp AB$ and $GH \perp AB$ (Given). 5. EF = GH (§ 15, § 193).

6. *Given* the quad. ABCD with EF joining the midpoints of AB and BC, and HG joining the midpoints of AD and DC.

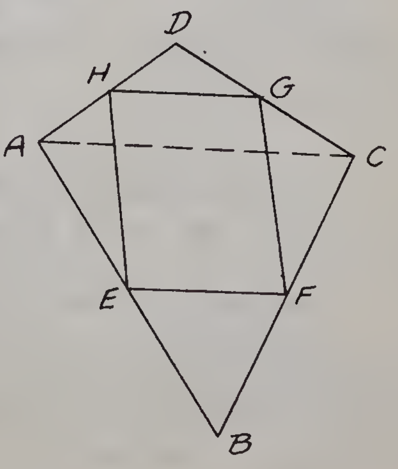


To prove that $HG \parallel EF$ and HG = EF.

Planning the Proof: 1. We can prove (a) lines $\parallel$ by §§160, 185, 204; (b) line segments = by §§97, 157, 188, 192-195, 202-204. 2. We shall use (a) § 204; (b) § 204.

Proof: 1. Draw AC (Asmt. 11). 2. EF joins the midpoints of AB and BC (Given). 3. $EF \parallel AC$ and EF = 1/2 AC (§ 204). 4. HG joins the midpoints of AD and DC (Given). 5. $HG \parallel AC$ and HG = 1/2 AC (§ 204). 6. $HG \parallel EF$ (§ 160e). 7. HG = EF (Asmt. 7).

7. *Given* the quad. ABCD with EF, FG, GH, and HE, joining the midpoints of AB, BC, CD, and DA in order.



To prove that EFGH is a ▱.

Planning the Proof: 1. We can prove that a quad. is a ▱ by §§ 185, 197-199. 2. We shall use § 198.

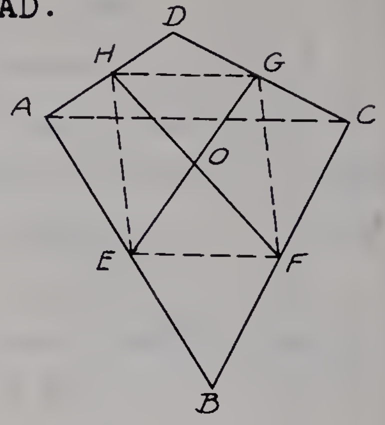
Proof: 1-7. (The proof follows Statements 1-7 in Ex. 6). 8. ∴ EFGH is a ▱ (§ 198).

8. Given the quad. ABCD with EG joining the midpoints of AB and CD, and HF joining the midpoints of BC and AD.

To prove that EG and HF bisect each other.

Planning the Proof: 1. We can prove line segments bisect each other by §§195, 202, 203. 2. We shall use §195.

Proof: 1-8. (The proof follows Statements 1-8 of Ex. 7.) 9. ∴ EG and HF bisect each other (§195).



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1. Median $= \frac{1}{2}(8 + 10) = 9$ (§207). 9 inches. Ans.

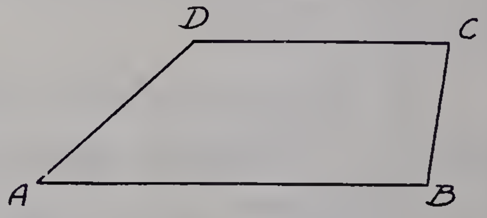
2. Median $= \frac{1}{2}(12 + 16) = 14$ (§207). 14 inches. Ans.

3. $m = \frac{1}{2}(b + b')$ (§207). $18 = \frac{1}{2}(b + \frac{38}{3})$ (Asmt. 6); whence $108 = 3b + 38$, and $b = 23\frac{1}{3}$. 23 feet 4 inches. Ans.

4. Given the trapezoid ABCD with AB ∥ DC and ∠s A and D adj. to AD, one of the nonparallel sides.

To prove that ∠s A and D are supp.

Planning the Proof: 1. We can prove ∠s supp. by §§53, 162, 190. 2. We shall use §162.

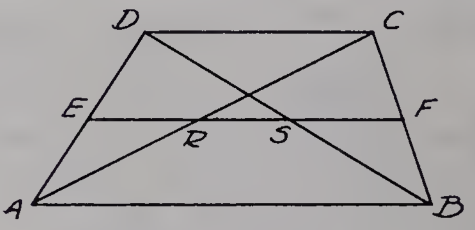


Proof: 1. AB ∥ DC (Given). 2. ∴ ∠s A and D are supp. (§162).

5. Given the trapezoid ABCD with AB and DC the ∥ sides, AC and BD the diagonals, and EF the median intersecting the diagonals in R and S respectively.

To prove that AR = RC and BS = SD.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 188, 192-195, 202-205. 2. We shall use §202.

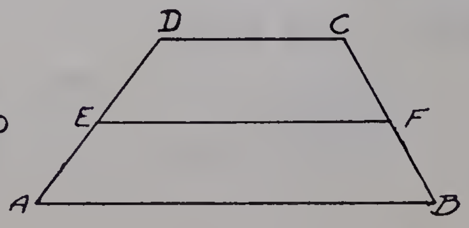


Proof: 1. EF is the median (Given). 2. AE = ED and BF = CF (§206, §22). 3. EF ∥ AB ∥ DC (§207, Given). 4. AR = RC and BS = SD (§202).

6. Given the trapezoid ABCD with AB and DC the ∥ sides, AE = ED, and EF ∥ DC and AB.

To prove that BF = FC.

Planning the Proof: 1. We can prove two line segments are equal by §§97, 157, 188, 192-195, 202-205. 2. We shall use §202.

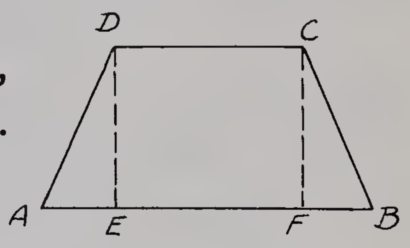


Proof: 1. $AB \parallel EF \parallel DC$ and EF bisects AD (Given). 2. $AE = ED$ (§22). 3. $BF = FC$ (§202).

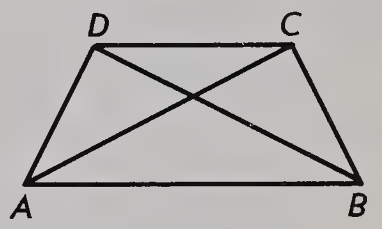
7. Since any two slats of the gate are so joined that they bisect each other, the figure formed by joining their endpoints is a ▱ (§199).

8. Given trapezoid ABCD having $AB \parallel DC$ and $AD = BC$.

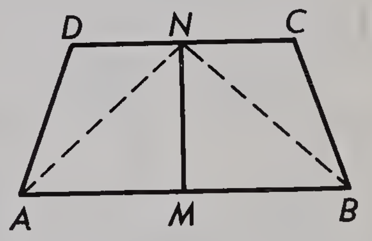
To prove: (a) $\angle A = \angle B$ and $\angle D = \angle C$; (b) diagonal AC = diagonal BD; and (c) MN, the $\perp$ bisector of AB, is also the $\perp$ bisector of DC. (Other statements can also be proved.)



Proof (a): 1. Draw $DE \perp AB$ and $CF \perp AB$ (§178a). 2. $\triangle ADE$ and $\triangle BCF$ are rt. $\triangle$s (§§14, 57). 3. $DE = CF$ (§193). 4. $AD = BC$ (Given). 5. $\triangle ADE \cong \triangle BCF$ (§156b). 6. $\angle A = \angle B$ (§98b). 7. $\angle ADC$ is the supp. of $\angle A$ and $\angle BCD$ is the supp. of $\angle B$ (§113). 8. $\angle ADC = \angle BCD$ (Asmt. 31).

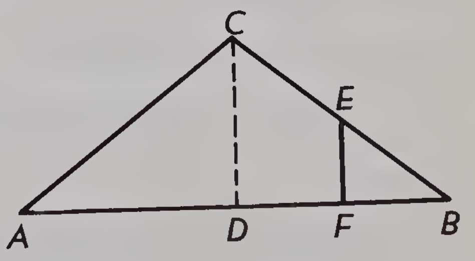


Proof (b): 1. $AD = BC$ (Given). 2. $\angle DAB = \angle CBA$ (Proved in (a)). 3. $AB = AB$ (Iden.). 4. $\triangle DAB \cong \triangle CBA$ (§95a). 5. $AC = DB$ (§97a).



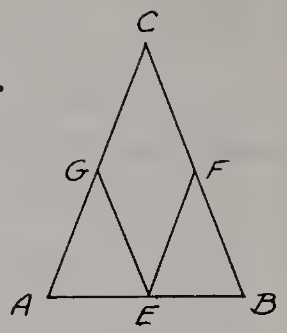
Proof (c): 1. Let N be the point in which MN, the $\perp$ bisector of AB, intersects CD (§110). 2. Draw NA and NB (Asmt. 11). 3. $NA = NB$ (§97b). 4. $\angle NAB = \angle NBA$ (§98a). 5. $\angle DAB = \angle CBA$ (Proved in (a)). 6. $\angle DAN = \angle CBN$ (Asmt. 2). 7. $AD = BC$ (Given). 8. $\triangle ADN \cong \triangle BCN$ (§95a). 9. $DN = NC$ (§97a). 10. MN bisects DC (§22). 11. $DC \parallel AB$ (Given). 12. $MN \perp DC$ (§159a). 13. MN is the $\perp$ bisector of CD (Statements 9, 11).

9. Planning the Proof: 1. We can prove one line segment a fractional part of another by Asmt. 4, §§203, 204, 207. 2. We shall use Asmt. 4.



Proof: 1. Draw $CD \perp AB$ (§178a). 2. $AD = BD$ (See proof of Ex. 5, p. 162). 3. $AD + DB = AB$ (Asmt. 9). 4. $2DB = AB$ (Asmt. 6). 5. $DB = 1/2\ AB$ (Asmt. 4). 6. $EF \perp AB$ (Given). 7. $EF \parallel CD$ (§160d). 8. $BE = EC$ (Given). 9. $FB = DF$ (§202). 10. $DF + FB = DB$ (Asmt. 9). 11. $2FB = DB$ (Asmt. 6). 12. $FB = 1/2\ DB$ (Asmt. 4). 13. $FB = 1/2\ (1/2\ AB) = 1/4\ AB$ (Asmt. 6).

10. Given the isos. $\triangle ABC$ with $AC = BC$, EF joining the midpoints of AB and BC, and EG joining the midpoints of AB and AC.



To prove that EFCG is a rhombus.

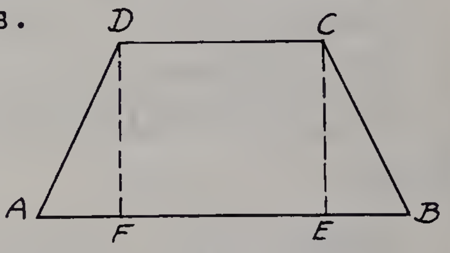
Planning the Proof: 1. We can prove that a quad. is a rhombus by §185. 2. We shall use §185.

Proof: 1. GE ∥ BC and FE ∥ AC (§204). 2. EFCG is a ▱ (§ 185). 3. EG = 1/2 BC and EF = 1/2 AC (§ 204). 4. AC = BC (Given). 5. 1/2 AC = 1/2 BC (Asmt. 4). 6. EG = EF (Asmt. 7). 7. EFCG is a rhombus (§ 185).

11. Planning the Proof: 1. We can prove line segments = by §§97, 157, 188, 192-195, 202-205. 2. We shall use §205.

Proof: 1. AD and BE are altitudes (Given). 2. ∠ AEB and ∠ BDA are rt. ∠s (§§90, 14). 3. △AEB and △ADB are rt. △s (§57). 4. M is the midpoint of AB (Given). 5. AM = BM (§ 22). 6. EM = AM = BM and DM = AM = BM (§ 205). 7. EM = DM (Asmt. 7).

12. If the nonparallel sides of a trapezoid make equal angles with either base, the trapezoid is isosceles.



Given the trapezoid ABCD with AB ∥ DC, and ∠A = ∠B or ∠D = ∠C.

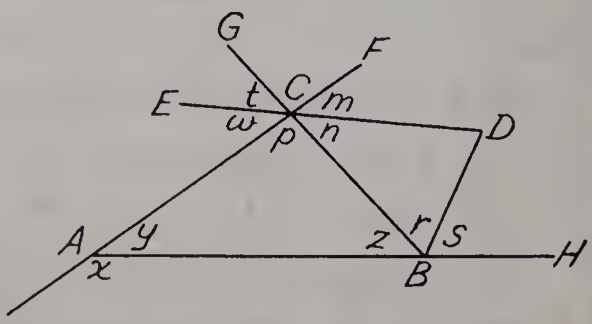
To prove that trapezoid ABCD is isos.

Planning the Proof: 1. We can prove that a trapezoid is isos. by § 206. 2. We shall use § 206.

Proof: Case I. When ∠A = ∠B. 1. Draw CE ⊥ AB and DF ⊥ AB (§178a). 2. ∴ ∠ DFA = ∠CEB (§§14, 50a). 3. DF = CE (§193). 4. ∠A = ∠B (Given). 5. ∴ ∠ ADF = ∠ BCE (§ 158c). 6. ∴ △ AFD ≅ △ BEC (§ 95b). 7. ∴ AD = BC (§ 97a). 8. ∴ Trapezoid ABCD is isos. (§ 206).

Case II. When ∠D = ∠C. 1. Draw CE ⊥ AB and DF ⊥ AB (§ 178a) 2. ∴ CE ⊥ DC and DF ⊥ DC (§ 159a). 3. ∴ ∠ DFA = ∠ CEB and ∠ FDC = ∠ECD (§§ 14, 50a). 4. ∠ADC = ∠BCD (Given). 5. ∴ ∠ADF = ∠ BCE (Asmt. 2). 6. DF = CE (§ 193). 7. ∴ △ ADF ≅ △ BCE (§95b). 8. ∴ AD = BC (§97a). 9. ∴ Trapezoid ABCD is isos. (§ 206).

13. DB = DC and DE = DC (§ 205). ∴ DB = DE = 12 in. (Asmt. 7 and 6).



14. Given △ABC, with EC bisecting ∠ ACG, BD bisecting ∠CBH, ext. ∠ x, and EC intersecting BD in D.

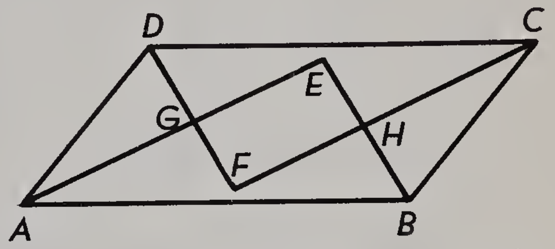
To prove that $\angle D = \frac{1}{2}\angle x$.

Planning the Proof: 1. We can prove angles = by §§ 50, 98, 158, 189. 2. We shall use § 50e.

Proof: 1. $\angle t = \angle w$ and $\angle r = \angle s$ (Given). 2. $\angle w = \angle m$, $\angle t = \angle n$ (§50d). 3. $\angle m = \angle n$ (Asmt. 6). 4. $\angle r + \angle s = \angle y + \angle p$ (§123). 5. $\angle m + \angle n = \angle y + \angle z$ (§123). 6. $\angle x = \angle p + \angle z$ (§123). 7. $2\angle r + 2\angle n + \angle x = 2\angle y + 2\angle p + 2\angle z$ (Asmt. 1). 8. $\angle r + \angle n + \frac{1}{2}\angle x = \angle y + \angle p + \angle z$ (Asmt. 4). 9. a st. $\angle$ = $\angle y + \angle p + \angle z$ (§120). 10. $\angle r + \angle n + \frac{1}{2}\angle x$ = a st. $\angle$ (Asmt. 6). 11. $\angle D + \angle n + \angle r$ = a st. $\angle$ (§120). 12. $\angle D + \angle n + \angle r = \angle n + \angle r + \frac{1}{2}\angle x$ (Asmt. 7). 13. $\angle D = \frac{1}{2}\angle x$ (Asmt. 2).

15. (a) Given the ▱ ABCD not a rhombus with AE, BE, CF, and DF bisecting $\angle$s BAD, ABC, BCD, and CDA, respectively.

To prove that FHEG is a rectangle.



Planning the Proof: 1. We can prove that a quad. is a rectangle by §185. 2. We shall use §185.

Proof: 1. $\angle DAB = \angle BCD$ (§189). 2. $\angle DAG = \angle BAG$ and $\angle DCH = \angle BCH$ (Given). 3. ∴ $\angle BAG = \angle DCH$ (Asmt. 9, 6, and 4). 4. $\angle BAG = \angle x$ (§158a). 5. $\angle DCH = \angle x$ (Asmt. 7). 6. ∴ GE ∥ FH (§160b). 7. Similarly, GF ∥ EH. 8. ∴ FHEG is a ▱ (§185). 9. $\angle BAD$ and $\angle ADC$ are supp. (§190). 10. $\angle DAG$ and $\angle ADG$ are comp. (Asmt. 6 and 4). 11. $\angle DAG + \angle AGD + \angle ADG$ = a st. $\angle$ (§120). 12. ∴ $\angle AGD$ = a rt. $\angle$ (Asmt. 2). 13. $\angle EGF = \angle AGD$ (§50d). 14. $\angle EGF$ = a rt. $\angle$ (Asmt. 6). 15. ∴ ▱ FHEG is a rect. (§185).

(b) In a rhombus the bisector of one $\angle$ is also the bisector of the opposite $\angle$, therefore, instead of four bisectors, a rhombus has only two. Two lines cannot form a rectangle (§185).

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1. Let n = the no. of sides in the polygon. (1) $\frac{360°}{n}$. (2) 360°.

2. (1) Triangle, 360°. (2) Quad., 360°. (3) Pentagon, 360°. (4) Hexagon, 360°. (5) Heptagon, 360°. (6) Octagon, 360°. (7) Nonagon, 360°. (8) Decagon, 360°.

3. Two $\angle$s of a rt. △ are acute (§122). Hence the 2 ext. $\angle$s that are adj. to the acute $\angle$s of a rt. △ are obtuse (§53b).

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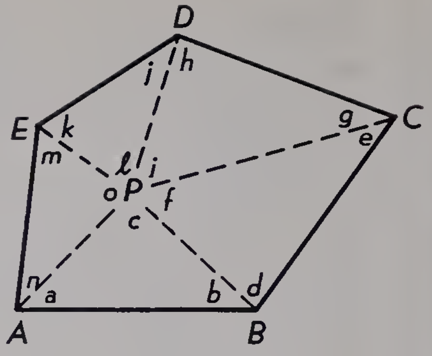
1. Given the polygon ABCDE with angles A, B, C, D, and E.

To prove $\angle A + \angle B + \angle C + \angle D + \angle E = (n - 2)$ st. $\angle$s.

Planning the Proof: 1. Same as for Th. 29. 2. We shall use §120, Asmt. 22.



Proof: 1. From P any point within ABCDE draw PA, PB, PC, PD, and PE forming $\angle$s a, b, c, $\cdots$ as shown (Asmt. 11). 2. $\angle a + \angle b + \angle c =$ a st. $\angle$, $\angle d + \angle e + \angle f =$ a st. $\angle$, etc. (§120). 3. $\angle a + \angle b + \angle c + \angle d + \angle e + \angle f + \angle g + \cdots = n$ st. $\angle$s (Asmt. 1). 4. $\angle c + \angle f + \angle i + \angle l + \angle o = 2$ st. $\angle$s (Asmt. 22). 5. $\angle a + \angle b + \angle d + \angle e + \angle g + \angle h + \angle j + \angle k + \angle m + \angle n = (n - 2)$ st. $\angle$s (Asmt. 2). 6. But $\angle a + \angle n = \angle A$, $\angle b + \angle d = \angle B$, etc. (Asmt. 9). 7. $\angle A + \angle B + \angle C + \angle D + \angle E = (n - 2)$ st. $\angle$s (Asmt. 6).

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1. (1) 360° (§208). (2) 360° (§208). (3) 360° (§208).

2. Let x = the no. of degrees in the $\angle$ in each case.
(1) $x = \frac{360}{n} = \frac{360}{3} = 120°$. (2) $x = \frac{360}{n} = \frac{360}{5} = 72°$.
(3) $x = \frac{360}{n} = \frac{360}{6} = 60°$.

3. Let S = the sum of the interior $\angle$s in each case. From §210 we have $S = (n - 2)$ st. $\angle$s. (1) $S = (4 - 2)$ st. $\angle$s $= 2$ st. $\angle$s; 360°. (2) $S = (5 - 2)$ st. $\angle$s $= 3$ st. $\angle$s; 540°. (3) $S = (6 - 2)$ st. $\angle$s $= 4$ st. $\angle$s; 720°. (4) $S = (7 - 2)$ st. $\angle$s $= 5$ st. $\angle$s; 900°. (5) $S = (8 - 2)$ st. $\angle$s $= 6$ st. $\angle$s; 1080°. (6) $S = (9 - 2)$ st. $\angle$s $= 7$ st. $\angle$s; 1260°. (7) $S = (10 - 2)$ st. $\angle$s $= 8$ st. $\angle$s; 1440°.

4. Let x = the no. of degrees in each int. $\angle$ in each case. From §211, we have $x = \frac{180(n - 2)}{n}$. (1) $x = \frac{180(5 - 2)}{5} = 108°$.
(2) $x = \frac{180(6 - 2)}{6} = 120°$. (3) $x = \frac{180(7 - 2)}{7} = 128\frac{4}{7}°$.
(4) $x = \frac{180(8 - 2)}{8} = 135°$. (5) $x = \frac{180(15 - 2)}{15} = 156°$.

5. $S = (n - 2)$ st. $\angle$s (§210). (1) $S = (4 - 2)180 = 360°$. (2) 360°. (3) 360°.

6. The sum of the int. $\angle$s of a trapezoid is 360° (Ex. 5). Then 360 − 290 = 70°.

7. (1) Decreases. (2) Increases.

8. The student should find that $\angle B + \angle C + \angle D + \angle A = 128° + 102° + 56° + 72° = 358°$. The sum of the int. $\angle$s of a

quad. = 360° (§208). Hence, there is an error in the size of at least one $\angle$.

9. Let x = the no. of degrees in the small $\angle$, and $3x$ = the no. of degrees in the large $\angle$. Then $x + 3x + 84 = 180$ (§120); $4x = 96$ (Asmt. 2); and $x = 24$ (Asmt. 4). $\therefore$ The small $\angle$ contains 24° and the large angle 72°.

10. The median to the hypotenuse is equal to 1/2 the hypotenuse (§205). The hypotenuse = 8 in. (Given). $\therefore$ The median = 1/2 of 8 in., or 4 in. (Asmt. 4, 6).

11. (1) $40 = \frac{360}{n}$ (§209); whence $n = 9$.
(2) $45 = \frac{360}{n}$; whence $n = 8$. (4) $51\frac{3}{7} = \frac{360}{7}$; $\frac{360}{7} = \frac{360}{n}$; $n = 7$.
(3) $60 = \frac{360}{n}$; whence $n = 6$. (5) $120 = \frac{360}{n}$; whence $n = 3$.

12. (1) $180° - 108° = 72°$. $72 = \frac{360}{n}$ (§209); whence $n = 5$.
(2) $180° - 144° = 36°$. $36 = \frac{360}{n}$; whence $n = 10$.
(3) $180° - 150° = 30°$. $30 = \frac{360}{n}$; whence $n = 12$.
(4) $180° - 162° = 18°$. $18 = \frac{360}{n}$; whence $n = 20$.

13. (1) $540 = (n - 2)180$ (§210); whence $n = 5$.
(2) $900 = (n - 2)180$; whence $n = 7$.
(3) $720 = (n - 2)180$; whence $n = 6$.
(4) $7 = n - 2$; whence $n = 9$.

14. The sum of the 3 ext. $\angle$s = 360° (§208). The sum of 2 ext. $\angle$s = 200° (Given). Then the 3rd ext. $\angle$ = 160° (Asmt. 2).

15. The sum of the int. $\angle$s of an octagon = $(8 - 2)180° = 1080°$ (§210). The sum of 7 $\angle$s of the octagon = 1000° (Given). Then the no. of degrees in the remaining $\angle$ = $1080 - 1000 = 80°$ (Asmt. 2).

16. $22.5 = \frac{360}{n}$ (§209); whence $n = 16$.

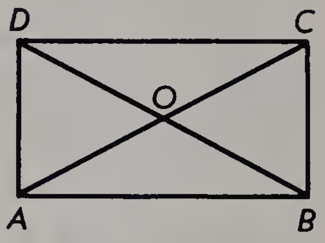
17. (1) 60°. (2) 120°.

18. The sum of the int. $\angle$s = $(n - 2)180$ (§210). The sum of the ext. $\angle$s = 360 (§208). Then $(n - 2)180 = 2(360)$; whence $n = 6$.

19. Let n = the no. of sides of the original polygon and S the sum of its $\angle$s. Then $n + 4$ = the no. of sides of the polygon having four additional sides. Let S' = the sum of the $\angle$s of this

polygon. S = (n − 2) st. ∠s and S' = (n + 4 − 2) st. ∠s (§120). S' − S = (n + 2) st. ∠s − (n − 2) st. ∠s (Asmt. 2, 6). Then S' − S = 4 st. ∠s or 720° (Asmt. 6). ∴ the sum is increased by 720°.

20. Planning the Proof: 1. We can prove that a △ is isosceles by §56. 2. We shall use §56.



Proof: 1. AD = BC, AC = BD, and AC and BD intersect in O (Given). 2. AB = AB (Iden.). 3. △ABD ≅ △ACB (§95c). 4. △CAB = ∠DBA (§98b). 5. AO = BO (§157a). 6. △AOB is isosceles (§56).

21. Given the polygon ABCDEF with n sides.

To prove that ∠A + ∠B + ∠C + ∠D + ∠E + ∠F = (n − 2) st. ∠s.

Planning the Proof: 1. We can find the values of sums of ∠s by Asmt. 22, §§52, 53, 120, 123, 161, 162, 208, 210. 2. We shall use §120 and Asmt. 9, 3.

Proof: 1. From any vertex as A, draw diagonals AC, AD, and AE (Asmt. 11). 2. There is a △ for each side except the 2 adj. to ∠A (Asmt. 11). 3. ∴ There are (n − 2) △s (Asmt. 6, 2). 4. The sum of the ∠s of each △ = a st. ∠ (§120). 5. ∴ The sum of the ∠s in all the △s = (n − 2) st. ∠s (Asmt. 9, 3). 6. ∴ ∠A + ∠B + ∠C + ∠D + ∠E + ∠F = (n − 2) st. ∠s (Asmt. 6).

22. (1) Four, since 4 x 90° = 360°.
(2) Six, since 6 x 60° = 360°.
(3) A hexagon, since 3 x 120° = 360°.

23. The int. ∠ of a regular octagon = $\frac{180(8 - 2)^\circ}{8} = 135^\circ$ (§211). The int. ∠ of a square = 90°. Then (2 x 135°) + 90° = 270° + 90° = 360°.

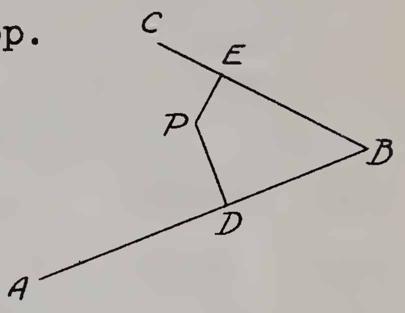
24. Planning the Proof: 1. We can prove line segments = by §§97, 157, 188, 192-195, 202-205. 2. We shall use §97a.

Proof: 1. Draw PH ⊥ AF (§178a). 2. AF ⊥ BC and PE ⊥ BC (Given). 3. HF ∥ PE (§107). 4. HP ⊥ AF (Const.). 5. HP ∥ FE (§107). 6. PEFH is a ▱ (§185). 7. PE = HF (§188). 8. DP ⊥ AC (Given). 9. ∠ADP and ∠AHP are rt. ∠s (§14). 10. △APD and △APH are rt. △s (§57). 11. AP = AP (Iden.). 12. AC = BC (Given). 13. ∠DAP = ∠B (§98a). 14. ∠APH = ∠B (§158a). 15. ∠APH = ∠DAP (Asmt. 7). 16. Rt. △APH ≅ rt. △APD (§156a). 17. AH = DP (§97a). 18. AH + HF = DP + PE (Asmt. 1). 19. AF = DP + PE (Asmt. 6, 9).

25. Given $\angle ABC$, P any point within $\angle ABC$, $PE \perp BC$, and $PD \perp AB$.

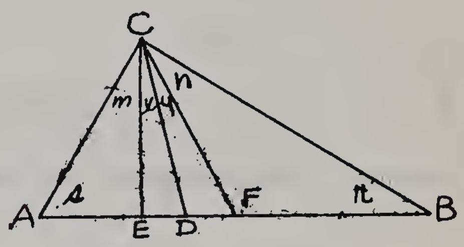
To prove that $\angle P$ and $\angle B$ are supp.

Planning the Proof: 1. We can prove $\angle$s supp. by §§ 53, 162, 190. 2. We shall use § 53a.



Proof: 1. $\angle P + \angle D + \angle B + \angle E = 2$ st. $\angle$s (§ 210). 2. $\angle E$ and $\angle D$ are rt. $\angle$s (Given). 3. $\therefore \angle P + \angle B =$ a st. $\angle$ (Asmt. 2). 4. $\therefore$ $\angle P$ and $\angle B$ are supp. (§ 53a).

26. Given the rt. $\triangle ABC$, CD bisecting the rt. $\angle ACB$, CE the altitude on AB, and CF the median on AB.



To prove that $\angle x = \angle y$.

Planning the Proof: 1. We can prove $\angle$s = by §§ 50, 98, 158, 189. 2. We shall use § 98a and Asmt.

Proof: 1. AF = FB = CF (§§ 89, 205). 2. $\angle n = \angle r$ (§ 98a). 3. $CE \perp AB$ (Given). 4. $\triangle CEB$ is a rt. $\triangle$ (§§ 14, 57). 5. $\angle ECB$ is comp. of $\angle r$ (§ 124). 6. $\angle m$ is comp. of $\angle ECB$ (§ 20). 7. $\angle m = \angle r$ (§ 50b). 8. $\angle m = \angle n$ (Asmt. 7). 9. $\angle m + \angle x = \angle y + \angle n$ (Given). 10. $\angle x = \angle y$ (Asmt. 2).

27. Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 188, 192-195, 202-205. 2. We shall use § 157 and Asmt. 1.

Proof: 1. $\angle EAD = \angle BAD$ and $\angle ABD = \angle FBD$ (Given). 2. $EF \parallel AB$ (Given). 3. $\angle BAD = \angle ADE$ and $\angle ABD = \angle FDB$ (§ 158a). 4. $\therefore \angle EAD = \angle ADE$ and $\angle FBD = \angle FDB$ (Asmt. 7). 5. $\therefore$ AE = ED and BF = FD (§ 157a). 6. $\therefore$ AE + BF = EF (Asmt. 1).

28. Let x and x + 80° represent the 2 consecutive $\angle$s. Then $x + x + 80° = 180°$ (§ 190); whence $2x = 100°$, and $x = 50°$. Then $x + 80° = 130°$. 130°, 50°, 130°, 50°. Ans.

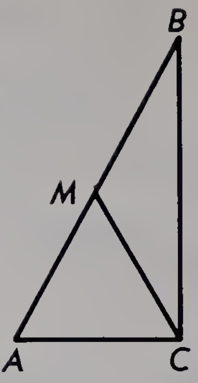
29. Planning the Proof: 1. We can prove line segments = as in Ex. 27). 2. We shall use: § 203, Asmt. 7.

Proof: 1. AE = ED and BF = FC (Given). 2. AD = BC (§ 188). 3. $\therefore$ 2AE = 2FC (Asmts. 9, 6). 4. $\therefore$ AE = FC (Asmt. 4). 5. $AE \parallel FC$ (§ 185). 6. $\therefore$ AFCE is a ▱ (§ 198). 7. $\therefore EC \parallel AF$ (§ 185). 8. $\therefore$ DM = MN and BN = MN (§ 203). 9. $\therefore$ DM = MN = BN (Asmt. 7).

30. If the midpoint of one side of a triangle is equidistant from the vertices of the triangle, the triangle is a right triangle.

Given $\triangle ABC$, M a point on AB, and MA = MC = MB.

To prove $\triangle ABC$ is a right $\triangle$.



Planning the Proof: 1. We can prove a $\triangle$ is a rt. $\triangle$ by § 57. 2. We shall use § 57.

Proof: 1. $\angle A + \angle ACB + \angle B = 180^\circ$ (§ 120). 2. $\angle ACB = \angle ACM + \angle MCB$ (Asmt. 9). 3. $MA = MC$ and $MC = MB$ (Given). 4. $\angle A = \angle ACM$ and $\angle B = \angle MCB$ (§98a). 5. $\angle ACM + \angle ACM + \angle MCB + \angle MCB = 180^\circ$ (Asmt. 6). 6. $2\angle ACM + 2\angle MCB = 180^\circ$ (Asmt. 6). 7. $\angle ACM + \angle MCB = 90^\circ$ (Asmt. 4). $\angle ACB = 90^\circ$ (Asmt. 6). 8. $\triangle ABC$ is a rt. $\triangle$ (§57).

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1. (1) Construct the $\perp$ bisector of AG (§ 171); (2) Using A as a center and GB as a radius draw an arc intersecting AB in H.

2. (1) (See Ex. 4, page 190). (2) Direct application of § 212.

3. Direct application of §212.

4. Use Prob. 10 to divide the given segment into 5 equal parts. The portion of the given segment containing 3 of these parts is 3/5 of the whole segment.

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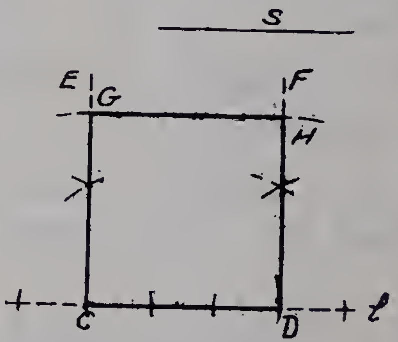
1. Given side s of a square.

To construct the square.

Construction: 1. On any line segment, ℓ, construct $CD = s$ (§ 9). 2. At C and D construct CE and DF $\perp \ell$ (§ 173). 3. Construct $CG = s$ and $DH = s$ (§ 9). 4. Draw GH (Asmt. 11).



Then CDHG is the required square.

Proof: 1. $CD = DH = CG = s$ (Const.). 2. $CG \perp \ell$ and $DH \perp \ell$ (Const.). 3. $\therefore CG \parallel DH$ (§160d). 4. $\therefore$ CDHG is a ▱ (§198). 5. CDHG is a rect. (§185). 6. $\therefore$ CDHG is a square (§ 185).

2. Given the perimeter p of a square.

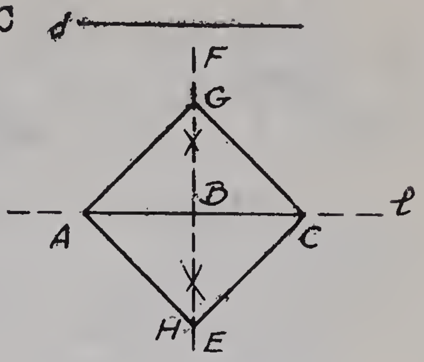
To construct the square.

Construction: 1. Divide p into 4 = parts (Ex. 4, page 190). 2. The remaining steps in the construction and proof are the same as in Ex. 1.

3. Given the diagonal d of a square.

To construct the square.

Construction: 1. On any line ℓ, construct $AC = d$ (§ 9).



2. Construct EF the ⊥ bisector of AC cutting AC at B (§171). 3. Construct GB = AB and BH = AB (§9). 4. Draw AH, HC, CG, and GA (Asmt. 11).

Then AHCG is the required square.

Proof: 1. AC = d (Const.). 2. AB = BC = BG = BH (Const.). 3. ∴ AHCG is a ▱ (§199). 4. △ ABG and CBG are rt. △ (Const.). 5. △ ABG and CBG are rt. △ (§57). 6. △ ABG ≅ △ CBG (§96d). 7. ∠BAG = ∠AGB = ∠BCG = ∠BGC (§98a and b). 8. ∠BAG + ∠AGB = a rt. ∠ (§124). 9. ∠AGB + ∠BGC = a rt. ∠ (Asmt. 9 and 7). 10. ∴ AHCG is a rect. (§185). 11. AG = GC (§97a). 12. ∴ AHCG is a square (§185).

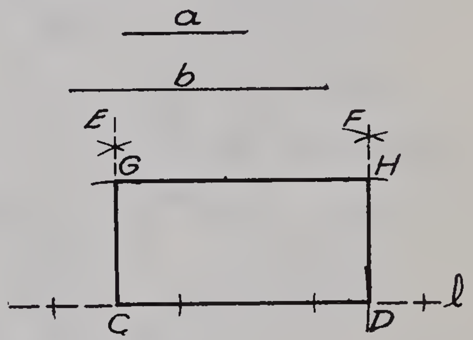
4. Given the line segment x = the sum of the diagonals of a square.

To construct the square.

Construction: (Since the diagonals of a square are = [Ex. 8, page 206], bisect x and proceed as in Ex. 3).

5. Given the 2 adj. sides a and b of a rect.

To construct the rect.



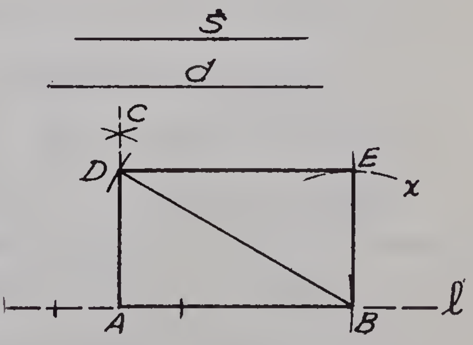
Construction: 1. On any line *l*, construct CD = b (§9). 2. At C, construct CE ⊥ CD and at D construct DF ⊥ CD (§173). 3. On CE construct CG = a and on DF construct DH = a (§9). 4. Draw GH (Asmt. 11).

Then CDHG is the required rect.

Proof: 1. CD = b and CG = a (Const.). 2. CG ⊥ CD and DH ⊥ CD (Const.). 3. ∴ CG ∥ DH (§160d). 4. CG = DH (Const.). 5. ∴ CDHG is a ▱ (§198). 5. ∴ CDHG is a rect. (§185).

6. Given side s and the diagonal d of a rect.

To construct the rect.

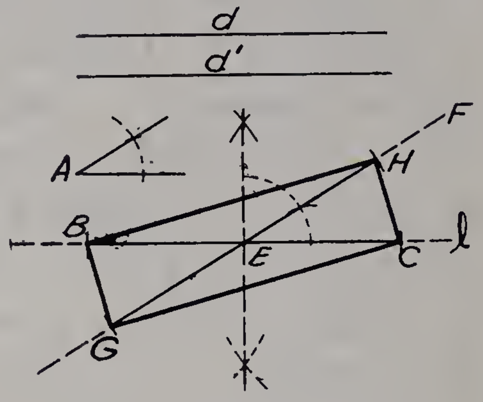


Construction: 1. On any line *l*, construct AB = s (§9). 2. At A construct AC ⊥ AB (§173). 3. With B as a center and d as a radius, draw an arc intersecting AC at D (Asmt. 15). 4. With B as a center and AD as a radius, draw an arc x (Asmt. 15). 5. With D as a center and s as a radius, draw an arc intersecting x at E (Asmt. 15). 6. Draw BD, DE, and BE (Asmt. 11).

Then ABED is the required rect.

Proof: 1. AB = s and BD = d (Const.). 2. DE = AB and BE = AD (Const.). 3. ∴ ABED is a ▱ (§197). 4. ∠BAD is a rt. ∠ (Const.). 5. ∴ ABED is a rect. (§185).

7. Given the diagonals d and d' of a rect. and ∠A formed by d and d'.



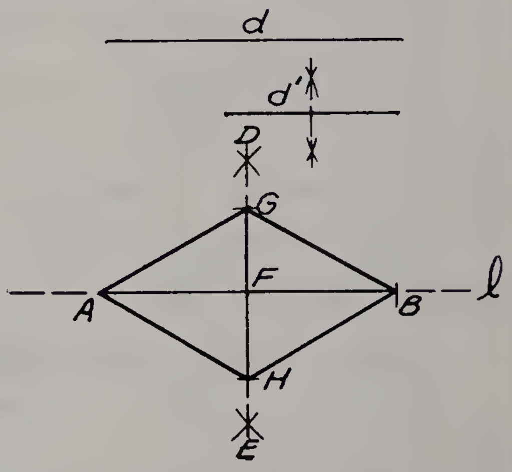
To construct the rect.

Construction: 1. The diagonals d and d' are = (Ex. 8, page 206). 2. On any line ℓ, construct BC = d (§9). 3. Bisect BC at E (§171). 4. At E construct ∠CEF = ∠A (§167). 5. Produce FE to G making EG = BE (Asmt. 13, § 9). 6. On EF construct EH = BE (§9). 7. Draw BG, GC, CH, and HB (Asmt. 11).

Then BGCH is the required rect.

Proof: 1. ∠CEH = ∠A and BC = d (Const.). 2. BE = EC = EH = EG (Const.). 3. ∴ BGCH is a ▱ (§199). 4. BC = GH (Asmt. 1). 5. ∴ BGCH is a rect. (Ex. 7, page 212).

8. Given the diagonals d and d' of a rhombus.



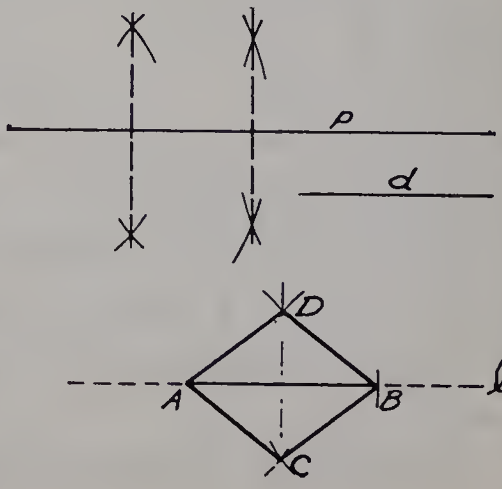
To construct the rhombus.

Construction: 1. On any line ℓ, construct AB = d (§9). 2. Construct DE the ⊥ bisector of AB, bisecting AB at F (§171). 3. Bisect d' (§171). 4. Construct FG = $\frac{1}{2}$d' and FH = $\frac{1}{2}$d' (§9). 5. Draw AG, GB, BH, and HA (Asmt. 11).

Then AHBG is the required rhombus.

Proof: 1. AB = d and GH = d' (Const.). 2. AF = FB and GF = HF (Const.). 3. ∴ AHBG is a ▱ (§199). 4. DE ⊥ AB (Const.). 5. AG = GB (Ex. 13, page 115). 6. ∴ ▱ AHBG is a rhombus (§185).

9. Given the perimeter p and diagonal d of a rhombus.



To construct the rhombus.

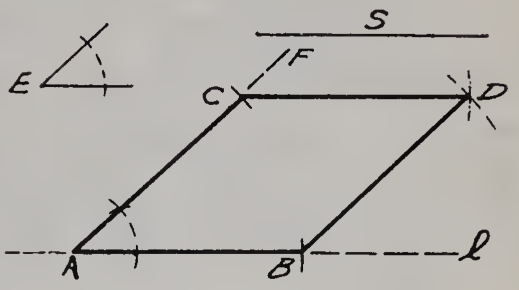
Construction: 1. Bisect p and then bisect one of the = segments (§171). 2. On any line ℓ construct AB = d (§9). 3. On AB as a base construct an isos. △ABC with AC = BC = $\frac{1}{4}$p (§165). 4. On AB as a base, construct an isos. △ADB with AD = BD = $\frac{1}{4}$p (§165).

Then ACBD is the required rhombus.

Proof: 1. AB = d, and AD = DB = BC = CA = $\frac{1}{4}p$ (Const.). 2. ∴ AD + DB + BC + CA = p (Asmt. 1). 3. ACBD is a ▱ (§197). 4. ACBD is a rhombus (§185).

10. Given side s and ∠E of a rhombus.

To construct the rhombus.



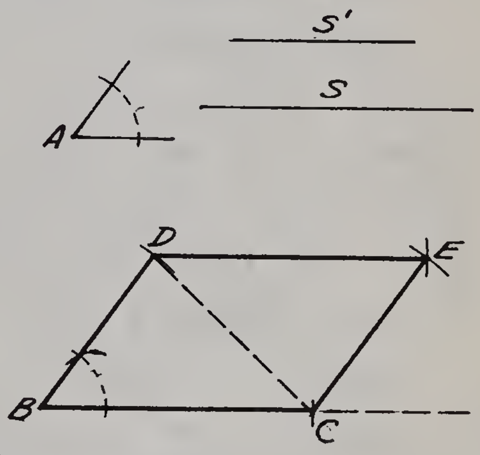
Construction: 1. On any line ℓ construct AB = s (§ 9). 2. At A construct ∠BAF = ∠E (§ 167). 3. On AF construct AC = s (§ 9). 4. With C and B as centers and a radius = s, draw arcs intersecting at D (Asmt. 15). 5. Draw CD and BD.

Then ABDC is the required rhombus.

Proof: 1. ∠A = ∠E and AC = BD = AB = CD (Const.). 2. ∴ ABDC is a ▱ (§197). 3. ∴ ABDC is a rhombus (§ 185).

11. Given 2 adj. sides s and s' of a ▱ and the ∠A included by s and s'.

To construct the ▱.



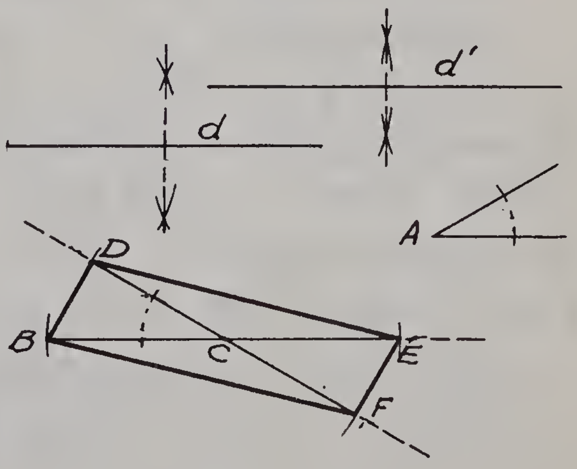
Construction: 1. Construct △BCD having BC = s, BD = s', and ∠B = ∠A (§169). 2. Construct △CDE having DE = s, and CE = s' (§165).

Then BCED is the required ▱.

Proof: 1. BC = s, BD = s', DE = s, CE = s', and ∠B = ∠A (Const.). 2. ∴ BC = DE and CE = BD (Asmt. 7). 3. ∴ BCED is a ▱ (§197).

12. Given the diagonals d and d' of a ▱ and the ∠A formed by d and d'.

To construct the ▱.



Construction: 1. Bisect d and d' (§171). 2. Construct △BCD having BC = $\frac{1}{2}$d, CD = $\frac{1}{2}$d', and ∠BCD = ∠A (§169). 3. Extend BC and construct CE = $\frac{1}{2}$d (Asmt. 13, §9). 4. Extend DC and construct CF = $\frac{1}{2}$d' (Asmt. 13, §9). 5. Draw ED, EF, and FB (Asmt. 11).

Then BFED is the required ▱.

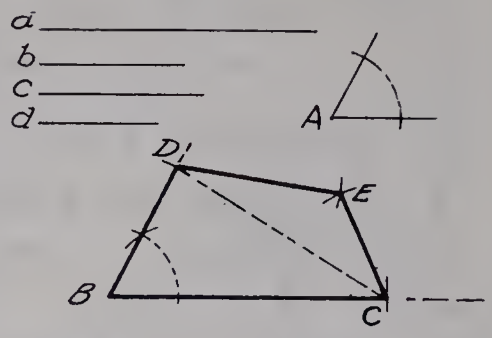
Proof: 1. BC = $\frac{1}{2}$d, CE = $\frac{1}{2}$d, DC = $\frac{1}{2}$d', CF = $\frac{1}{2}$d', and ∠BCD = ∠A (Const.). 2. ∴ BE = d and DF = d' (Asmt. 1). 3. BC = CE and DC = CF (Asmt. 7). 4. ∴ BFED is a ▱ (§199).

13. Given the 4 sides a, b, c, and d of a quad., and $\angle A$ the $\angle$ formed by a and b.

To construct the quad.

Construction: 1. Construct $\triangle BCD$ having $BC = a$, $BD = b$, and $\angle B = \angle A$ (§169). 2. Construct $\triangle DCE$ having $DE = c$, and $CE = d$ (§ 165).

Then BCED is the required quad.



Proof: 1. $BC = a$, $BD = b$, $\angle B = \angle A$, $DE = c$, and $CE = d$ (Const.). 2. Hence BCED is the required quad.

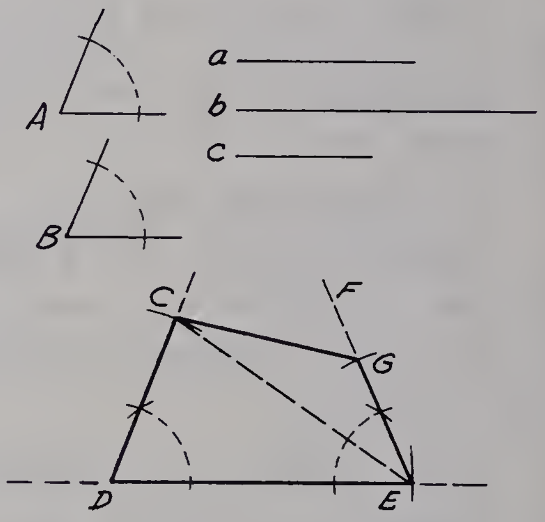
Discussion. $c + d$ must be $> DC$ or the construction is impossible.

14. Given $\angle A$ and $\angle B$, 2 adj. $\angle$s of a quad., and a, b, and c, the 3 sides which include $\angle$s A and B respectively.

To construct the quad.

Construction: 1. Construct $\triangle DEC$ having $DC = a$, $DE = b$, and $\angle D = \angle A$ (§169). 2. At E construct $\angle DEF = \angle B$ (§167). 3. On EF construct $EG = c$ (§9). 4. Draw GC (Asmt. 11).

Then DEGC is the required quad.



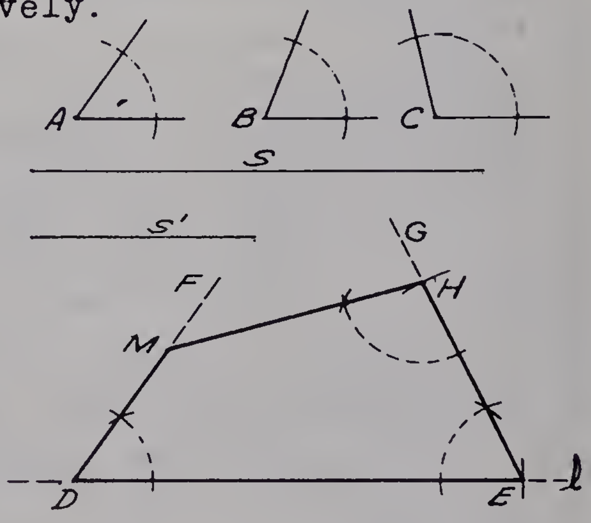
Proof: 1. $DC = a$, $\angle D = \angle A$, $DE = b$, $\angle DEG = \angle B$, and $EG = c$ (Const.). 2. Hence DEGC is the required quad.

15. Given $\angle A$, $\angle B$, and $\angle C$, 3 $\angle$s of a quad., and sides s and s', included by $\angle$s A, B, and C respectively.

To construct the quad.

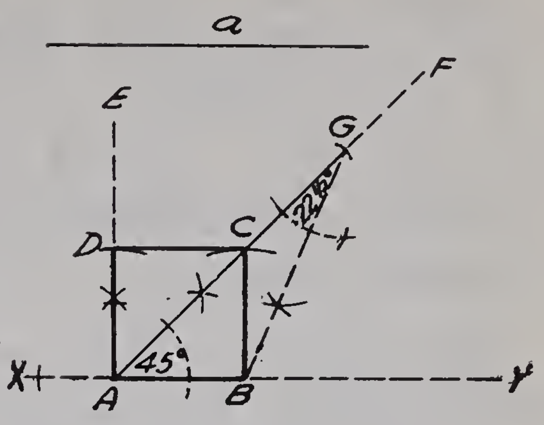
Construction: 1. On any line l, construct $DE = s$ (§ 9). 2. At D, construct $\angle FDE = \angle A$ and at E construct $\angle DEG = \angle B$ (§ 167). 3. On EG construct $EH = s'$ (§ 9). 4. At H construct $\angle EHM = \angle C$ (§ 167).

Then DEHM is the required quad.



Proof: 1. $\angle D = \angle A$, $DE = s$, $\angle E = \angle B$, $EH = s'$, and $\angle H = \angle C$ (Const.). 2. Hence DEHM is the required quad.

16. Given the line segment a = the sum of the diagonal and a side of a square.



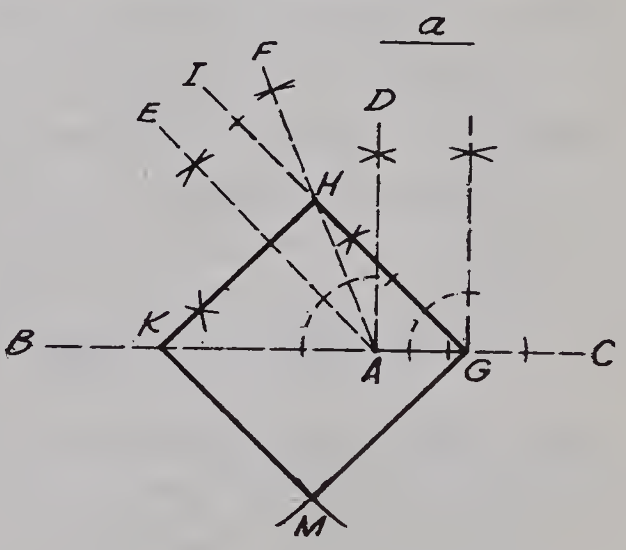
To construct the square.

Construction: 1. At any point A in any line XY, construct AE $\perp$ XY (§173). 2. Construct $\angle$YAF = 45° (Ex. 1, page 192). 3. On AF construct AG = a (§9). 4. At G construct a line intersecting XY at B and making $\angle$AGB = $22\frac{1}{2}°$ (§§167, 166). 5. With B as a center and AB as a radius draw an arc intersecting AG in C (Asmt. 15). 6. With A as a center and AB as a radius, draw an arc intersecting AE in D (Asmt. 15). 7. Draw BC, AD, and DC.

Then ABCD is the required square.

Proof: 1. AE $\perp$ AB (Const.). 2. $\therefore$ $\angle$BAD = 90° (§14, Asmt. 6). 3. $\angle$BAC = 45° (Const.). 4. $\therefore$ $\angle$ DAC = 45° (Asmt. 2). 5. AB = BC (Const.). 6. $\therefore$ $\angle$ ACB = $\angle$ BAC (§ 98a). 7. $\therefore$ $\angle$ ACB = $\angle$ DAC (Asmt. 7). 8. $\therefore$ BC $\parallel$ AD (§160b). 9. BC = AD (Const.). 10. $\therefore$ ABCD is a ▱ (§198). 11. ABCD is a rect. (§ 185). 12. DC = AB (§ 188). 13. $\therefore$ AB = BC = CD = AD (Asmt. 7). 14. $\therefore$ ABCD is a square (§ 185). 15. $\angle$ABC = 90° (§ 191). 16. $\angle$ BCG = $\angle$BAC + $\angle$ ABC (§ 123). 17. $\therefore$ $\angle$ BCG = 135° (Asmt. 6). 18. $\angle$ CGB + $\angle$ BCG + $\angle$CBG = 180° (§ 120). 19. $\angle$ CGB = $22\frac{1}{2}°$ (Const.). 20. $\therefore$ $\angle$CBG = $22\frac{1}{2}°$ (Asmt. 6, 2). 21. $\therefore$ $\angle$CGB = $\angle$CBG (Asmt. 7). 22. $\therefore$ BC = CG (§157a). 23. AC + CG = a (Const.). 24. $\therefore$ AC + BC = a (Asmt. 6).

17. Given the line segment a = the difference between the diagonal and a side of a square.



To construct the square.

Construction: 1. At any point A, in any line BC, construct AD $\perp$ BC (§ 173). 2. Construct AE the bisector of $\angle$BAD and AF the bisector of $\angle$EAD (§ 166). 3. On AC construct AG = a (§ 9). 4. At G construct $\angle$ AGI = 45° (Ex. 1, page 192). 5. At H, the intersection of GI and AF, construct a line $\perp$ HG (§ 173) intersecting BC at K. 6. Construct $\triangle$KMG with KM = HG and MG = KH (§ 165).

Then KMGH is the required square.

Proof: 1. $\angle$ CAF = $112\frac{1}{2}°$ and $\angle$AGH = 45° (Const.). 2. $\angle$CAF + $\angle$AGH + $\angle$AHG = 180° (§120). 3. $\therefore$ $\angle$AHG = $22\frac{1}{2}°$ (Asmt. 6, 2). 4. $\angle$ KHG = 90° (Const.). 5. $\therefore$ $\angle$KHA = $67\frac{1}{2}°$ (Asmt. 2). 6. $\angle$ HKG

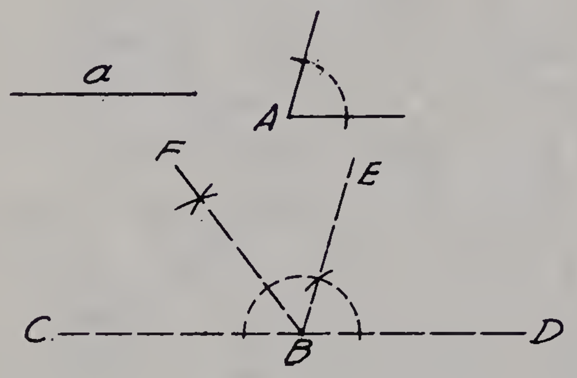
+ ∠HGK = 90° (§161). 7. ∠HKG = 45° (Asmt. 6, 2). 8. ∠HKA + ∠KHA + ∠KAH = 180° (§120). 9. ∠KAH = $67\frac{1}{2}^\circ$ (Asmt. 6, 2). 10. KH = KA (§157a). 11. AG = KG - KA (Asmt. 9). 12. AG = KG - KH (Asmt. 6). 13. KH = HG (§157a). 14. KH = MG and KM = HG (Const.). 15. KMGH is a ▱ (§197). 16. ∴ KMGH is a rect. and a square (§185).

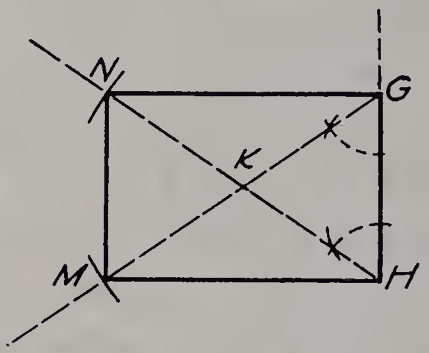
18. Given one side, a, of a rect. and ∠A, the ∠ formed by the diagonals of the rect.

To construct the rect.

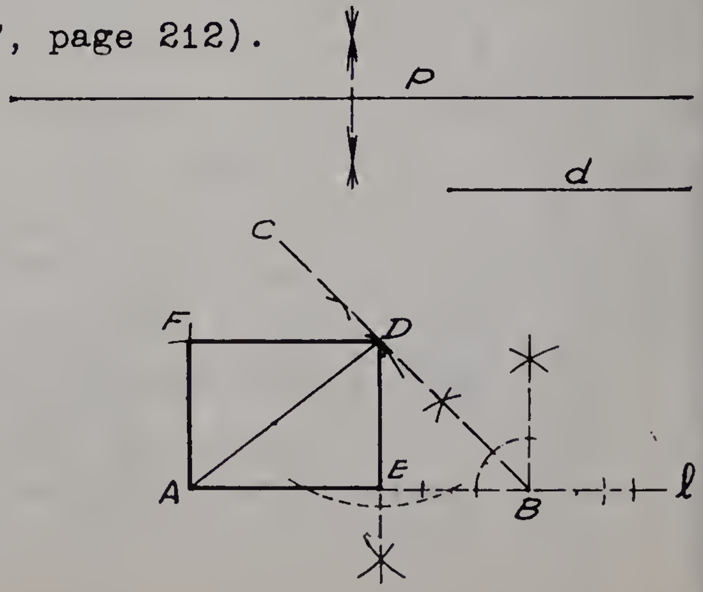
Construction: 1. At any point B, in any line CD, construct ∠DBE = ∠A (§167). 2. Construct BF the bisector of ∠CBE (§166). 3. Construct △GHK having ∠KGH = ∠CBF, ∠KHG = ∠EBF and GH = a (§170). 4. Extend GK making KM = GK and extend HK making KN = HK (Asmt. 13, §9). 5. Draw MH, MN, and NG (Asmt. 11).

Then MHGN is the required rect.





Proof: 1. GH = a (Const.). 2. ∠CBF + ∠FBE + ∠EBD = 180° (§16). 3. ∠KGH + ∠KHG + ∠GKH = 180° (§120). 4. ∴ ∠CBF + ∠FBE + ∠EBD = ∠KGH + ∠KHG + ∠GKH (Asmt. 7). 5. ∠CBF = ∠KGH and ∠EBF = ∠KHG (Const.). 6. ∴ ∠CBF + ∠EBF = ∠KGH + ∠KHG (Asmt. 1). 7. ∴ ∠GKH = ∠EBD (Asmt. 2). 8. ∠A = ∠EBD (Const. 9. ∠GKH = ∠A (Asmt. 7). 10. ∠KHG = ∠KGH (Asmt. 7). 11. GK = HK (§157a). 12. KM = GK and KN = HK (Const.). 13. ∴ MHGN is a ▱ (§199). 14. KM = GK = KN = HK (Asmt. 7). 15. NH = MG (Asmt. 1). 16. ∴ MHGN is a rect. (Ex. 7, page 212).



19. Given the perimeter p and a diagonal d of a rect.

To construct the rect.

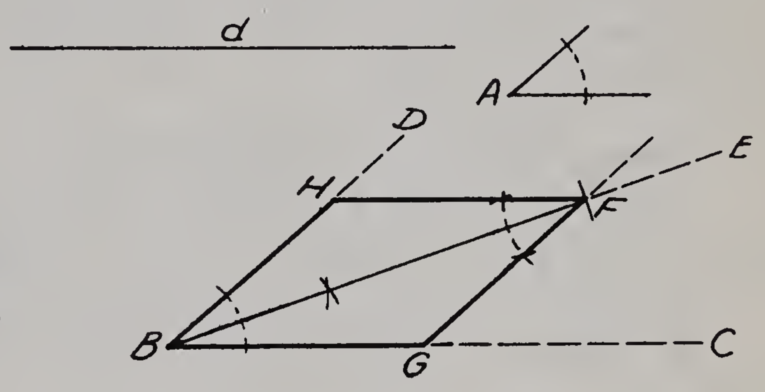
Construction: 1. Bisect p (§171). 2. On any line ℓ, construct AB = $\frac{1}{2}$p (§9). 3. At B, construct ∠ABC = 45° (Ex. 1, page 192). 4. With A as a center and a radius = d, draw an arc intersecting BC at D (Asmt. 15). 5. From D, construct DE ⊥ AB (§ 172). 6. Construct △ADF with AF = DE and DF = AE (§165).

Then AEDF is the required rect.

Proof: 1. AD = d and DE ⊥ AB (Const.). 2. ∠EDB + ∠DBE = 90° (§161). 3. ∠DBE = 45° (Const.). 4. ∴ ∠EDB = 45° (Asmt. 2). 5. EB = ED (§157a). 6. AE = AE (Iden.). 7. AE + EB = AE + ED (Asmt. 1). 8. AE + EB = $\frac{1}{2}$p (Const.). 9. ∴ AE + ED = $\frac{1}{2}$p (Asmt. 7). 10. ED = FA and AE = FD (Const.). 11. ∴ AE + ED + FA + FD = p (Asmt. 1, 6). 12. AEDF is a ▱ (§ 197). 13. ∠ AED is a rt. ∠ (Const.). 14. ∴ ▱ AEDF is a rect. (§ 185).

20. Given the diagonal d and ∠A an ∠ of a rhombus.

To construct the rhombus.



Construction: 1. At B in any line BC, construct ∠CBD = ∠A (§ 167). 2. Construct BE the bisector of ∠CBD (§ 166). 3. On BE, construct BF = d (§ 9). 4. Through F construct a line ∥ BD intersecting BC at G and a line ∥ BG intersecting BD at H (§ 174).

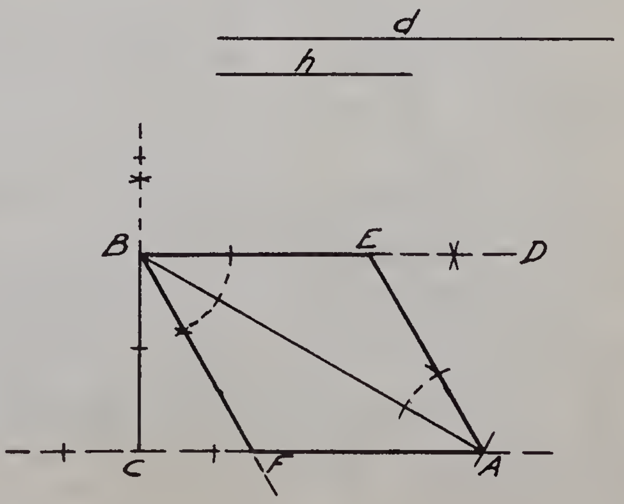
Then BGFH is the required rhombus.

Proof: 1. BF = d and ∠GBH = ∠A (Const.). 2. FH ∥ BG and FG ∥ HB (Const.). 3. ∴ BGFH is a ▱ (§ 185). 4. ∠HBF = ∠GBF (Const.). 5. ∠HBF = ∠BFG (§ 158a). 6. ∴ ∠GBF = ∠BFG (Asmt. 7). 7. BG = GF (§ 157a). 8. ∴ ▱ BGFH is a rhombus (§ 185).

Discussion. A different rhombus is possible if the given ∠ is taken opp. the diagonal. To construct the rhombus, subtract the given ∠ from a st. ∠ and follow the steps in the above construction.

21. Given the altitude h and the diagonal d of a rhombus.

To construct the rhombus.



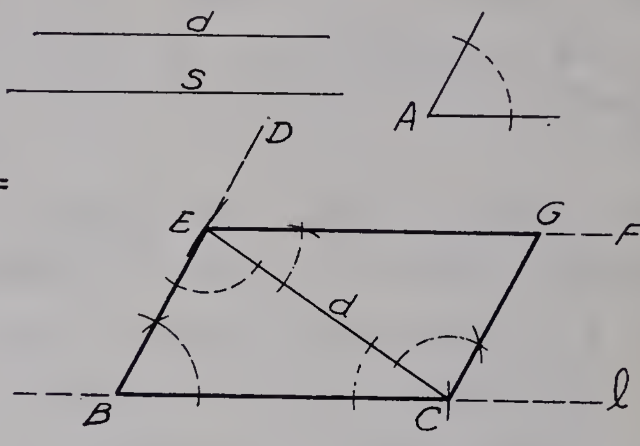
Construction: 1. Construct rt. △ ACB with AB = d, and BC = h (Ex. 13, page 195). 2. Through B, construct BD ∥ CA (§ 174). 3. At A, construct ∠BAE = ∠DBA (§ 167). 4. Through B, construct a line ∥ EA, intersecting CA at F (§ 174).

Then FAEB is the required rhombus.

Proof: 1. BA = d and BC = h (Const.). 2. BE ∥ FA and BF ∥ EA (Const.). 3. ∴ FAEB is a ▱ (§185). 4. ∠BAE = ∠EBA (Const.) 5. ∴ BE = EA (§157a). 6. ∴ FAEB is a rhombus (§185).

22. Given a diagonal d, a side s, and an ∠A of a ▱.

To construct the ▱.



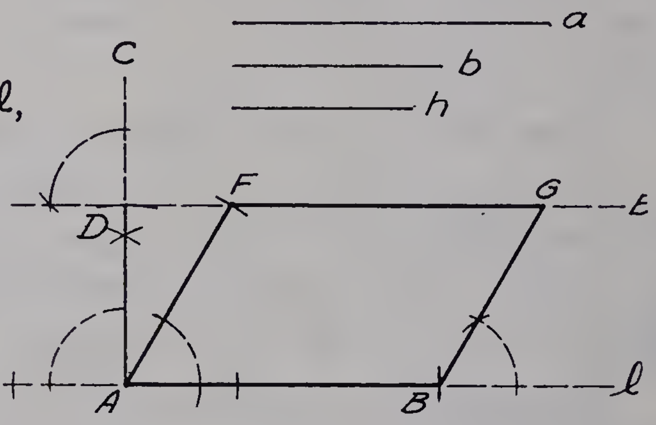
Construction: 1. On any line ℓ construct BC = s (§9). 2. At B, construct ∠CBD = ∠A (§167). 3. With C as a center and a radius = d, draw an arc intersecting BD at E (Asmt. 15). 4. Through E, construct EF ∥ BC (§174). 5. Through C construct a line ∥ BE, intersecting EF at G (§174).

Then BCGE is the required ▱.

Proof: 1. BC = s, EC = d, and ∠B = ∠A (Const.). 2. EG ∥ BC and CG ∥ BE (Const.). 3. ∴ BCGE is a ▱ (§185).

23. Given 2 adj. sides a and b of a ▱ and h the altitude to side a.

To construct the ▱



Construction: 1. On any line ℓ, construct AB = a (§9). 2. At A, construct AC ⊥ AB (§173). 3. On AC, construct AD = h (§9). 4. Through D construct DE ∥ AB (§174). 5. With A as a center and b as a radius, draw an arc intersecting DE at F (Asmt. 15). 6. Draw AF (Asmt. 11). 7. Through B, construct a line ∥ AF, intersecting DE at G (§174).

Then ABGF is the required ▱.

Proof: 1. AD = h, AB = a, and AF = b (Const.). 2. FG ∥ AB and BG ∥ AF (Const.). 3. ∴ ABGF is a ▱ (§185).

Discussion. Another ▱ can be constructed if the altitude is taken to side b.

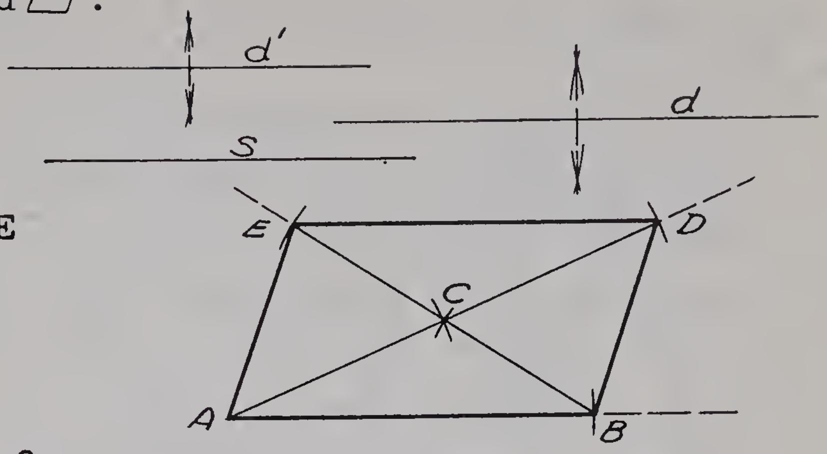
24. Given the diagonals d and d' and a side s of a ▱.

To construct the ▱.

Construction: 1. Bisect d and d' (§171). 2. Construct △ABC with AB = s, AC = $\frac{1}{2}$d, and BC = $\frac{1}{2}$d' (§165). 3. Extend AC making CD = AC and extend BC making CE = BC (Asmt. 13, §9). 4. Draw BD, DE, and EA (Asmt. 11).

Then ABDE is the required ▱.

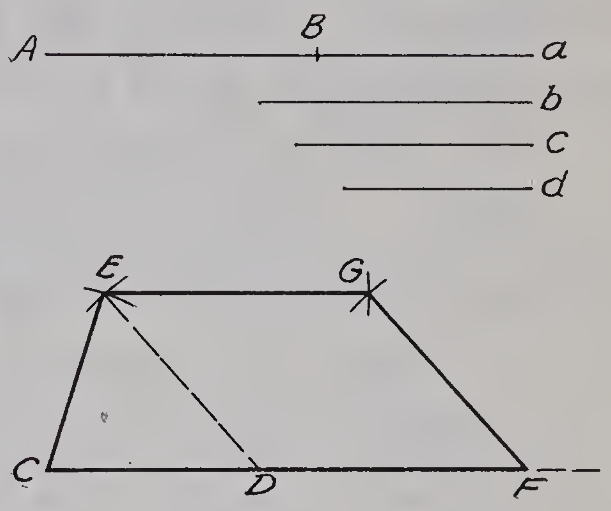
Proof: 1. AB = s, AC = $\frac{1}{2}d$, and BC = $\frac{1}{2}d'$ (Const.). 2. CD = AC and CE = BC (Const.). 3. ∴ ABDE is a ▱ (§199). 4. AD = d and BE = d' (Asmt. 6 and 1).



25. Given the sides a, b, c, and d of a trapezoid with a ∥ b.

To construct the trapezoid.

Construction: 1. On a, construct AB = b (§9). 2. Construct △CDE with CD = a − b, CE = d, and DE = c (§165). 3. Extend CD making CF = a (§9). 4. With E and F as centers and b and c as radii respectively draw arcs intersecting at G (Asmt. 15). 5. Draw EG and FG.



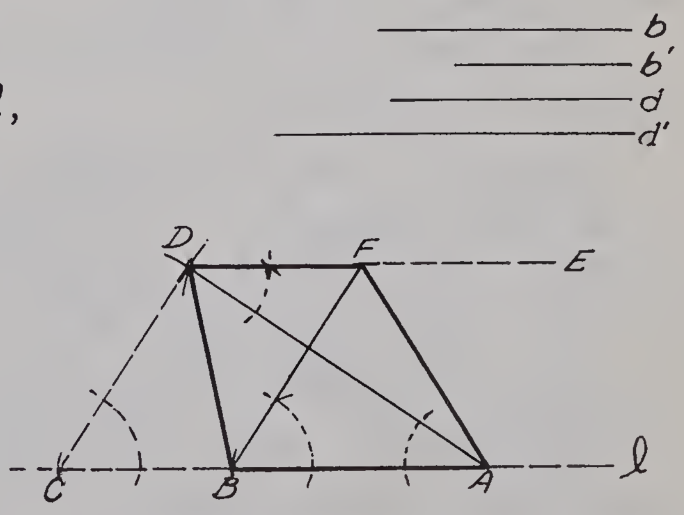
Then CFGE is the required trapezoid.

Proof: 1. CE = d, CF = a, FG = c, and GE = b (Const.). 2. EDFG is a ▱ (§197). 3. ∴ EG ∥ DF or CF (§185). 4. ∴ CFGE is a trapezoid (§206).

26. Given the bases b and b' and the diagonals d and d' of a trapezoid.

To construct the trapezoid.

Construction: 1. On any line ℓ, construct AB = b and BC = b' (§9). 2. Construct △CAD with CD = d and DA = d' (§165). 3. Through D, construct DE ∥ CA (§174). 4. Through B, construct a line ∥ CD, intersecting DE at F (§174). 5. Draw DB and FA (Asmt. 11).

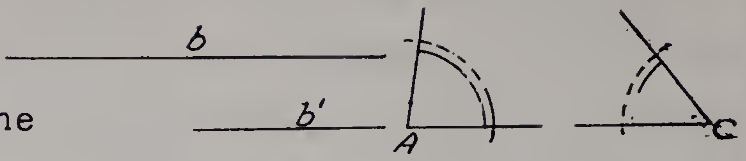


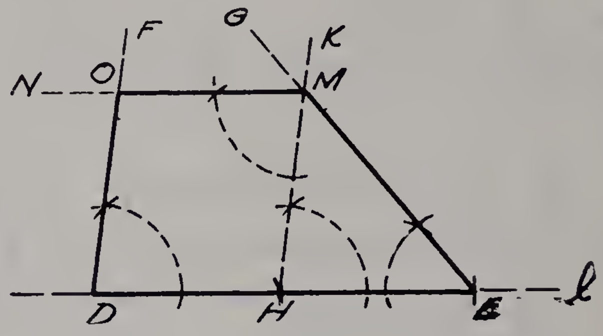
Then BAFD is the required trapezoid.

Proof: 1. DF ∥ CA (Const.). 2. ∴ DBAF is a trapezoid (§206). 3. BF ∥ CD (Const.). 4. ∴ CBFD is a ▱ (§185). 5. ∴ CB = DF and BF = CD (§188). 6. CB = b' and CD = d (Const.). 7. ∴ DF = b' and BF = d (Asmt. 7). 8. BA = b and DA = d' (Const.).

27. Given the bases b and b' and the lower base ∠s A and C of a trapezoid.

To construct the trapezoid.





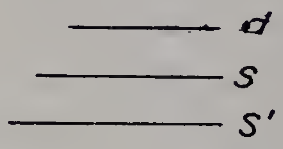
Construction: 1. On any line ℓ, construct DE = b (§9). 2. At D, construct ∠EDF = ∠A and at E, construct ∠DEG = ∠C (§167). 3. On ℓ, construct DH = b' (§9). 4. Through H construct HK ∥ DF and intersecting EG in M (§174). 5. Through M, construct MN ∥ ℓ and intersecting DF in O (§174).

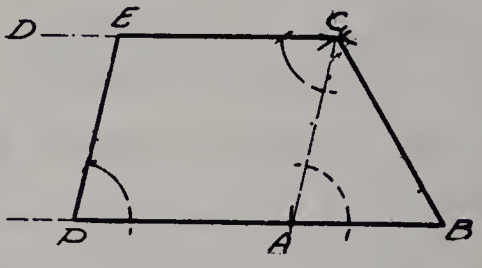
Then DEMO is the required trapezoid.

Proof: 1. DE = b, ∠ODE = ∠A, and ∠DEM = ∠C (Const.). 2. HM ∥ DO and OM ∥ DE (Const.). 3. ∴ ODHM is a ▱ (§185). 4. ∴ OM = DH (§188). 5. DH = b' (Const.). 6. ∴ OM = b' (Asmt. 7). 7. DEMO is a trapezoid (§206).

28. Given s and s' the nonparallel sides of a trapezoid and d the difference between the bases.

To construct the trapezoid.





Construction: 1. Construct △ABC with AB = d, AC = s, and BC = s' (§165). 2. Through C, construct CD ∥ BA (§174). 3. Through P any point in BA extended (Asmt. 13), construct a line ∥ AC intersecting DC at E (§174).

Then PBCE is a trapezoid fulfilling the given conditions.

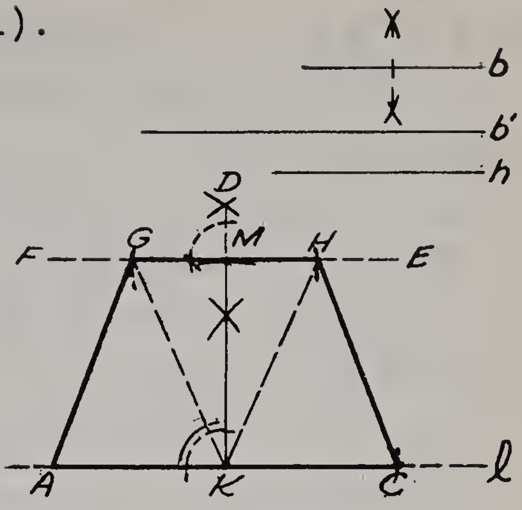
Proof: 1. CE ∥ BP (Const.). 2. ∴ PBCE is a trapezoid (§206). 3. PE ∥ AC (Const.). 4. ∴ PACE is a ▱ (§185). 5. ∴ PA = EC (§188). 6. PA + AB = PB (Asmt. 9). 7. PA = PB − AB (Asmt. 2). 8. ∴ EC = PB − AB (Asmt. 6). 9. BC = s', and AC = s (Const.). 10. EP = AC (§188). 11. ∴ EP = s (Asmt. 7).

Discussion. Since the lengths of sides EC and PB are undetermined, an infinite number of trapezoids fulfilling the given conditions can be constructed.

29. Given the bases b and b' and the altitude h of an isos. trapezoid.

To construct the isos. trapezoid.

Construction: 1. On any line ℓ, construct AC = b' (§9).



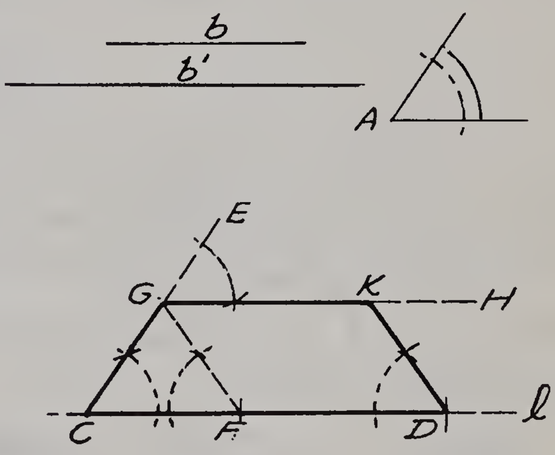
2. Construct KD the ⊥ bisector of AC (§171).
3. On KD, construct MK = h (§9).
4. Through M, construct EF ∥ ℓ (§174).
5. Bisect b (§171). 6. On EF, construct $MH = \frac{1}{2}b$ and $MG = \frac{1}{2}b$ (§9).
7. Draw GA and HC (Asmt. 11).

Then ACHG is the required isos. trapezoid.

Proof: 1. AC = b', GH = b, and KM = h (Const.). 2. GH ∥ AC (Const.). 3. Draw GK and KH (Asmt. 11). 4. MK ⊥ GH (§159a). 5. ∴ △s GMK and HMK are rt. △s (§57). 6. MK = MK (Iden.). 7. MG = MH (Const.). 8. ∴ △GMK ≅ △HMK (§96d). 9. ∴ GK = KH (§97a). 10. ∠GKM = ∠HKM (§98b). 11. ∠MKA = ∠MKC (Const.). 12. ∴ ∠GKA = ∠HKC (Asmt. 2). 13. AK = KC (Const.). 14. ∴ △AKG ≅ △CKH (§95a). 15. ∴ AG = HC (§97a). 16. ∴ ACHG is an isos. trapezoid (§206).

30. Given the bases b and b' and ∠A of an isos. trapezoid.

To construct the isos. trapezoid.



Construction: 1. On any line ℓ, construct CD = b' (§9). 2. At C, construct ∠DCE = ∠A (§167). 3. On DC, construct DF = b (§9). 4. At F, construct a line meeting CE at G and making ∠CFG = ∠A (§167). 5. Through G, construct GH ∥ CD, and through D construct DK ∥ FG (§174).

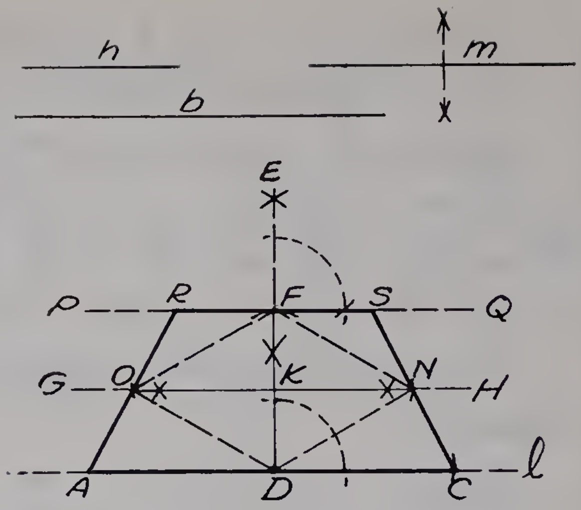
Then CDKG is the required isos. trapezoid.

Proof: 1. GK ∥ CD and DK ∥ FG (Const.). 2. ∴ FDKG is a ▱ (§185). 3. ∴ DK = FG and GK = FD (§188). 4. FD = b (Const.). 5. ∴ GK = b (Asmt. 7). 6. CD = b' (Const.). 7. ∠GCF = ∠A and ∠GFC = ∠A (Const.). 8. ∴ ∠GCF = ∠GFC (Asmt. 7). 9. ∴ CG = FG (§157a). 10. ∴ CG = DK (Asmt. 7). 11. ∴ CDKG is an isos. trapezoid (§206).

31. Given a base b, the median m, and the altitude h of an isos. trapezoid.

To construct the isos. trapezoid.

Construction: 1. On any line ℓ, construct AC = b (§9). 2. Construct DE the ⊥ bisector of AC (§171). 3. On DE, construct



DF = h (§ 9). 4. Construct GH the ⊥ bisector of DF, intersecting DF in K (§ 171). 5. Bisect m (§ 171). 6. On GH, construct KN = $\frac{1}{2}$m and KO = $\frac{1}{2}$m (§9). 7. Through F, construct PQ ∥ ℓ (§174). 8. Draw CN and extend CN to intersect PQ in S (Asmt. 11, 13). 9. Draw AO and extend AO to intersect PQ in R (Asmt. 11, 13).

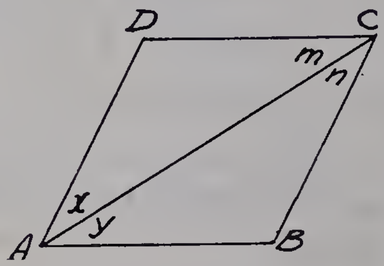
Then ACSR is the required isos. trapezoid.

Proof: 1. KO = $\frac{1}{2}$m and KN = $\frac{1}{2}$m (Const.). 2. ∴ ON = m (Asmt. 9, 1). 3. DF = h and AC = b (Const.). 4. RS ∥ AC (Const.). 5. Draw DO and DN (Asmt. 11). 6. ON ⊥ DF (Const.). 7. ∴ △s OKD and NKD are rt. △s (§57). 8. KO = KN (Asmt. 7). 9. KD = KD (Iden.). 10. ∴ △ OKD ≅ △ NKD (§96d). 11. ∴ OD = ND (§97a). 12. ∠KDO = ∠KDN (§98b). 13. ∠KDA = ∠KDC (Const.). 14. ∴ ∠ODA = ∠NDC (§ 50b). 15. AD = DC (Const.). 16. ∴ △ ADO ≅ △ CDN (§95a). 17. ∴ ∠DAO = ∠DCN (§98b). 18. ∴ ACSR is an isos. trapezoid (Ex. 12, page 217).

PAGES 225-226

1. Given the ▱ ABCD with diagonal AC bisecting ∠BAD.

To prove that ABCD is a rhombus.



Planning the Proof: 1. We can prove that a ▱ is a rhombus by § 185. 2. We shall use § 185.

Proof: 1. ∠BAD = ∠BCD (§ 189). 2. ∠ x = ∠ y (Given). 3. ∠ m = ∠ y and ∠ n = ∠ x (§ 158a). 4. ∴ ∠ x = ∠ m (Asmt. 7). 5. ∴ AD = DC (§157a). 6. ∴ ▱ ABCD is a rhombus (§185).

2. Given the st. lines DE, CG, DG, and EC; F the midpoint of CE; and FG = DF.

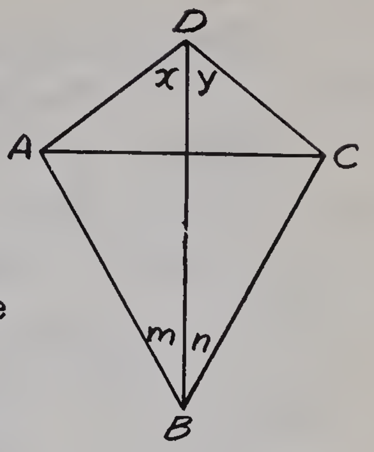
To prove that CG ∥ AB.

Planning the Proof: 1. We can prove lines ∥ by §§ 160, 185, 204, 207. 2. We shall use § 185.

Proof: 1. CF = FE and DF = FG (Given). 2. Draw CD and GE (Asmt. 11). 3. ∴ CDEG is a ▱ (§199). 4. CG ∥ DE (§185).

3. Given the quad. ABCD with diagonals AC and BD, and BD bisecting ∠s ADC and ABC.

To prove that DB is the ⊥ bisector of AC.

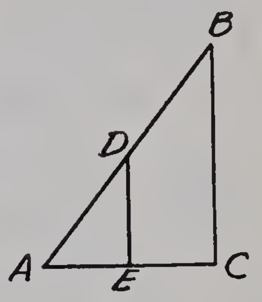


Planning the Proof: 1. We can prove that one line is the ⊥ bisector of another by § 140. 2. We shall use § 140.

Proof: 1. ∠x = ∠y and ∠m = ∠n (Given). 2. DB = DB (Iden.). 3. ∴ △ ADB ≅ △CDB (§95b). 4. ∴ AD = DC and AB = BC (§97a). 5. ∴ DB is the ⊥ bisector of AC (§ 140).

4. Given the rt. △ACB with ∠C the rt. ∠ and DE the ⊥ bisector of AC.

To prove that AD = DB.

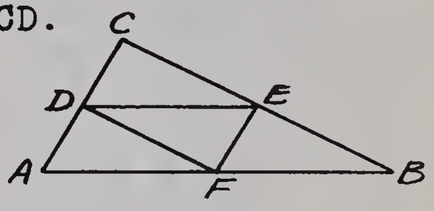


Planning the Proof: 1. We can prove line segments equal by §§97, 157, 188, 192-195, 203-205. 2. We shall use § 203.

Proof: 1. DE ⊥ AC, BC ⊥ AC, and AE = EC (Given). 2. ∴ DE ∥ BC (§ 160d). 3. ∴ AD = DB (§ 203).

5. Given the △ABC with DE, EF, and FD joining the midpoints of AC, BC, and AB.

To prove that △ AFD ≅ △ FBE ≅ △ FED ≅ △ ECD.

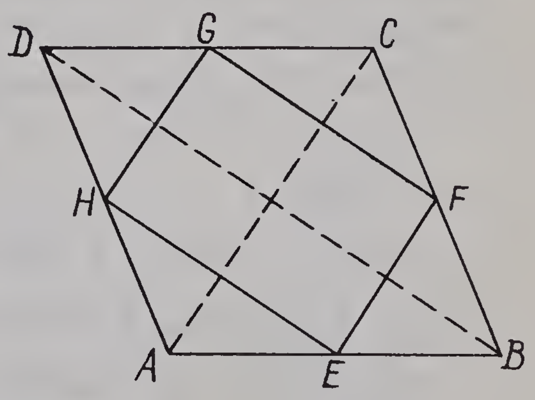


Planning the Proof: 1. We can prove △s ≅ by §§ 95, 187. 2. We shall use § 187.

Proof: 1. DE ∥ AB, FE ∥ AC, and DF ∥ CB (§204). 2. AFED, FBED, and FECD are ▱s (§185). 3. ∴ △ FED ≅ △ AFD, △ FED ≅ △ FBE, and △ FED ≅ △ ECD (§187). 4. ∴ △ AFD ≅ △ FBE ≅ △ FED ≅ △ ECD (Asmt. 7).

6. Given the rhombus ABCD with EF, FG, GH, and HE joining the midpoints of AB, BC, CD, and DA in order.

To prove that EFGH is a rect.



Planning the Proof: 1. We can prove that a quad. is a rect. by §185. 2. We shall use § 185.

Proof: 1. Draw DB (Asmt. 11). 2. GF ∥ DB and = $\frac{1}{2}$DB (§ 204). 3. HE ∥ DB and = $\frac{1}{2}$DB (§204). 4. ∴ GF ∥ HE and GF = HE (§ 160e, Asmt. 7). 5. ∴ HEFG is a ▱ (§198). 6. DC = CB = AB = AD (Given). 7. Draw AC (Asmt. 11). 8. AC ⊥ DB (§159b). 9. ∴ AC ⊥ HE (§159a). 10. GH ∥ CA (§204). 11. ∴ GH ⊥ HE (§159a). 12. ∴ HEFG is a rect. (§185).

7. Given the △ABC with the median CD, AE ⊥ CD, and BF ⊥ CD.

To prove that AE = BF.

Planning the Proof: 1. We can prove line segments = as in Ex. 4. 2. We shall use §97a.

Proof: 1. AE ⊥ CD and BF ⊥ CD (Given). 2. ∴ ◬ ADE and BDF are rt. ◬ (§57). 3. ∠ADE = ∠BDF (§50d). 4. AD = DB (Given). 5. ∴ △ADE ≅ △BDF (§156a). 6. ∴ AE = BF (§97a).

8. Planning the Proof: 1. We can prove that a quad. is a ▱ by §§185, 197, 198, 199. 2. We shall use §198.

Proof: 1. AE = BF (Ex. 7). 2. AE ∥ BF (§160d). 3. ∴ AFBE is a ▱ (§198).

9. The number of diagonals that can be drawn from one vertex is (n - 3) since a diagonal cannot be drawn from a vertex to itself or from a vertex to an adj. vertex (§182). From n vertices n(n - 3) diagonals can be drawn. But if diagonals are drawn from each vertex, each diagonal is drawn twice. Hence $\frac{n(n - 3)}{2}$ is the greatest number of diagonals that can be drawn.

10. Given the kite ABCD with AD = CD and AB = CB.

To prove that △ADB ≅ △CDB and that ◬ ADC and ABC are isos.

Planning the Proof: 1. We can prove (a) ◬ ≅: §§95, 187. (b) ◬ isos.: § 56. 2. We shall use (a) § 95c. (b) § 56.

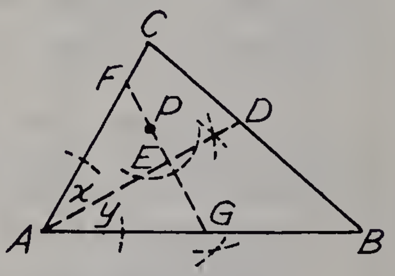
Proof: 1. AD = DC and BA = BC (Given). 2. BD = BD (Iden.). 3. ∴ △ADC ≅ △CDB (§95c). 4. ◬ ADC and ABC are isos. (§ 56).

11. Given any point P within △ABC.

To construct a line through P making equal angles with AB and AC.

Construction: 1. Construct AD bisecting ∠A (§177a). 2. Through P, construct PE ⊥ AD, meeting AC in F and AB in G (§178a).

Then FG makes equal angles with AB and AC.

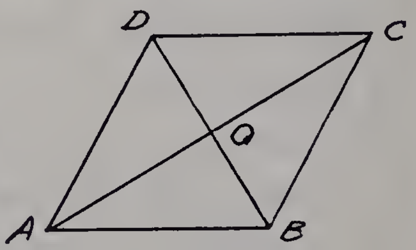


Proof: 1. FG ⊥ AD (Const.). 2. ∠AEF = ∠AEG (§14). 3. AE = AE (Iden.). 4. ∠x = ∠y (Const.). 5. △ AEF ≅ △AEG (§95b). 6. ∠AFG = ∠AGF (§ 98b).

12. Given the ▱ABCD with diagonal AC ⊥ diagonal DB at O.

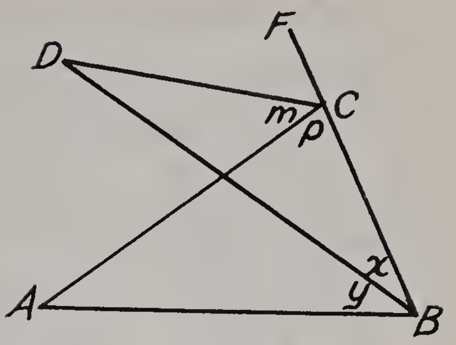
To prove that ABCD is a rhombus.

Planning the Proof: 1. We can prove that a ▱ is a rhombus by § 185. 2. We shall use §185.



Proof: 1. AO = OC (§195). 2. OD = OD (Iden.). 3. ∠ AOD = ∠COD (§50a). 4. ∴ △AOD ≅ △COD (§95a). 5. ∴ AD = DC (§97a). 6. ∴ ▱ABCD is a rhombus (§185).

13. Given △ABC with DB bisecting ∠B and DC bisecting ext. ∠ACF

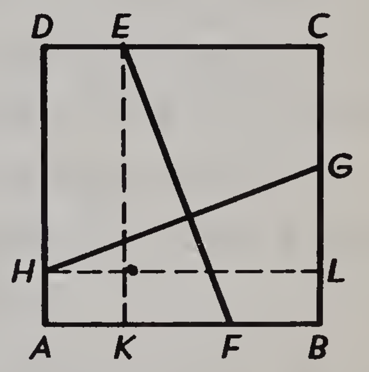


To prove that $\angle D = \frac{1}{2}\angle A$.

Planning the Proof: 1. We can prove angles = by §§50, 98, 158, 189. 2. We shall use §50e.

Proof: 1. DB bisects $\angle ABC$ and DC bisects $\angle ACF$ (Given). 2. $\angle A + \angle B + \angle C = 180°$ (§120). 3. $\angle x = \angle y$ (§22). 4. $\angle A + \angle x + \angle y + \angle p = 180°$ (Asmt. 6). 5. $\angle A + 2\angle x + \angle p = 180°$ (Asmt. 6). 6. $\angle A = 180° - p - 2x$ (Asmt. 2). 7. $\angle D + \angle x + \angle p + \angle m = 180°$ (§120, Asmt. 6). 8. But $\angle DCF + \angle m + \angle p = 180°$ (Asmt. 23). 9. $\angle m + \angle m + \angle p = 180°$ (Asmt. 6). 10. $2\angle m + \angle p = 180°$ (Asmt. 6). 11. $2\angle m = 180° - \angle p$ (Asmt. 2). 12. $\angle m = 90° - \angle\frac{p}{2}$ (Asmt. 4). 13. From (7 and 12), $\angle D + \angle x + \angle p + 90° - \angle\frac{p}{2} = 180°$ (Asmt. 6). 14. $\angle D + \angle x + \angle\frac{p}{2} = 90°$ (Asmt. 6, 2). 15. $\angle D = 90° - \angle x - \angle\frac{p}{2}$ (Asmt. 2). 16. From 6, $\frac{1}{2}\angle A = 90° - \angle\frac{p}{2} - x$ (Asmt. 4). 17. $\angle D = \frac{1}{2}\angle A$ (Asmt. 6).

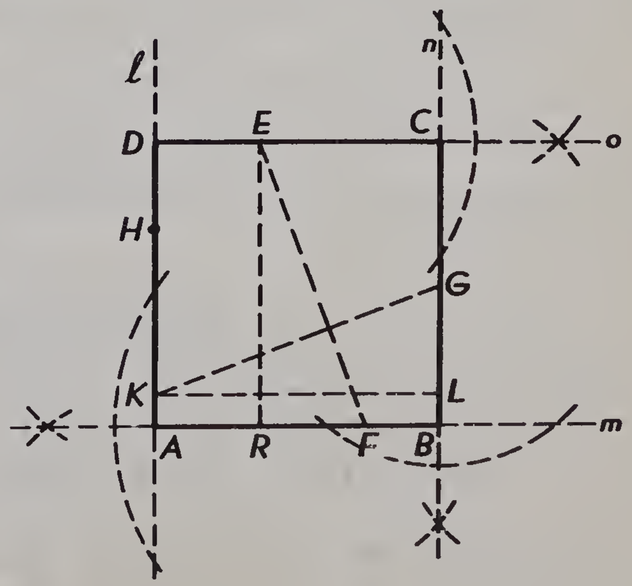
14. Planning the Proof: 1. We can prove line segments = by §§97, 157, 188, 192-195, 203-205. 2. We shall use §97a.



Proof: 1. ABCD is a square (Given). 2. DC $\parallel$ AB and DA $\parallel$ BC (§185). 3. Draw HL $\parallel$ AB and EK $\parallel$ DA (§174). 4. EK = DA and HL = AB (§192). 5. DA = AB (§§185, 194). 6. EK = HL (Asmt. 7). 7. $\angle A$ and $\angle B$ are rt. $\angle$s (§191). 8. DA $\perp$ AB (§14). 9. EK $\perp$ AB (§114). 10. EK $\perp$ HL (§114). 11. GH $\perp$ EF (Given). 12. $\angle KEF = \angle LHG$ (§127). 13. $\angle EKB$ is a rt. $\angle$ (§14). 14. AB $\perp$ BC (§14). 15. HL $\perp$ BC (§114). 16. $\angle HLC$ is a rt. $\angle$ (§14). 17. $\angle EKB = \angle HLC$ (Asmt. 19). 18. $\triangle KEF \cong \triangle LHG$ (§95b). 19. EF = HG (§97a).

15. Given points E, F, G, and H.

To construct a square such that each of the sides (or its extension) passes through one of the points.



Construction: 1. Draw EF (Asmt. 11). 2. Construct GK $\perp$ EF and = EF (§§178a, 9). 3. Draw line ℓ through K and H (Asmt. 11). 4. From F construct line m $\perp$ ℓ at A, from G construct line n $\perp$ m at B, and from E construct line o $\perp$ n at C (§178a). 5. Extend o to meet ℓ at D (§115).

Then ABCD is a required square.

Proof: 1. DA ⊥ AB and CB ⊥ AB (Const.). 2. DA ∥ CB (§107). 3. AB ⊥ CB and DC ⊥ BC (Const.). 4. DC ∥ AB (§107). 5. ABCD is a ▱ (§185). 6. ∠DAB is a rt. ∠ (§14). 7. ABCD is a rectangle (§185). 8. Construct KL ⊥ BC and ER ⊥ AB (§178a). 9. ∠ERB and ∠KLC are rt. ∠s (§14). 10. △KLG and △REF are rt. △s (§57). 11. KL ∥ AB (§107). 12. ER ⊥ KL (§114). 13. GK ⊥ EF (Const.). 14. ∠REF = ∠LKG (§127). 15. GK = EF (Const.). 16. △LKG ≅ △REF (§156a). 17. ER = KL (§97a). 18. ER ∥ AD (§107). 19. ER = AD and KL = AB (§192). 20. AD = AB (Asmt. 7). 21. Rectangle ABCD is a square (§185).

Discussion: The square is not determined. By joining the points in other orders, other squares can be obtained. The given points may be in a straight line.

PAGE 227 (Practical Problems)

1. It is assumed that the table is a quadrilateral. Measure the sides, and if the opposite sides are equal, the quadrilateral is a parallelogram. Then measure the diagonals, and if they are equal the parallelogram is a rectangle (Ex. 7, page 212).

2. Both pairs of opposite sides must be equal (§197) and the angles must be right ∠s (§185).

3. 72 ft. (§204).

4. (1) 1. ABCD is a ▱ (Const.). 2. DC ∥ ground (Const.). 3. AB ∥ DC (§185). 4. AB ∥ ground (§160e).
(2) 1. ABCD is a ▱ (§198). 2. ∴ AD ∥ BC (§185).

5. The periscope consists of two parallel plane mirrors (see Ex. 5, page 169). The mirrors are kept in parallel planes by two series of flexible rhombuses (see Ex. 7, page 217).

6. The bars are joined to form flexible parallelograms. The top and bottom of the jack will always remain ∥ to the screw, and therefore ∥ to each other (§160e).

PAGES 228-229 (Space Geometry)

4. (1) F-AB-D and A-CD-G; E-AB-D and A-CD-H.
(2) M-AB-F and G-CD-N; M-AB-E and H-DC-N.

5. M-AB-F and A-CD-H; M-AB-E and A-CD-G.

6. Any two of the following pairs:
(1) M-AB-F and E-AB-D; (2) A-CD-H and G-CD-N;
(3) M-AB-E and F-AB-D; (4) A-CD-G and H-CD-N.

7. (1) Yes. (2) Yes. 8. An infinite number. 9. One. 10. One. 11. Not necessarily. 12. Yes. 13. Yes. 14. Not necessarily. 15. Not necessarily. 16. Yes. 17. Yes. 18. Yes. 19. None. 20. An infinite number. 21. An infinite number. 22. Yes. 23. When the line is parallel to the plane or lies in the plane. 24. One. 25. Yes. 26. The ⊥. 27. Yes.

PAGE 230

1. This is reasoning from a special case. 2. This is reasoning from a special case. 3. This is reasoning from a special case. 4. This is reasoning by analogy. 5. This is reasoning by analogy.

PAGES 231-232 (Review Questions)

1. (1) §§187, 188, 189, 190, 195. (2) §191. (3) §207. 2. §§185, 197, 198, 199. 3. Rhombuses, squares. 4. Rectangles, squares, isosceles trapezoids. 5. Parallelograms. 6. Rhombuses, squares.

7. They bisect each other and are =.

8. They bisect each other, are ⊥, and bisect the ∠s of the rhombus.

9. They bisect each other, are ⊥, bisect the ∠s of the square, and are =.

10. Neither. The sum of the ext. ∠s is constant and = 2 st. ∠s.

11. (1) 360° (§210). (2) 180°. (3) 540°. (4) 720°.

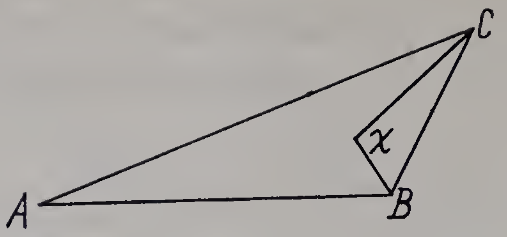
12. (1) 360° (§208). (2) 360°. (3) 360°.

13. It is ∥ the bases and = $\frac{1}{2}$ their sum.

14. It is ∥ the third side and = $\frac{1}{2}$ of it.

15. Quadrilateral. 16. §§165-174, 212. 17. Trapezoid. 18. 96 in. (§204). 19. They are supp. 20. See Ex. 7, p. 212.

21. $\angle B + \angle C + 20 = 180$ (§120).
$\angle B + \angle C = 160$ (Asmt. 2).
$\frac{1}{2}B + \frac{1}{2}C = 80$ (Asmt. 4).
$x + \frac{1}{2}B + \frac{1}{2}C = 180$ (§120).
$\therefore x = 100°$ (Asmt. 2).



22. $\angle B = 45°$ (§98a). $\angle C + 45° + 45° = 180°$ (§120). $\angle C = 90°$ (Asmt. 2).

23. $\angle B + \angle C = 180°$ (§162). $110° + \angle C = 180°$ (Asmt. 6). $\therefore \angle C = 70°$ (Asmt. 2).

24. $4\angle D + \angle D = 180°$ (§162). $5\angle D = 180°$ (Asmt. 6). $\therefore \angle D = 36°$ (Asmt. 4). $\angle B = \angle D = 36°$ (§189).

25. Sum of ext. $\angle$s $= 360°$ (§208).
Each ext. $\angle = \frac{360°}{18} = 20°$ (Asmt. 4). Each int. $\angle = 160°$ (§53b).

26. $360° \div 24° = 15$, the no. of sides (§208).

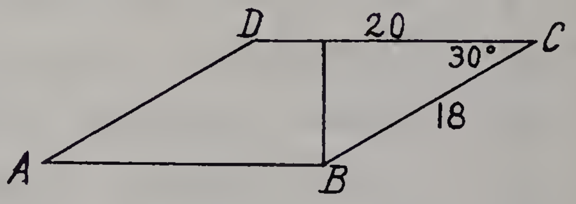
27. AB = 20 in. (§136). BD = 10 in. (Asmt. 4).

28. CD = BD = AD (§205). AB = 16 in. (Asmt. 3). $\angle B = 30°$ (§161). CA = 8 in. (§136).

29. CD = 8 in. (§205).

30. Let x and y = the number of degrees in each of the two $\angle$s respectively. Then $x + y = 108$ and $x - y = 16$; $2x = 124$ (Asmt. 1); $x = 62$ (Asmt. 4). Or $2y = 92$ (Asmt. 2); $y = 46$ (Asmt. 4). Then $62 + 46 + z = 180$ (§120); $z = 72$ (Asmt. 2). 62°, 46°, 72°. Ans.

31. $\angle C = \angle A$ (§189).
Alt. = 9 in. (§136).



32. AC = AB = BC. $\therefore \triangle ABC$ is equilateral. $\therefore \angle ACB = 60°$ (§84). In like manner, $\angle ACD = 60°$. $\therefore \angle BCD = 120°$ (Asmt. 1).

33. (1) EF = 10 (§207). (2) ER = 4 (§204). (3) SF = 4 (§204). (4) RS + 4 + 4 = 10 (Asmt. 9). $\therefore$ RS = 2 (Asmt. 2).

34. (1) DC = 18 (§207). (2) ER = 9 (§204). (3) SF = 9 (§204). (4) RS + 9 + 9 = 24 (Asmt. 9). $\therefore$ RS = 6 (Asmt. 2).

35. (1) DC = 10 (§204). (2) AB = 14 (§204). (3) SF = 5 (§204). (4) RS + 5 = 7 (Asmt. 9). $\therefore$ RS = 2 (Asmt. 2).

36. $x + 4x = 180$ (§162); $5x = 180$ (Asmt. 6); $x = 36$ (Asmt. 4). Then $4x = 144$ (Asmt. 3). 36°, 144°, 36°, 144°. Ans.

37. $AB = AD$ (Given). $\angle ABD = \angle ADB$ (§98a). $\angle ABD + \angle ADB + 60 = 180$ (§120). $2\angle ADB + 60 = 180$ (Asmt. 6). $2\angle ADB = 120$ (Asmt. 2). $\therefore \angle ADB = 60$ (Asmt. 4). $BD = AB = 10$ in. (§157a).

38. $\angle A + \angle B = 180°$ (§162). $\angle A + 150° = 180°$ (Asmt. 6). $\angle A = 30°$ (Asmt. 2). $\therefore$ Alt. = 6 in. (§136).

39. Let n = number of sides in a polygon. From each vertex $n - 3$ diagonals may be drawn. From the n vertices $n(n - 3)$ diagonals may be drawn, but each diagonal is duplicated. Hence there are $\frac{n(n - 3)}{2}$ diagonals. In a polygon of 10 sides there are $\frac{10 \times 7}{2}$, or 35 diagonals.

40. (1) 2. (2) 9. (3) $\frac{n(n - 3)}{2}$.

PAGE 234 (Test 14)

1. 8 sides. 2. is not. 3. 720°. 4. supplementary.
5. equilateral △. 6. two parallels. 7. 4 right angles.
8. one half.

PAGE 235 (Test 15)

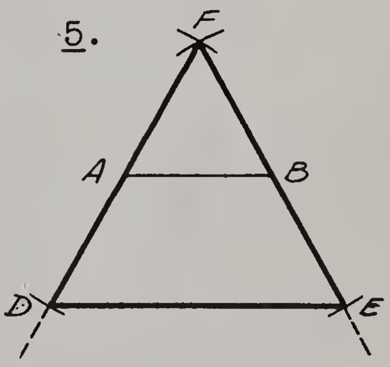
1. See §212.

2. See Ex. 3, page 224.

3. See Ex. 11, page 224.

4. See Ex. 10, page 224.

5.



PAGE 235 (Test 16)

1. 50°, 130°, 130°. 2. 10 in. 3. 12 in. 4. 120°.
5. 8. 6. 9 in. 7. 2 in. 8. 32. 9. 18. 10. 95.

Chapter 8. Areas of Polygons

PAGE 238

1. $S = bh = 50 \times 135 = 6750$ (sq.ft.).

2. $S = bh = 300 \times 160 = 48{,}000$ (sq.ft.).
$48{,}000 \div 9 = 5333\ 1/3$ (sq.yd.). $5333\ 1/3 \times \$0.36 = \$1920$.

3. $S = bh = 50 \times 80 = 4000$ (sq.rd.). $4000 \div 160 = 25$ (A.).
$25 \times \$80 = \$2000$.

4. $S = bh = 36 \times 78 = 2808$ (sq.ft.). $2808 \div 9 = 312$ (sq.yd.).
$312 \times \$0.40 = \$124.80$.

5. $S = bh$. $16.5h = 1300.3$. $h = 78.8$ (in.).

6. $S = bh = (2x - 1)(x + 4) = 2x^2 + 7x - 4$.

7. $8 \times \frac{11}{8} = 11$. $12 - (2 \times 1\frac{3}{8}) = 12 - 2\frac{3}{4} = 9\frac{1}{4}$, or $\frac{37}{4}$.
$\frac{37}{4} \times 2 = 18\frac{1}{2}$. $11 + 11 + 18\frac{1}{2} = 40\ 1/2$ (sq.in.).

8. Doubled. 9. Quadrupled.

PAGE 239

1. a. $\frac{2}{3}$. b. $\frac{1}{4}$. c. $\frac{1}{2x}$. d. $\frac{x}{2y}$. e. $\frac{1}{2a}$. f. $(a + b) : (a - b)$.

2. a. $1 : 3$. b. $1 : 2$. c. $1 : 6$. d. $1 : 2$.

3. We have $5x = 7y$. D_{5y}, $\frac{5x}{5y} = \frac{7y}{5y}$; whence $\frac{x}{y} = \frac{7}{5}$. Then $\frac{y}{x} = \frac{5}{7}$.

4. $5x$.

5. $2x + 5x = 105$; whence $7x = 105$, and $x = 15$. Then $2x = 30$, and $5x = 75$.
30, 75. Ans.

6. Let $3x$, $5x$, and $7x$ represent the 3 $\angle$s. Then $3x + 5x + 7x = 180$; whence $15x = 180$, and $x = 12$. Then $3x = 36$, $5x = 60$, and $7x = 84$.
36°, 60°, 84°. Ans.

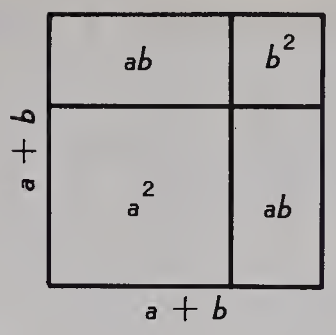
PAGE 240 (Areas of Rectangles)

1. 12 ft. 4 in. = 148 in. and 10 ft. 8 in. = 128 in.
148 in. ÷ 4 in. = 37 (tiles for length).
128 in. ÷ 4 in. = 32 (tiles for width).
$32 \times 37 = 1184$ (tiles for area).

2. $S = (x - 4)^2 = x^2 - 8x + 16$.

3. a. 2 : 3.
   b. 1 : 2.
   c. 1 : 2.
   d. 2 : 1.
   e. 3 : 2.
   f. 1 : 4.

4.



PAGE 240 (Bottom)

1. Yes. 2. No. 3. No.

4. Since congruent figures are figures that can be made to coincide, it follows that they must have the same size and shape. The symbol for congruent contains the symbol for equal to show that congruent figures are equal.

PAGE 242

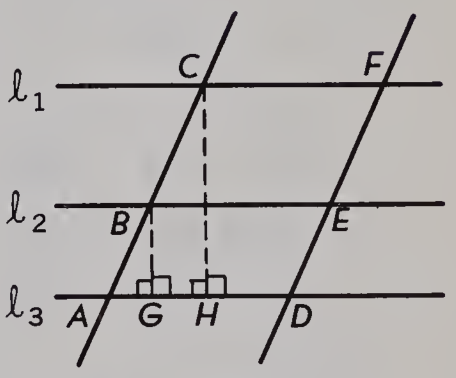
1. $S = bh = 18 \times 12 = 216$ (sq.in.).

2. $S = bh = 17.4 \times 11.4 = 198.36$.

3. $S = bh = 48 \times 33 = 1584$ (sq.in.).

4. All have the same area (§236).

5. $b = \frac{S}{h} = 8430 \div 150 = 56.2$ (ft.).



6. Construct $BG \perp \ell_3$ and $CH \perp \ell_3$ (§172). Since ∠CAD = 30° and AB = 2 in., BG = 1 in. (§136). Since AC = 5 in. and ∠CAD = 30°, CH = $2\frac{1}{2}$ in. (§136). BG is the altitude of ▱ADEB and CH is the altitude of ▱ADFC (§186). AD is the base of both ▱s (§186).

$\frac{\text{Area ADEB}}{\text{Area ADFC}} = \frac{BG}{CH}$ (§237). $\frac{\text{Area ADEB}}{\text{Area ADFC}} = \frac{1}{2\frac{1}{2}} = \frac{2}{5}$ (Asmt. 6).

7. 2 ft. = 24 in.; 5 ft. 4 in. = 64 in. $24 \times 64 = 1536$ (sq.in.). 90% of 1536 sq.in. = 1382.4 sq.in. Each tile face contains 2×1, or 2 sq.in. $1382.4 \div 2 = 691.2$. 692. Ans.

8. Let $3x$ = the altitude (a). Then $2x$ = the base (b). $3x + 2x = 40$. $5x = 40$. $x = 8$. Thus $a = 3x = 24$ (in.) and $b = 2x = 16$ (in.). $24 \times 16 = 384$. The area is 384 sq.in.

9. In rt. △ABD, ∠ABD = 30° and hypotenuse BD = 14 in. AD = 7 in. (§136). $12 \times 7 = 84$. The area is 84 sq.in.

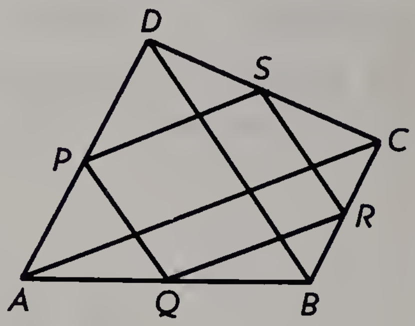
10. Given rectangle ABCD, diagonal AC, square S, and rectangle R.

To prove S = R.

Planning the Proof: 1. We can prove areas = by § 236 and Asmt. 2. We shall use Asmt.

Proof: 1. R and S are rectangles (Given). 2. The opposite sides of R are ∥ (§185). 3. The other two quadrilaterals are ▱s (§185). 4. ∴ x = y and m = n (§187). 5. △ABC ≅ △ACD (§187). 6. y + R + n = x + S + m (Asmt. 6, 9). 7. From 4 and 6, R = S (Asmt. 2).

11. Let P, Q, R, and S be midpoints of AD, AB, BC, and CD, respectively. AB = AD and BC = DC, so AC ⊥ BD (§ 159b). PS ∥ AC and = $\frac{1}{2}$AC, and QR ∥ AC and = $\frac{1}{2}$AC (§ 218a). Consequently, PQRS is a ▱ (§223c). Also, PQ ∥ BD and = $\frac{1}{2}$BD (§218a). Then PQ ⊥ AC and consequently PQ ⊥ QR (§159a). So ∠PQR is a rt. ∠ (§14) and PQRS is a rectangle (§ 185). Since AC = 12, QR = 6, and since BD = 8, PQ = 4, ∴ area PQRS = 24 (Asmt. 40).



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1. $S = \frac{1}{2}bh = \frac{1}{2} \times 17 \times 12 = 102.$

2. $S = \frac{1}{2}bh = \frac{1}{2} \times 21.4 \times 14.8 = 158.36.$

3. $\frac{1}{2}bh = S.$ $\frac{1}{2}b \times 9 = 576.$ b = 128 (ft.).

4. $S = \frac{1}{2}dd' = \frac{1}{2} \times 32 \times 40 = 640$ (sq.ft.).

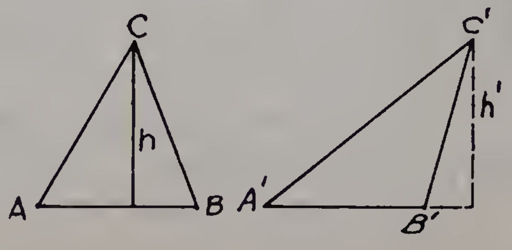
5. $\frac{1}{2}dd' = S.$ $\frac{1}{2} \times 56d = 2352.$ d = 84 (in.) = 7 ft.

6. $\frac{\triangle ACD}{\triangle BCD} = \frac{AD}{BD}$ (§240). $\frac{\triangle ACD}{\triangle BCD} = 1.$ Then △ACD = △BCD. Likewise, △AOD = △BOD. △AOC = △BOC (Asmt. 2). Similarly, △ABO = △BOC. ∴ △ABO = △BOC = △AOC = $\frac{1}{3}$△ABC = $\frac{1}{3} \times 108$ = 36 sq.in. Then △ODB = △BOE = △AOF = $\frac{1}{2} \times 36$ = 18 sq.in.

7. Given △s ABC and A'B'C' with = altitudes h and h' and = bases AB and A'B'.

To prove that △ABC = △A'B'C'.

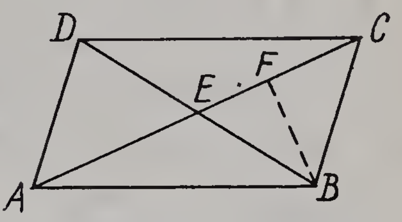
Planning the Proof: 1. We can compare areas of △s by §§ 95, 96, 156, 216, 239-241. 2. We shall use § 241.



Proof: 1. $\frac{\triangle ABC}{\triangle A'B'C'} = \frac{AB \times h}{A'B' \times h'}$ (§241). But AB = A'B' and h = h' (Given). 3. $\triangle ABC = \triangle A'B'C'$ (Asmt. 6, 3).

8. Given ▱ABCD with diagonals AC and BD intersecting in E.

To prove that $\triangle ABE = \triangle BEC$.



Planning the Proof: Same as in Ex. 7.

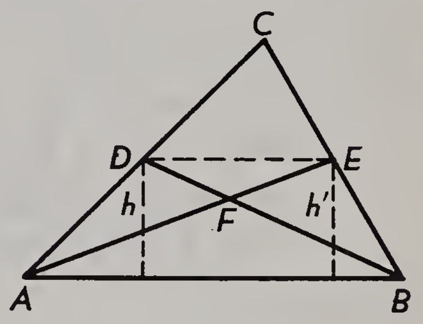
Proof: 1. Draw BF ⊥ AC (§178a). 2. BF is the altitude of $\triangle$s ABE and EBC (§90). 3. AE = EC (§217c). 4. $\therefore \triangle ABE = \triangle EBC$ (§241).

9. 1 : 1; 1 : 2; 1 : 2; 1 : 4.

10. h = 8 (§136). S = bh = 19 × 8 = 152.

11. Let h = the altitude from C to AB. Then the area of $\triangle ABC = \frac{1}{2}h \times 16 = 8h$. But the area of ABC $= \frac{1}{2} \times 15 \times 18 = 135$. $8h = 135$. $h = 16\frac{7}{8}$. The altitude from C to AB = $16\frac{7}{8}$ in.

12. Planning the Proof: 1. We can prove two geometric figures = by Asmt. 2. We shall use Asmt. 6, 2.



Proof: 1. BD and AE are medians (Given). 2. D and E are midpoints of AC and CB respectively (§89). 3. Draw DE (Asmt. 11). 4. DE ∥ AB (§204). 5. Construct h ⊥ AB from D and h' ⊥ AB from E (§178a). 6. h = h' (§193). 7. $\triangle ABD = \triangle ABE$ (See Ex. 7). 8. $\triangle AFB = \triangle AFB$ (Iden.). 9. $\triangle ABD - \triangle AFB = \triangle ABE - \triangle AFB$ (Asmt. 2). 10. $\triangle ADF = \triangle BFE$ (Asmt. 6). 11. $\triangle ACE = \triangle ABE$ (See Ex. 7). 12. $\triangle ACE - \triangle ADF = \triangle ABE - \triangle BFE$ (Asmt. 2). 13. Quadrilateral DFEC = $\triangle ABF$ (Asmt. 6).

PAGES 245-246

1. $S = \frac{1}{2}h(b + b') = \frac{1}{2} \times 18(9 + 13) = 198$ (sq.in.).

2. $S = \frac{1}{2}h(b + b') = \frac{1}{2} \times 15.7(24.3 + 16.9) = 323.42$.
323.4. Ans.

3. $\frac{1}{2}h(b + b') = S$. $\frac{1}{2}h(68 + 84) = 4256$. $h = 56$ (in.).

4. $\frac{h}{2}(b + b') = S$. $\frac{18}{2}(b + 24) = 504$. $9b + 216 = 504$. $9b = 288$. $b = 32$ (in.).

5. §240.

6. $\frac{h}{2}(b + b') = S$. $\frac{h}{2}(27 + 35) = 651$. $h = 21$ (ft.).

7. $\frac{h}{2}(b + b') = S$. $\frac{h}{2}(8 + 20) = 96$. $h = 6\frac{6}{7}$ (in.).

8. $S = \frac{h}{2}(b + b') = \frac{17}{2}(35) = 297.5$.

9. Planning the Proof: 1. We can compare the areas of $\triangle$s by §§95, 96, 156, 216, 239-241. 2. We shall use § 239, Asmt. 2.
Proof: 1. The $\perp$s from D and C to AB are = (§ 217b). 2. $\frac{\triangle ABD}{\triangle ABC} = 1$ (§ 239). 3. $\triangle ABD = \triangle ABC$ (Asmt. 3). 4. $\triangle ABE = \triangle ABE$ (Iden.). 5. $\therefore$ $\triangle ADE = \triangle BEC$ (Asmt. 2).

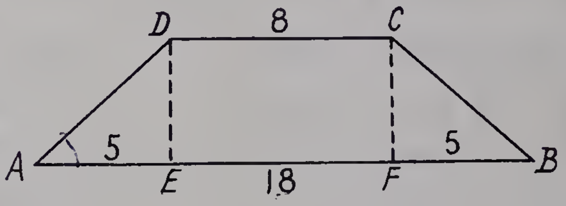
10. $S = \frac{h}{2}(b + b') = \frac{ab}{2}(2a - b + 2a + b) = 2a^2b$.

11. $\frac{h}{2}(b + b') = S$. $\frac{34.5}{2}(b + 155) = 3105$. $b = 25$ (in.).

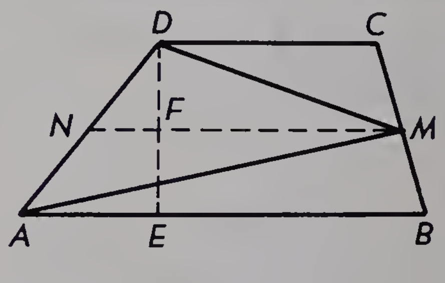
12. Area of $\triangle$ $= \frac{1}{2}bh = \frac{1}{2} \times 26.5 \times 10 = 132.5$. $S = \frac{h}{2}(b + b')$. $132.5 = \frac{h}{2}(25 + 28)$. $h = 5$.

13. The alt. from D to AB $= \frac{1}{2}(12) = 6$ (§136). $S = \frac{h}{2}(b + b') = \frac{6}{2}(24 + 10) = 102$.

14. $AE = FB = \frac{1}{2}(18 - 8) = 5$.
$AE = DE$ (§ 128). Then $DE = 5$.
$S = \frac{h}{2}(b + b') = \frac{5}{2}(8 + 18) =$ 65 (sq.in.).



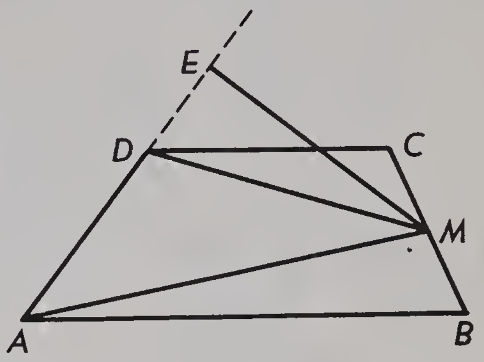
15. Given trapezoid ABCD with AB ∥ DC, M the midpoint of BC, and DM and AM drawn.
To prove area $\triangle AMD = \frac{1}{2}$ area trapezoid ABCD.
Planning the Proof: 1. We can compare areas by Asmt.; §§ 95, 96, 156, 216, 239-241. 2. We shall use Asmt. 1, 4, 6, 7, 9.



Proof: 1. Draw MN to N the midpoint of AD (§ 171, Asmt. 11). 2. MN is the median of trapezoid ABCD (§ 206). 3. Construct DE $\perp$ AB (§172). 4. DE is an altitude of ABCD (§ 206). 5. Area trapezoid ABCD $= \frac{1}{2}DE(AB + DC)$ (§ 244). 6. $AB + DC = 2MN$ (§ 207). 7. Area trapezoid ABCD $= \frac{1}{2}DE\ (2MN)$ (Asmt. 6). 8. Area trapezoid ABCD $= DE \times MN$ (Asmt. 6). 9. $\frac{1}{2}$ area trapezoid ABCD $= \frac{1}{2}DE \times MN$ (Asmt. 4). 10. MN ∥ AB ∥ DC (§ 207). 11. Let F be the intersection of DE with MN (§ 110). 12. $BM = MC$ (§ 22). 13. $EF = FD$ (§ 202). 14. Area $\triangle$ NMD $= \frac{1}{2}DF \times MN$ and area $\triangle$ AMN $= \frac{1}{2}EF \times MN$ (§ 238

15. Area $\triangle NMD$ + area $\triangle AMN = \frac{1}{2}(DF + EF)MN$ (Asmt. 1). 16. Area $\triangle NMD$ + area $\triangle AMN$ = area $\triangle AMD$ and $DF + EF = DE$ (Asmt. 9). 17. Area $\triangle AMD = \frac{1}{2}DE \times MN$ (Asmt. 6). 18. Area AMD = $\frac{1}{2}$ area trapezoid ABCD (Asmt. 7).

16. Given trapezoid ABCD with $AB \parallel DC$, $\triangle AMD$ with M the midpoint of BC, and $ME \perp AD$.

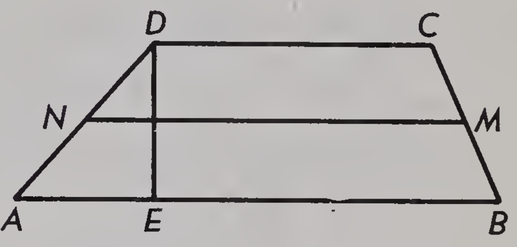


To prove area trapezoid ABCD = AD × ME.

Planning the Proof: 1. We can find the area of a trapezoid by § 244 and Asmt. 2. We shall use Asmt. 3.

Proof: 1. ME is an altitude of $\triangle AMD$ (§ 90). 2. Area $\triangle AMD = \frac{1}{2}AD \times ME$ (§ 238). 3. Area $\triangle AMD = \frac{1}{2}$ area trapezoid ABCD (Proved in Ex. 15). 4. $\frac{1}{2}$ area trapezoid ABCD $= \frac{1}{2}AD \times ME$ (Asmt. 7). 5. Area trapezoid ABCD = AD × ME (Asmt. 3).

17. Given trapezoid ABCD with $\parallel$ sides AB and DC, median MN, and altitude DE.



To prove area trapezoid ABCD = MN × DE.

Planning the Proof: 1. We can find the area of a trapezoid by § 244. 2. We shall use § 244.

Proof: Proof given in statements 1-8 of Ex. 15.

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1. $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$
2. $\sqrt{25} = 5$
3. $\sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$
4. $\sqrt{60} = \sqrt{4}\sqrt{15} = 2\sqrt{15}$
5. $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$
6. $\sqrt{81} = 9$
7. $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$
8. $\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$
9. $2\sqrt{16} = 2 \times 4 = 8$
10. $3\sqrt{8} = 3\sqrt{4}\sqrt{2} = 6\sqrt{2}$

11. $5\sqrt{75} = 5\sqrt{25}\sqrt{3} = 25\sqrt{3}$

12. $\sqrt{16} = 4$

13. $\sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

14. $\sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}$

15. $\sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{3} = \frac{1}{3}\sqrt{6}$

16. $\sqrt{\frac{9}{25}} = \frac{3}{5}$.

17. $\sqrt{\frac{3}{5}} = \frac{\sqrt{15}}{\sqrt{25}} = \frac{\sqrt{15}}{5} = \frac{1}{5}\sqrt{15}$

18. $\sqrt{\frac{1}{7}} = \frac{\sqrt{7}}{\sqrt{49}} = \frac{\sqrt{7}}{7} = \frac{1}{7}\sqrt{7}$

19. $\sqrt{\frac{1}{a}} = \frac{\sqrt{a}}{\sqrt{a^2}} = \frac{\sqrt{a}}{a} = \frac{1}{a}\sqrt{a}$

20. $\sqrt{\frac{1}{a^3}} = \frac{\sqrt{a}}{\sqrt{a^4}} = \frac{\sqrt{a}}{a^2} = \frac{1}{a^2}\sqrt{a}$

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1.
$$\begin{aligned} x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

2.
$$\begin{aligned} 4x^2 &= 100 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

3.
$$\begin{aligned} x^2 + 25 &= 75 \\ x^2 &= 50 \\ x &= \pm 5\sqrt{2} \end{aligned}$$

4.
$$\begin{aligned} 3y^2 - 192 &= 0 \\ 3y^2 &= 192 \\ y^2 &= 64 \\ y &= \pm 8 \end{aligned}$$

5.
$$\begin{aligned} x^2 - \frac{x^2}{4} &= 100 \\ 4x^2 - x^2 &= 400 \\ 3x^2 &= 400 \\ x^2 &= \frac{400}{3} \\ x &= \pm\frac{20}{3}\sqrt{3} \end{aligned}$$

6.
$$\begin{aligned} 4x^2 - \frac{1}{9} &= 0 \\ 36x^2 - 1 &= 0 \\ 36x^2 &= 1 \\ x^2 &= \frac{1}{36} \\ x &= \pm\frac{1}{6} \end{aligned}$$

7.
$$\begin{aligned} \frac{2x^2 - 7}{5} - \frac{x^2 - 4}{3} + \frac{8}{5} &= 0 \\ 6x^2 - 21 - 5x^2 + 20 + 24 &= 0 \\ 6x^2 - 5x^2 &= 21 - 20 - 24 \\ x^2 &= -23 \\ x &= \pm\sqrt{-23} = \pm i\sqrt{23} \end{aligned}$$

8.
$$\begin{aligned} \frac{x^2 - 1}{4} + \frac{3x^2 + 3}{10} &= 5 \\ 5x^2 - 5 + 6x^2 + 6 &= 100 \\ 5x^2 + 6x^2 &= 5 - 6 + 100 \\ 11x^2 &= 99 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

9.
$$\begin{aligned} (x - 2)^2 + 4x &= 16 \\ x^2 - 4x + 4 + 4x &= 16 \\ x^2 &= 16 - 4 \\ x^2 &= 12 \\ x &= \pm 2\sqrt{3}. \end{aligned}$$

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1.
$$\begin{gathered} x^2 - x - 6 = 0 \\ (x - 3)(x + 2) = 0 \\ \begin{array}{r|r} x - 3 = 0 & x + 2 = 0 \\ x = 3 & x = -2 \end{array} \end{gathered}$$

2.
$$\begin{gathered} x^2 + x - 30 = 0 \\ (x + 6)(x - 5) = 0 \\ \begin{array}{r|r} x + 6 = 0 & x - 5 = 0 \\ x = -6 & x = 5 \end{array} \end{gathered}$$

3.
$$\begin{gathered} 2x^2 + 5x - 3 = 0 \\ (2x - 1)(x + 3) = 0 \\ \begin{array}{r|r} 2x - 1 = 0 & x + 3 = 0 \\ 2x = 1 & x = -3 \\ x = \frac{1}{2} & \end{array} \end{gathered}$$

4.
$$\begin{gathered} 3x^2 + 14x = 5 \\ 3x^2 + 14x - 5 = 0 \\ (3x - 1)(x + 5) = 0 \\ \begin{array}{r|r} 3x - 1 = 0 & x + 5 = 0 \\ 3x = 1 & x = -5 \\ x = \frac{1}{3} & \end{array} \end{gathered}$$

5.
$$\begin{gathered} x^2 - 25 = 0 \\ (x + 5)(x - 5) = 0 \\ \begin{array}{r|r} x + 5 = 0 & x - 5 = 0 \\ x = -5 & x = 5 \end{array} \end{gathered}$$

6.
$$\begin{gathered} y^2 - 2y - 24 = 0 \\ (y - 6)(y + 4) = 0 \\ \begin{array}{r|r} y - 6 = 0 & y + 4 = 0 \\ y = 6 & y = -4 \end{array} \end{gathered}$$

7. $x^2 - 8x = 0$

$x(x - 8) = 0$

$$\begin{array}{r|r} x = 0 & x - 8 = 0 \\ & x = 8 \end{array}$$

8. $3x^2 - 15x = 0$

$3x(x - 5) = 0$

$$\begin{array}{r|r} 3x = 0 & x - 5 = 0 \\ x = 0 & x = 5 \end{array}$$

9. $12x^2 + 11x - 5 = 0$

$(4x + 5)(3x - 1) = 0$

$$\begin{array}{r|r} 4x + 5 = 0 & 3x - 1 = 0 \\ 4x = -5 & 3x = 1 \\ x = -1\frac{1}{4} & x = \frac{1}{3} \end{array}$$

10. $6x^2 - x - 1 = 0$

$(3x + 1)(2x - 1) = 0$

$$\begin{array}{r|r} 3x + 1 = 0 & 2x - 1 = 0 \\ 3x = -1 & 2x = 1 \\ x = -\frac{1}{3} & x = \frac{1}{2} \end{array}$$

11. $2y^2 - 3y = 35$

$2y^2 - 3y - 35 = 0$

$(2y + 7)(y - 5) = 0$

$$\begin{array}{r|r} 2y + 7 = 0 & y - 5 = 0 \\ 2y = -7 & y = 5 \\ y = -3\frac{1}{2} & \end{array}$$

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1. $x^2 + 2x = 8$

$x^2 + 2x + 1 = 9$

$x + 1 = \pm 3$

$x = -1 \pm 3$

$x = 2, \text{ or } -4$

2. $x^2 + 16x = 36$

$x^2 + 16x + 64 = 100$

$x + 8 = \pm 10$

$x = -8 \pm 10$

$x = 2, \text{ or } -18$

3. $4x^2 + 20x = 11$

$x^2 + 5x = \frac{11}{4}$

$x^2 + 5x + \frac{25}{4} = 9$

$x + \frac{5}{2} = \pm 3$

$x = -\frac{5}{2} \pm 3$

$x = \frac{1}{2}, \text{ or } -5\frac{1}{2}$

4. $x^2 - 6x = 7$

$x^2 - 6x + 9 = 16$

$x - 3 = \pm 4$

$x = 3 \pm 4$

$x = 7, \text{ or } -1$

5. $x^2 - 40x = 41$

$x^2 - 40x + 400 = 441$

$x - 20 = \pm 21$

$x = 20 \pm 21$

$x = 41, \text{ or } -1$

6. $m^2 + 8m = 9$

$m^2 + 8m + 16 = 25$

$m + 4 = \pm 5$

$m = -4 \pm 5$

$m = 1, \text{ or } -9$

7. $x^2 + 4x = 7$

$x^2 + 4x + 4 = 11$

$x + 2 = \pm\sqrt{11}$

$x = -2 \pm \sqrt{11}$

$x = -2 \pm 3.3$

$x = 1.3, \text{ or } -5.3$

8. $2x^2 + 5x + 1 = 0$

$x^2 + \frac{5}{2x} = -\frac{1}{2}$

$x^2 + \frac{5}{2x} + \frac{25}{16} = \frac{17}{16}$

$x + \frac{5}{4} = \pm\frac{\sqrt{17}}{4}$

$x = \frac{-5 \pm \sqrt{17}}{4}$

$x = -.2, \text{ or } -2.3$

9.
$$\begin{aligned} y^2 + 3y &= 1 \\ y^2 + 3y + \frac{9}{4} &= \frac{13}{4} \\ y + \frac{3}{2} &= \pm\frac{\sqrt{13}}{2} \\ y &= \frac{-3 \pm \sqrt{13}}{2} \\ y &= .3, \text{ or } -3.3 \end{aligned}$$

10.
$$\begin{aligned} x^2 + 10x + 24 &= 0 \\ x &= \frac{-10 \pm \sqrt{100 - 96}}{2} \\ x &= \frac{-10 \pm 2}{2} \\ x &= -4, \text{ or } -6 \end{aligned}$$

11.
$$\begin{aligned} x^2 - 11x + 30 &= 0 \\ x &= \frac{11 \pm \sqrt{121 - 120}}{2} \\ x &= \frac{11 \pm 1}{2} \\ x &= 6, \text{ or } 5. \end{aligned}$$

12.
$$\begin{aligned} x^2 - 2x - 40 &= 0 \\ x &= \frac{2 \pm \sqrt{4 + 160}}{2} \\ x &= \frac{2 \pm 2\sqrt{41}}{2} \\ x &= 1 \pm \sqrt{41} \end{aligned}$$

13.
$$\begin{aligned} 2x^2 + x - 3 &= 0 \\ x &= \frac{-1 \pm \sqrt{1 + 24}}{4} \\ x &= \frac{-1 \pm 5}{4} \\ x &= 1, \text{ or } -1\tfrac{1}{2} \end{aligned}$$

14.
$$\begin{aligned} 3y^2 + y - 10 &= 0 \\ y &= \frac{-1 \pm \sqrt{1 + 120}}{6} \\ y &= \frac{-1 \pm 11}{6} \\ y &= 1\tfrac{2}{3}, \text{ or } -2 \end{aligned}$$

15.
$$\begin{aligned} 6h^2 - h - 12 &= 0 \\ h &= \frac{1 \pm \sqrt{1 + 288}}{12} \\ h &= \frac{1 \pm 17}{12} \\ h &= 1\tfrac{1}{2}, \text{ or } -1\tfrac{1}{3} \end{aligned}$$

16.
$$\begin{aligned} p^2 - p - 5 &= 0 \\ p &= \frac{1 \pm \sqrt{1 + 20}}{2} \\ p &= \frac{1 \pm \sqrt{21}}{2} \end{aligned}$$

17.
$$\begin{aligned} x^2 + 6x + 1 &= 0 \\ x &= \frac{-6 \pm \sqrt{36 - 4}}{2} \\ x &= \frac{-6 \pm 4\sqrt{2}}{2} \\ x &= -3 \pm 2\sqrt{2} \end{aligned}$$

18.
$$\begin{aligned} x^2 - x - 1 &= 0 \\ x &= \frac{1 \pm \sqrt{1 + 4}}{2} \\ x &= \frac{1 \pm \sqrt{5}}{2} \end{aligned}$$

19.
$$\begin{aligned} x^2 - x - 90 &= 0 \\ (x - 10)(x + 9) &= 0 \end{aligned}$$
$$\begin{array}{r|r} x - 10 = 0 & x + 9 = 0 \\ x = 10 & x = -9 \end{array}$$

20.
$$\begin{aligned} 2x^2 - 5x - 12 &= 0 \\ (2x + 3)(x - 4) &= 0 \end{aligned}$$
$$\begin{array}{r|r} 2x + 3 = 0 & x - 4 = 0 \\ x = -\frac{3}{2} & x = 4 \end{array}$$

21.
$$\begin{aligned} x^2 - 42x - 400 &= 0 \\ (x - 50)(x + 8) &= 0 \end{aligned}$$
$$\begin{array}{r|r} x - 50 = 0 & x + 8 = 0 \\ x = 50 & x = -8 \end{array}$$

22.
$$\begin{aligned} y^2 + 2y &= 10 \\ y^2 + 2y + 1 &= 11 \\ y + 1 &= \pm\sqrt{11} \\ y &= -1 \pm \sqrt{11} \\ y &= -1 \pm 3.3 \\ y &= 2.3, \text{ or } -4.3 \end{aligned}$$

23.
$$\begin{aligned} m^2 - 3m - 88 &= 0 \\ (m - 11)(m + 8) &= 0 \end{aligned}$$
$$\begin{array}{r|r} (m - 11) = 0 & (m + 8) = 0 \\ m = 11 & m = -8 \end{array}$$

24. $p^2 - 4p - 11 = 0$

$$p = \frac{4 \pm \sqrt{16 + 44}}{2}$$

$$p = \frac{4 \pm \sqrt{60}}{2}$$

$$p = \frac{4 \pm 2\sqrt{15}}{2}$$

$$p = 2 \pm \sqrt{15}$$

$$p = 2 \pm 3.9$$

$$p = 5.9, \text{ or } -1.9$$

25. Let x = the no.

$$x^2 = x + 110$$

$$x^2 - x - 110 = 0$$

$$(x - 11)(x + 10) = 0$$

$$\begin{array}{r|r} x - 11 = 0 & x + 10 = 0 \\ x = 11 & x = -10 \end{array}$$

26. Let x = first no., then
$15 - x$ = second no.

$$x(15 - x) = 54$$

$$x^2 - 15x + 54 = 0$$

$$(x - 9)(x - 6) = 0$$

$$\begin{array}{r|r} x - 9 = 0 & x - 6 = 0 \\ x = 9 & x = 6 \\ 15 - x = 6 & 15 - x = 9 \end{array}$$

When the first no. is 9, the second is 6; and when the first no. is 6, the second is 9.

27. Let x = the length of the altitude, then $x + 3$ = the length of the base

$$x(x + 3) = 130$$

$$x^2 + 3x = 130$$

$$x^2 + 3x - 130 = 0$$

$$(x + 13)(x - 10) = 0$$

$$\begin{array}{r|r} x + 13 = 0 & x - 10 = 0 \\ x = -13 & x = 10 \\ & x + 3 = 13 \end{array}$$

13 in. Ans.

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1. 64. 2. 289. 3. (1) 49. (2) 81. 4. 24 in.
5. 256 sq.in.

6. SEBR = BFGC = 144; BE = DE = 16; SE = 9; DS = 7.

7. ACHK = 1296; DEBA = 1521; BFGC = 1521 - 1296 = 225.

8. AB = 16; AD = 16; ACHK = DSRA = 12 x 16 = 192.

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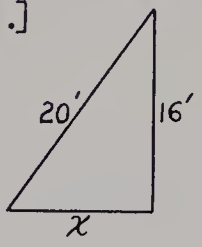
1. (a) $c^2 = 15^2 + 20^2 = 225 + 400 = 625$. $c = 25$.
(b) $a^2 + 12^2 = 13^2$. $a^2 + 144 = 169$. $a^2 = 25$. $a = 5$.
(c) $33^2 + b^2 = 44^2$. $1089 + b^2 = 1936$. $b^2 = 847$.
$b = 29.1$.
(d) $c^2 = 14.4^2 + 6^2 = 207.36 + 36 = 243.36$. $c = 15.6$

2. One side = 10. $d^2 = 10^2 + 10^2 = 200$. $d = 10\sqrt{2}$, or 14.14.

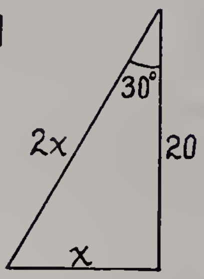
3. $x^2 + x^2 = 100$. $2x^2 = 100$. $x^2 = 50$. $x = \pm 7.07$.
Side = 7.1 in. Ans.

4. Let x = the number of feet from the foot of the ladder to the house. Then $x^2 + 256 = 400$; $x^2 = 144$; $x = \pm 12$. 12 ft. Ans.

[4.]



[6.]



5. Let x = a side of the square. Then $x^2 + x^2 = 50$; $2x^2 = 50$; $x^2 = 25$; $x = 5$.

6. $4x^2 = x^2 + 400$. $3x^2 = 400$. $x^2 = \frac{400}{3}$. $x = \sqrt{\frac{400}{3}} = \frac{20}{3}\sqrt{3}$, or 11.55.

7. Let x = a side of the square. Then $x^2 + x^2 = 27$; $2x^2 = 27$; $x^2 = \frac{27}{2}$; $x = \frac{3}{2}\sqrt{6}$.

8. Following Example 1, $x^2 + (4\sqrt{2})^2 = (8\sqrt{2})^2$. Then $x^2 + 32 = 128$; $x^2 = 96$; $x = 4\sqrt{6}$, or 9.80.

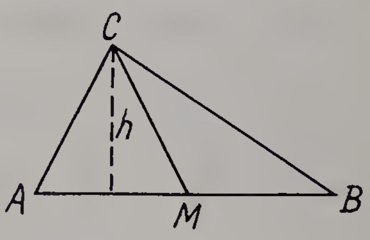
9. $d^2 = 24^2 + 45^2 = 576 + 2025 = 2601$; $d = 51$ (in.).

10. Let x = one side. Then $x^2 = \frac{x^2}{4} + 9216$; $4x^2 = x^2 + 36{,}864$; $3x^2 = 36{,}864$; $x^2 = 12{,}288$; $x = 110.85$ (in.).

11. Given $\triangle ABC$ with median CM.

To prove that $\triangle AMC = \triangle MBC$.

Planning the Proof: 1. We can compare areas of $\triangle$s by §§95, 96, 156, 216, 239, 240, 241. 2. We shall use §239, Asmt. 3.

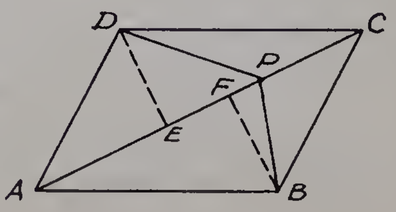


Proof: 1. Draw the altitude h from C to AB (§178a). 2. h is the altitude of $\triangle$s AMC and MBC (§90). 3. AM = MB (Given). 4. $\frac{\triangle AMC}{\triangle MBC} = \frac{1}{1}$ (§239). 5. $\triangle AMC = \triangle MBC$ (Asmt. 3).

12. Given ▱ABCD, P any point in diagonal AC, and lines PB and PD.

To prove that $\triangle ABP = \triangle ADP$.

Planning the Proof: 1. We can compare the areas of $\triangle$s by (Same as in Ex. 11). 2. We shall use §239, Asmt. 6, 3.



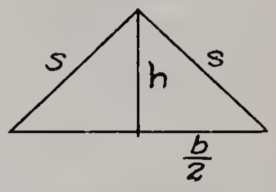
Proof: 1. Draw DE $\perp$ AC and BF $\perp$ AC (§178a). 2. AB $\parallel$ DC (§185). 3. $\angle$ACD = $\angle$CAB (§158a). 4. AB = DC (§217a). 5. $\therefore$ Rt. $\triangle$ABF = rt. $\triangle$DEC (§156a). 6. $\therefore$ BF = DE (§97a). 7. $\triangle$s ABP and ADP have the same base (AP is common to both). 8. $\frac{\triangle ABP}{\triangle ADP} = \frac{BF}{DE}$ (§239). 9. $\frac{\triangle ABP}{\triangle ADP} = \frac{BF}{BF} = 1$ (Asmt. 6). 10. $\therefore$ $\triangle$ABP = $\triangle$ADP (Asmt. 3).

13. Other side of $\triangle = \sqrt{52^2 - 40^2} = \sqrt{1104} = 33.23$ (ft.). By § 238, area = $\frac{1}{2} \times 33.2 \times 40 = 664.0$ (sq.ft.).

14. $A = (a - b)(a + b) - b^2 = a^2 - b^2 - b^2 = a^2 - 2b^2.$

15. $h = \sqrt{s^2 - \frac{b^2}{4}}.$

Area $= \frac{1}{2}b\sqrt{s^2 - \frac{b^2}{4}} = \frac{b}{4}\sqrt{4s^2 - b^2}.$

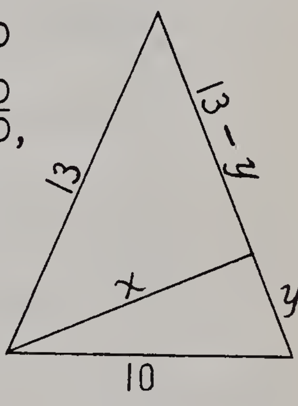


16. See §254.

17. Let x = the altitude and x + 91 = the base. Then $x^2 + (x + 91)^2 = 221^2$ (§250), $2x^2 + 182x = 40,560$; $x^2 + 91x = 20,280$. $x^2 + 91x + 2070.25 = 22,350.25$; $x + 45.5 = 149.5$; $x = 104$ (Asmt. 2). $x + 91 = 195$. Altitude, 104; base, 195. Ans.

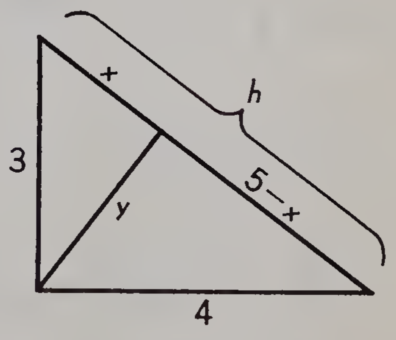
18. (1) $x^2 + y^2 = 100$
$x^2 + (13 - y)^2 = 169$

$$\begin{array}{rl} x^2 + y^2 & = 100 \\ x^2 - 26y + y^2 & = 0 \\ \hline 26y & = 100, \\ \text{and } y & = \frac{50}{13}. \end{array}$$



Substituting $y = \frac{50}{13}$ in $x^2 + y^2 = 100$, we have $x^2 + \frac{2500}{169} = \frac{16,900}{169}$, or $x^2 = \frac{14,400}{169}$, and $x = \frac{120}{13} = 9.2$. (2) $A = \frac{1}{2} \times 13 \times \frac{120}{13} = 60$ (square units).

19. Let h = the length of the hypotenuse. $h^2 = 3^2 + 4^2 = 9 + 16 = 25$. $h = 5$. Let x = the length of the segment of the hypotenuse adjacent to the 3-inch side. Then 5 − x = the length of the segment of the hypotenuse adjacent to the 4-inch side. Let y = the length of the altitude to the hypotenuse. Then $y^2 + (5 - x)^2 = 4^2$, whence $y^2 + 25 - 10x + x^2 = 16$, and $y^2 = -x^2 + 10x - 9$. Also $y^2 + x^2 = 3^2$, whence $y^2 = 9 - x^2$. Then, $-x^2 + 10x - 9 = 9 - x^2$, so $10x = 18$, and $x = 1.8$. $5 - x = 3.2$. The segment of the hypotenuse adjacent to the 3-inch side is 1.8 in. long and the other segment is 3.2 in. long.



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1. $d = s\sqrt{2}$
$d = 6\sqrt{2}$

2. $d = s\sqrt{2}$
$d = \sqrt{2}\sqrt{2} = 2$

3. $d = s\sqrt{2}$
$d = 8\sqrt{2}$

4. $d = s\sqrt{2}$
$d = 2\sqrt{3}\sqrt{2}$
$d = 2\sqrt{6}$

5. $h = \frac{s}{2}\sqrt{3}$
$h = \frac{10}{2}\sqrt{3} = 5\sqrt{3}$

6. $h = \frac{s}{2}\sqrt{3}$
$h = \frac{18}{2}\sqrt{3} = 9\sqrt{3}$

7. $h = \frac{s}{2}\sqrt{3}$
$h = \frac{15}{2}\sqrt{3}$

8. $h = \frac{s}{2}\sqrt{3}$
$h = \frac{14\sqrt{2}}{2}\sqrt{3}$
$h = 7\sqrt{6}$

9. $A = \frac{s^2}{4}\sqrt{3}$
$A = \frac{64}{4}\sqrt{3} = 16\sqrt{3}$

10. $A = \frac{s^2}{4}\sqrt{3}$
$A = \frac{144}{4}\sqrt{3} = 36\sqrt{3}$

11. $A = \frac{s^2}{4}\sqrt{3}$
$A = \frac{49}{4}\sqrt{3}$

12. $A = \frac{s^2}{4}\sqrt{3}$
$A = \frac{5}{4}\sqrt{3}$

13. $\frac{s^2}{4}\sqrt{3} = 9\sqrt{3}$. $s^2\sqrt{3} = 36\sqrt{3}$; $s^2 = 36$; $s = 6$.

14. $\frac{s^2}{4}\sqrt{3} = 16\sqrt{3}$. $s^2\sqrt{3} = 64\sqrt{3}$; $s^2 = 64$; $s = 8$.

15. $\frac{s^2}{4}\sqrt{3} = \frac{1}{4}\sqrt{3}$. $s^2\sqrt{3} = \sqrt{3}$; $s^2 = 1$; $s = 1$.

16. $\frac{s^2}{4}\sqrt{3} = \sqrt{3}$. $s^2\sqrt{3} = 4\sqrt{3}$; $s^2 = 4$; $s = 2$.

17. $\frac{s^2}{4}\sqrt{3} = 16$. $s^2\sqrt{3} = 64$; $s^2 = \frac{64\sqrt{3}}{3}$; $s = \frac{8\sqrt{3}\sqrt[4]{3}}{3}$.

18. $\frac{s^2}{4}\sqrt{3} = 20$. $s^2\sqrt{3} = 80$; $s^2 = \frac{80\sqrt{3}}{3}$; $s = \frac{4\sqrt{15}\sqrt[4]{3}}{3}$.

19. $\frac{s^2}{4}\sqrt{3} = 64$. $s^2\sqrt{3} = 256$; $s^2 = \frac{256\sqrt{3}}{3}$; $s = \frac{16\sqrt{3}\sqrt[4]{3}}{3}$.

20. $\frac{s^2}{4}\sqrt{3} = 100$. $s^2\sqrt{3} = 400$; $s^2 = \frac{400\sqrt{3}}{3}$; $s = \frac{20\sqrt{3}\sqrt[4]{3}}{3}$.

21. The hexagon consists of 6 equal equilateral triangles. Hence $A = 6 \times \frac{s^2\sqrt{3}}{4} = \frac{6 \times 8^2\sqrt{3}}{4} = 96\sqrt{3} = 166.272$ (sq.in.).

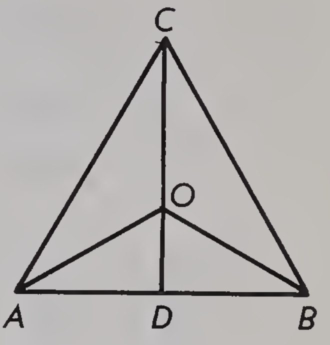
22. $h = \frac{3}{2}\sqrt{3}$. $15 = \frac{s}{2}\sqrt{3}$; $30 = s\sqrt{3}$; $s = \frac{30}{\sqrt{3}}$; $s = 10\sqrt{3} = 17.32$ (in.).

23. $h = \frac{s}{2}\sqrt{3}$. $2h = s\sqrt{3}$; $s = \frac{2h}{\sqrt{3}}$; $s = \frac{2\sqrt{3}h}{3}$.

24. $s = \frac{2\sqrt{3}h}{3}$. $s = \frac{2\sqrt{3}(9)}{3} = 6\sqrt{3}$; $A = \frac{(6\sqrt{3})^2\sqrt{3}}{4} = 27\sqrt{3} = 46.76$ (sq.in.).

25. $s = \frac{2\sqrt{3}h}{3}$. $A = \frac{1}{2}bh$. $A = \frac{1}{2}(\frac{2\sqrt{3}h}{3})h = \frac{h^2\sqrt{3}}{3}$.

26. $A = 6(\frac{s^2\sqrt{3}}{4})$. $54\sqrt{3} = 6(\frac{s^2\sqrt{3}}{4})$; $\frac{s^2}{4} = 9$; $s^2 = 36$; $s = 6$ (ft.).



27. CD is an altitude (Given). $CD \perp AB$ (§90). $\angle ADC = \angle BDC$ (§14). $\angle CAD = \angle CBD$ (§84). $\angle ACD = \angle BCD$ (§158c). $AC = BC$ (§56). $CO = CO$ (Iden.). $\triangle ACO = \triangle BCO$ (§95a). Area $\triangle ACO$ + area $\triangle BCO$ = area quad. AOBC (Asmt. 9). 2 area $\triangle ACO$ = area quad. AOBC (Asmt. 6). Area $\triangle ACO$ = $\frac{1}{2}$ quad. AOBC (Asmt. 4). Area $\triangle AOB = \frac{1}{2}AB \times OD$ (§238). $CO = \frac{2}{3}CD$ (Given). $OD = \frac{1}{3}CD$ (Asmt. 2). Area $\triangle AOB = \frac{1}{2}AB \times \frac{1}{3}CD = \frac{1}{6}AB \times CD$ (Asmt. 6). Area $\triangle ABC = \frac{1}{2}AB \times CD$ (§238). $\frac{1}{3}$ area $\triangle ABC = \frac{1}{6}AB \times CD$ (Asmt. 4). Area $\triangle AOB = \frac{1}{3}$ area $\triangle ABC$ (Asmt. 7). Area $\triangle ABC$ − area $\triangle AOB = \frac{2}{3}$ area $\triangle ABC$ (Asmt. 2). Area quad. AOBC = $\frac{2}{3}$ area $\triangle ABC$ (Asmt. 6). Area ACO = $\frac{1}{2}(\frac{2}{3}$ area $\triangle ACB)$ = $\frac{1}{3}$ area $\triangle ACB$ (Asmt. 6). Area $\triangle BCO = \frac{1}{3}$ area $\triangle ABC$ (Asmt. 6). Area $\triangle ACO$ = area $\triangle BCO$ = area $\triangle ABO$ (Asmt. 7).

28. From Ex. 25, $\frac{h^2\sqrt{3}}{3} = 60\sqrt{3}$. $\therefore \frac{h^2}{3} = 60$; $h^2 = 180$; $h = 6\sqrt{5}$, or 13.42.

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1. If a triangle is a right triangle, the sum of the squares of two sides is equal to the square of the third side.

2. Yes.

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1. $A = \sqrt{s(s-a)(s-b)(s-c)}$. $s = \frac{1}{2}(13 + 20 + 21) = 27$.
$A = \sqrt{27 \times 14 \times 7 \times 6} = 126$.

2. $A = \sqrt{s(s-a)(s-b)(s-c)}$. $s = \frac{1}{2}(13 + 14 + 15) = 21$.
$A = \sqrt{21 \times 8 \times 7 \times 6} = 84$.

3. $A = \sqrt{s(s-a)(s-b)(s-c)}$. $s = \frac{1}{2}(25 + 17 + 28) = 35$.
$A = \sqrt{35 \times 10 \times 18 \times 7} = 210$.

4. $A = \sqrt{s(s-a)(s-b)(s-c)}$. $s = \frac{1}{2}(29 + 36 + 25) = 45$.
$A = \sqrt{45 \times 16 \times 9 \times 20} = 360$.

5. $A = \sqrt{s(s - a)(s - b)(s - c)}$. $\quad s = \frac{1}{2}(5 + 6 + 7) = 9$.
$A = \sqrt{9 \times 4 \times 3 \times 2} = 6\sqrt{6}$.

6. $A = \sqrt{s(s - a)(s - b)(s - c)}$. $\quad s = \frac{1}{2}(5 + 12 + 15) = 16$.
$A = \sqrt{16 \times 11 \times 4 \times 1} = 8\sqrt{11}$.

7. $A = \sqrt{s(s - a)(s - b)(s - c)}$. $\quad s = \frac{1}{2}(200 + 300 + 400) = 450$.
$A = \sqrt{450 \times 250 \times 150 \times 50} = 7500\sqrt{15}$.

8. $A = \sqrt{s(s - a)(s - b)(s - c)}$. $\quad s = \frac{1}{2}(a + b + a - b + 2c) = a + c$.
$A = \sqrt{(a + c)(c - b)(c + b)(a - c)} = \sqrt{(a^2 - c^2)(c^2 - b^2)}$.

9. $A = \sqrt{s(s - a)(s - b)(s - c)}$. $\quad s = \frac{1}{2}(m + n + m + p + n + p)$
$= m + n + p$.
$A = \sqrt{(m + n + p) \cdot p \cdot n \cdot m} = \sqrt{mnp(m + n + p)}$.

10. Let S represent the side of the equilateral triangle.
$A = \sqrt{s(s - a)(s - b)(s - c)}$.
$s = \frac{1}{2}(S + S + S) = \frac{3S}{2}$. $\quad A = \sqrt{\frac{3S}{2} \times \frac{S}{2} \times \frac{S}{2} \times \frac{S}{2}} = \frac{S^2}{4}\sqrt{3}$.

11. By Ex. 4, $\sqrt{s(s - a)(s - b)(s - c)} = 360$.
For altitude to side 36: $h = \frac{2}{36}(360) = 20$. (§ 258).
For altitude to side 29: $h = \frac{2}{29}(360) = 24\frac{24}{29}$ (§ 258).
For altitude to side 25: $h = \frac{2}{25}(360) = 28\frac{4}{5}$ (§ 258).

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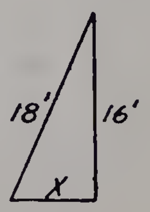
1. Area = bh = 17.5 x 11.5 = 201.25 (sq.ft.), or 201 sq.ft., approx.

2. Let x = length of the rope in feet. Then $x^2 = 22^2 + 34^2$; $x = \sqrt{1640} = 40.49$ (ft.), or 40 ft., approx.

3. Area of floor = 12 x 24 = 288 (sq.ft.) = 32 sq.yd. Cost of carpet = 3.25(32) = 104. 108 - 104 = 4. The carpet would be $4 cheaper.

4. Length of one side in rods = $\sqrt{1847} = 42.976$ (rd.), or 42.98 rd., approx.

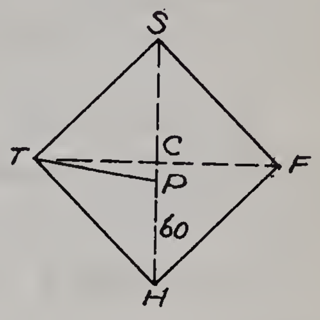
5. Let x = no. of ft. in distance between foot of ladder and the house. Then $x^2 + 16^2 = 18^2$; $x^2 = 68$; $x = 8.246$ (ft.), or 8.2 ft., approx.



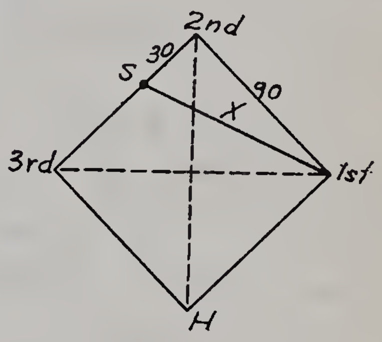
6. The required distance = the diagonal of the field. Then $d^2 = 90^2 + 90^2 = 2(90)^2$; $d = 90\sqrt{2} = 127.26$ (ft.), or 130 ft., approx.

7. (1) Since the diagonals of a square are = (Ex. 8, p. 206), the distance from home plate to second base = d = 127.26 ft., or 127 ft., approx. Then 127 − 60 = 67 (ft.), approx.

(2) $TC = \frac{1}{2}d = \frac{1}{2}(127.26) = 63.63$ (ft.), or 64 ft., approx. (§217c, Asmt. 4, 6). PC = 64 − 60 = 4 ft., approx. Then $\overline{PT}^2 = \overline{TC}^2 + \overline{PC}^2$ (§250). $\overline{PT}^2 = 64^2 + 4^2 = 4112$. $PT = \sqrt{4112} = 64.1$ (ft.), or 64 ft., approx.



8. Let x = no. of ft. shortstop must throw. $x^2 = 90^2 + 30^2$; $x = \sqrt{9000} = 94.8$ (ft.), or 95 ft., approx.

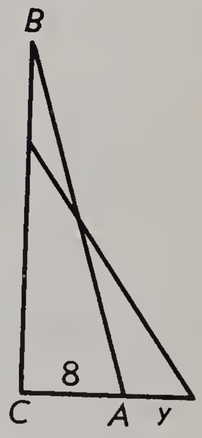


9. $A = \sqrt{s(s-a)(s-b)(s-c)}$.

$s = \frac{1}{2}(17.2 + 12.3 + 8.5) = 19.0$.

$A = \sqrt{(19.0)(1.8)(6.7)(10.5)} = \sqrt{2405.97}$. A = 49.05 sq.in., or 49.1 sq.in., approx.

10. Let CB represent the height on the wall to which the ladder originally reaches. Then $\overline{CB}^2 = 50^2 - 8^2 = 2436$. $CB = \sqrt{2436} = 49.3$ (ft.). The height of the ladder after the top is lowered = 49.3 − 6 = 43.3 (ft.). Let y = no. of ft. foot of ladder moves when top is lowered 6 ft. Then $(43.3)^2 + (y + 8)^2 = 50^2$. $1874.89 + x^2 + 16x + 64 = 2500$; $x^2 + 16x - 561.11 = 0$; $x = \frac{-16 \pm \sqrt{16^2 + 4(561.11)}}{2}$; x = 17.002 (ft.), or 20 ft., approx.



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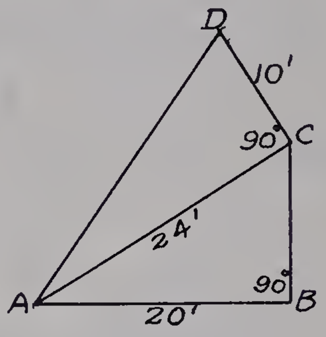
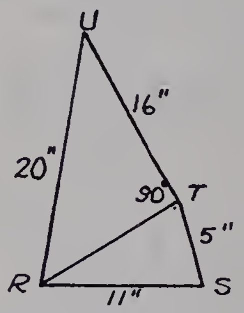
1. Area of $\triangle AFG = \frac{1}{2}(2)(3.5) = 3.5$, or 4, approx. HK = HL − KL = 3; then area of trapezoid EFGK = $\frac{1}{2}(3.5 + 5)(5) = 21.25$, or 20, approx. Area of $\triangle DEK = \frac{1}{2}(5)(4) = 10$. AH = AG + GH = 4; then area of $\triangle ABH = \frac{1}{2}(4)(4.5) = 9$. Area of trapezoid BCLH = $\frac{1}{2}(4.5 + 6)(5) = 26.25$, or 30, approx. LD = KD − KL = 2. Area of $\triangle CDL = \frac{1}{2}(6)(2) = 6$. Area of ABCDEF = 4 + 20 + 10 + 9 + 30 + 6 = 79, or 80, approx. (If the given data were exact, the result obtained would be 76.)

2. In $\triangle ADC$, $s = \frac{1}{2}(15 + 16 + 22) = 26.5$. Area $\triangle ADC = \sqrt{(26.5)(10.5)(11.5)(4.5)} = \sqrt{14399.415} = 119.99$ (sq.ft.), or 120 sq.ft., approx. In $\triangle ABC$, $s = \frac{1}{2}(22 + 20 + 25) = 33.5$. Area $\triangle ABC = \sqrt{(33.5)(11.5)(13.5)(8.5)} = \sqrt{44207.395} = 210.25$ (sq.ft.), or 210 sq.ft., approx. Area polygon 120 + 210 = 330 (sq.ft.), or 330 sq.ft., approx. (If the given data were exact, the result obtained would be 320.24^+ sq.ft.).

3. $BC = \sqrt{24^2 - 20^2} = 13.3$. Area $\triangle ABC = \frac{1}{2}(20)(13.3) = 133$ (sq.ft.), or 130 sq.ft., approx. Area $\triangle ACD = \frac{1}{2}(10)(24) = 120$ (sq.ft.). Area polygon ABCD = 130 + 120 = 250 (sq.ft.), approx. (If the data were exact, the result obtained would be 253 sq.ft.).



4. $RT = \sqrt{20^2 - 16^2} = 12$. Area $\triangle RTU = \frac{1}{2}(12)(16) = 96$ (sq.in.), or 100 sq.in., approx. In $\triangle RST$, $S = \frac{1}{2}(12 + 11 + 5) = 14$. Area $\triangle RST = \sqrt{14(2)(9)(3)} = \sqrt{756} = 27.49$, or 30 sq.in., approx. Total area = 100 + 30 = 130 (sq.in.), approx. (If the data were exact, the result obtained would be 123.49 sq.in.).



5. In $\triangle ACE$, $s = \frac{1}{2}(18 + 60 + 70) = 74$. Area $\triangle ACE = \sqrt{74(14)(4)(56)} = \sqrt{232064} = 481.7^+$ (sq.rd.), or 480 sq.rd., approx. Area $\triangle ABC = \frac{1}{2}(22)(60) = 660$ (sq.rd.). Area $\triangle ECD = \frac{1}{2}(14)(70) = 490$ (sq.rd.). Area field = 480 + 660 + 490 = 1630 (sq.rd.). 1630 ÷ 160 = 10.2, no. of acres. $\frac{324}{10.2} = 31.7$, or 32 bu. per acre, approx.

6. Area EABF $= \frac{45}{2}(60 + 80) = 3150$, or 3200, approx.

Area FBCH $= \frac{33}{2}(80 + 56) = 2244$, or 2200, approx.

$\therefore$ Area EABCH = 3200 + 2200 = 5400, approx.

Area EADG $= \frac{53}{2}(60 + 40) = 2650$, or 2600, approx.

Area GDCH $= \frac{25}{2}(40 + 56) = 1200$, approx.

$\therefore$ Area EADCH = 2600 + 1200 = 3800, approx.

$\therefore$ Area ABCD = 5400 - 3800 = 1600, approx. (If the data were exact, the result obtained would be 1544.)

7. Approximately 85,000 sq.mi.

8. Area of first quadrilateral at the left = $\frac{x}{2}(y_0 + y_1)$

Area of second quadrilateral = $\frac{x}{2}(y_1 + y_2)$

Area of third quadrilateral = $\frac{x}{2}(y_2 + y_3)$

Area of fourth quadrilateral = $\frac{x}{2}(y_3 + y_n)$

$$\therefore S = \frac{x}{2}(y_0 + y_1) + \frac{x}{2}(y_1 + y_2) + \frac{x}{2}(y_2 + y_3) + \frac{x}{2}(y_3 + y_n)$$

$$S = \frac{x}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + y_n)$$

9. $S = \frac{x}{2}(y_0 + y_n + 2y_1 + 2y_2 + 2y_3) = x\left(\frac{y_0 + y_n + 2y_1 + 2y_2 + 2y_3}{2}\right)$ $= x\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + y_3\right)$ (Asmt. 6).

10. No matter how many trapezoids are drawn upon AB, each of the $\perp$s thus formed (except the first and last), will be the side of two trapezoids and will appear in the formula of Ex. 8 as $2y_1$, $2y_2$, . . . For n trapezoids there are n + 1 $\perp$s. If we subtract the first and last $\perp$s from the total number of $\perp$s, we have n + 1 - 2, that is, n - 1 intermediate $\perp$s. Proceeding as in Ex. 8, we then have $S = \frac{x}{2}(y_0 + 2y_1 + 2y_2 + 2y_3 + \cdots + 2y_{n-1} + y_n)$. Whence $S = x\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + y_3 + \cdots + y_{n-1}\right)$.

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1. Let S = the area of the tract.

By §261 $S = 100\left(\frac{190 + 150}{2} + 200 + 180 + 140 + 100\right)$

$S = 7900$ (sq.ft.), approximately.

2. Since the least precise measurement made by the botanist is to the nearest inch, we must round off all his more precise measurements to the same unit of measure. Thus $1\frac{7}{8}$ in. becomes 2 in., $2\frac{7}{8}$ in. becomes 3 in., and $2\frac{1}{4}$ in. becomes 2 in.

Let S = the area of the leaf.

By §261 $S = 1\left(\frac{0 + 0}{2} + 2 + 3 + 3 + 2 + 1\right) = 11$ (sq.in.), approx.

If the botanist wanted a closer approximation to the true area he would have to use a unit of measurement more precise than one inch for all his measurements.

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1. Let s denote the side of the square, then $s^2 = \frac{1}{2} \times 144 \times 72 = 72^2$. Then s = 72 (in.).

2. Let d = the length of the short diagonal. Then 2d = the length of the long diagonal. By § 242 $\frac{1}{2} \times d \times 2d = 2304$. $\therefore d^2 = 2304$, $d = 48$, and $2d = 96$. The short diagonal is 48 in. and the long diagonal is 96 in.

3. Let s = a side of the equilateral $\triangle$. Then $s^2 = 15^2 + 20^2 = 625$. Then $A = \frac{s^2}{4}\sqrt{3} = \frac{625}{4}\sqrt{3}$, or 270.625.

4. Doubled. 5. Nine times as large. 6. Halved.

7. Since $3^2 + 4^2 = 5^2$, that is, since $9 + 16 = 25$, the $\triangle$ is a rt. $\triangle$ (§255).

8. $A_{\triangle} = \sqrt{s(s-a)(s-b)(s-c)}$. $s = \frac{1}{2}(16 + 30 + 34) = 40$.

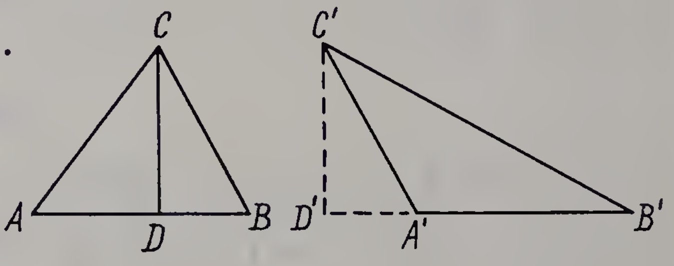
$A = \sqrt{40 \times 24 \times 10 \times 6} = 240$. Then $17x = 240$;

$x = 14\frac{2}{17}$ (in.). The area may also be found by showing that the $\triangle$ is a rt. $\triangle$ with base 16 and altitude 30, in which case $A_{\triangle} = \frac{1}{2} \times 16 \times 30 = 240$.

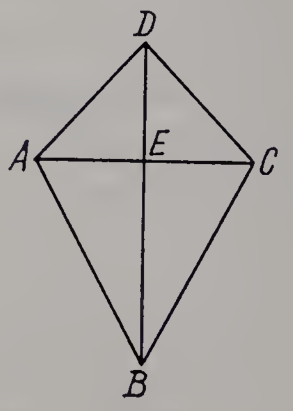
9. Given $\triangle$s ABC and A'B'C' with AB = A'B', AC = A'C', and $\angle$BAC supp. $\angle$B'A'C'.

To prove that $\triangle ABC = \triangle A'B'C'$.

Planning the Proof: 1. We can compare areas of $\triangle$s by §§ 95, 96, 156, 216, 239-241 and Asmt. 2. We shall use § 239.

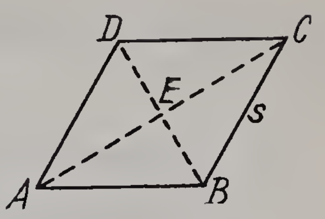


Proof: 1. AB = A'B', AC = A'C', and $\angle$BAC is supp. $\angle$B'A'C' (Given). 2. Draw CD $\perp$ AB and C'D' $\perp$ A'B' extended (§178a, Asmt. 3). 3. $\angle$D'A'C' is supp. $\angle$B'A'C' (Asmt. 24). 4. $\therefore \angle$DAC = $\angle$D'A'C' (§50c). 5. $\therefore$ Rt. $\triangle$ ACD $\cong$ rt. $\triangle$ A'C'D' (§156a). 6. $\therefore$ CD = C'D' (§97a). 7. $\frac{\triangle ABC}{\triangle A'B'C'} = \frac{CD}{C'D'}$ (§239). 8. $\therefore \frac{\triangle ABC}{\triangle A'B'C'} = 1$ (Asmt. 6). 9. $\therefore \triangle ABC = \triangle A'B'C'$ (Asmt. 3).



10. 1. AD = DC and BA = BC (Given). 2. BD $\perp$ AC (§159b). 3. $\triangle ACD = \frac{1}{2}DE \times AC$ and $\triangle ABC = \frac{1}{2}BE \times AC$ (§238). 4. $\triangle ACD + \triangle ABC = \frac{1}{2}DE \times AC + \frac{1}{2}BE \times AC$ (Asmt. 1). 5. $\therefore ABCD = \frac{1}{2}AC(DE + BE) = \frac{1}{2}dd'$ (Asmt. 6).

11. 1. Let $\angle B = 120°$. Then $\angle A = 60°$ (§222). 2. AD = AB (§194). 3. $\therefore \angle$ABD = $\angle$ADB (§98a). 4. $\angle$ABD +



$\angle ADB + 60° = 180°$ (§120). 5. $\angle ABD + \angle ADB = 120°$ (Asmt. 2). 6. $2\angle ABD = 120°$ (Asmt. 6). 7. $\angle ABD = 60°$ (Asmt. 4). 8. $\angle ADB = 60°$ (Asmt. 6). 9. $\therefore \triangle ABD$ is equilateral (§129). 10. $\therefore$ Area of $\triangle ABD = \frac{s^2}{4}\sqrt{3}$ (§254). 11. In the same manner, it can be shown that the area of $\triangle BCD = \frac{s^2}{4}\sqrt{3}$ (Statements 1-10). 12. $\triangle ABD + \triangle BCD = \frac{s^2}{4}\sqrt{3} + \frac{s^2}{4}\sqrt{3}$ (Asmt. 1). 13. $\therefore ABCD = \frac{s^2}{2}\sqrt{3}$ (Asmt. 6).

12. Let s = a side of the $\triangle$. Then $\frac{s^2}{4}\sqrt{3} = 18^2 = 324$. $s^2\sqrt{3} = 1296$; $s^2 = 432\sqrt{3}$; $s = 12\sqrt{3}\sqrt[4]{3}$, or 27.35 (in.).

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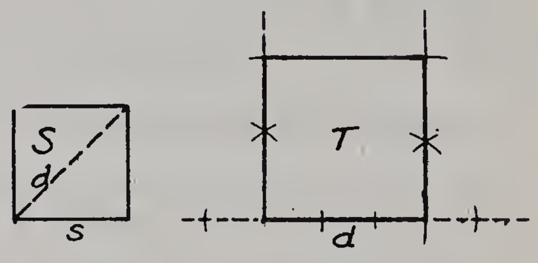


1. **Given** the square S with side s.

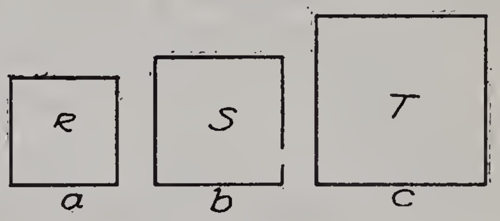
To construct a square T = 2S.

Construction: 1. Draw d, a diagonal of square S (Asmt. 11). 2. With a side = d, construct square T.

Then T is the required square.

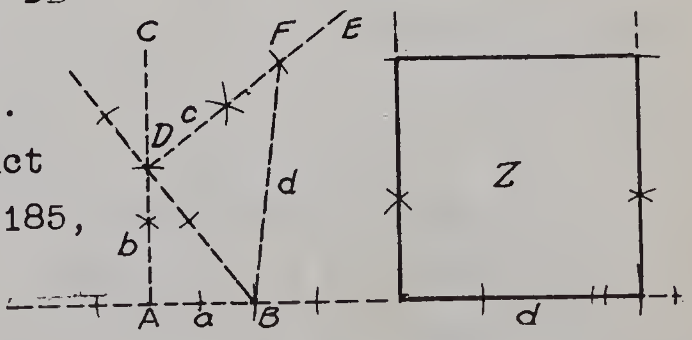
Proof: 1. $S = s^2$ (§233). 2. $d^2 = s^2 + s^2 = 2s^2$ (§250, Asmt. 6). 3. T is a square with side = d (Const.). 4. $T = d^2$ (§233). 5. $\therefore T = 2S$ (Asmt. 6).

2. **Given** the squares R, S, and T with sides a, b, and c respectively.



To construct a square Z = R + S + T.

Construction: 1. Construct AB = a (§9). 2. Construct AC ⊥ AB (§178b). 3. Construct AD = b (§9). 4. Draw DB (Asmt. 11). 5. Construct DE ⊥ BD (§178b). 6. Construct DF = c (§9). 7. Draw FB (Asmt. 11). 8. Construct square Z having a side = BF (d) (§§185, 178b).

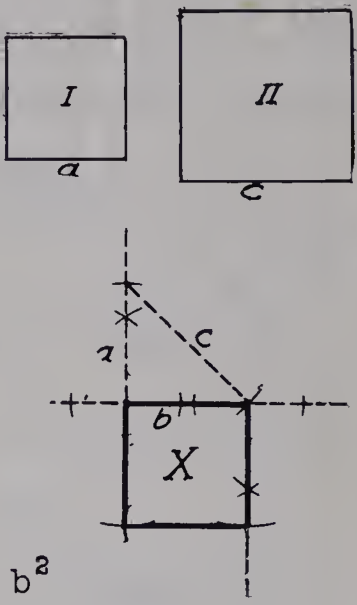


Then Z is the required square.

Proof: 1. ABD is a rt. $\triangle$ (Const.). 2. $\overline{AB}^2 + \overline{AD}^2 = \overline{BD}^2$ (§250). 3. AB = a, AD = b (Const.). 4. $a^2 + b^2 = \overline{BD}^2$ (Asmt. 6). 5. Similarly, $\overline{BD}^2 + c^2 = d^2$ (Statements 1-4). 6. $a^2 + b^2 + c^2 = d^2$ (Asmt. 6). 7. $R = a^2$, $S = b^2$, $T = c^2$ (§233). 8. Z is a square with side = d (Const.). 9. $Z = d^2$ (§233). 10. ∴ $Z = R + S + T$ (Asmt. 6).

3. Given square I with side a and square II with side c.

To construct a square X = square II - square I.



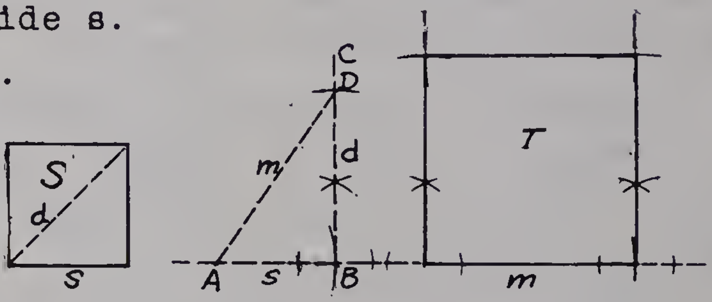
Construction: 1. Construct a rt. △ with hypotenuse = c and a leg = a (Ex. 13, page 195). 2. On b, the other leg of the △ as a side, construct a square X (§§185, 178b).

Then X is the required square.

Proof: 1. X is a square with b as a side (Const.). 2. a is a side of square I and c is a side of square II (Const.). 3. c is the hypotenuse, and b and a are legs of a rt. △ (Const.). 4. $c^2 = a^2 + b^2$ (§§250, 233). 5. Square I = a^2, square II = c^2, and square X = b^2 (§233). 6. ∴ Square II = square I + square X (Asmt. 6). 7. ∴ Square X = square II - square I (Asmt. 2).

4. Given the square S with side s.

To construct a square T = 3S.



Construction: 1. Draw d, a diagonal of square S (Asmt. 11). 2. Construct AB = s (§9). 3. Construct BC ⊥ AB (§178b). 4. Construct BD = d (§9). 5. Draw AD (Asmt. 11). 6. Construct square T having a side = m (§§185, 178b).

Then T is the required square.

Proof: 1. $d^2 = 2s^2$ (§250, Asmt. 6). 2. △ ABD is a rt. △ (Const.). 3. $\overline{AB}^2 + \overline{BD}^2 = m^2$ (§250). 4. AB = s, BD = d (Const.). 5. $s^2 + 2s^2 = m^2$, or $3s^2 = m^2$ (Asmt. 6). 6. T is a square with side = m (Const.). 7. $T = m^2$, $S = s^2$ (§233). 8. ∴ T = 3S (Asmt. 6).

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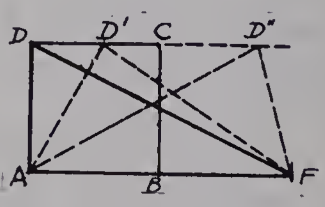
1. Given the square ABCD.

To construct a △ = ABCD.



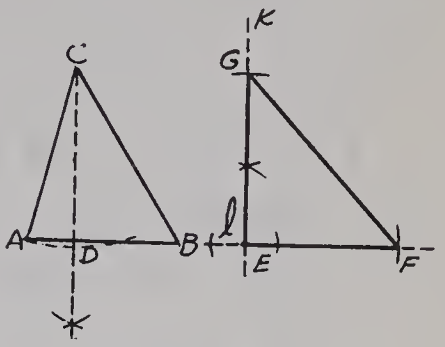
Construction: 1. On the side AB extended, construct BF = AB (Asmt. 13, §9). 2. From any point (D, D', etc.) on DC or DC extended, draw lines to A and F, forming the required △ (Asmt. 11).

Proof: 1. BF = AB (Const.). 2. AB + BF = 2AB (Asmt. 1). 3. AF = 2AB (Asmt. 9, 6). 4. △ AFD' = $\frac{1}{2}$AF x AD (§238).

5. △AFD' = AB × AD (Asmt. 6). 6. Square ABCD = AB × AD (§228). 7. ∴ △AFD = square ABCD (Asmt. 6).

2. Given △ABC.

To construct a rt. △ = △ABC.



Construction: 1. Construct CD, the altitude of △ABC on base AB (§§90, 178a). 2. On a line ℓ, take EF = AB (§9). 3. At E construct EK ⊥ EF (§178b). 4. On EK take EG = CD (§9). 5. Draw GF (Asmt. 11).

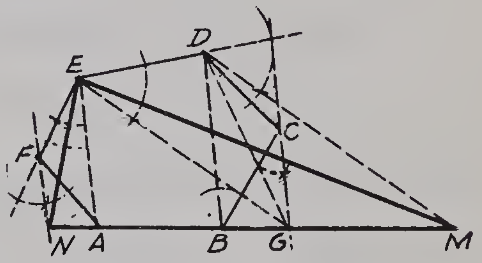
Then EFG is the required △ .

Proof: 1. EFG is a rt. △ (Const., §57). 2. EG = CD and EF = AB (Const.). 3. CD is the altitude of △ABC on base AB and GE is the altitude of △EFG on base EF (§90). 4. △ ABC = $\frac{1}{2}$ × CD × AB and △ EFG = $\frac{1}{2}$ × GE × EF (§238). 5. But $\frac{1}{2}$ × GE × EF = $\frac{1}{2}$ × CD × AD (Asmt. 6). 6. ∴ △ ABC = △ EFG (Asmt. 7).

3. The construction is the same as for the △AFD in Ex. 1, above, since ∠BAD is a rt. ∠ (§185) and hence △AFD is a rt. △ (§57).

4. Given the hexagon ABCDEF.

To construct a △ = ABCDEF.



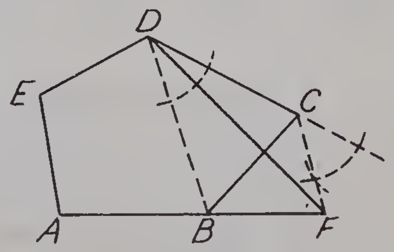
Construction: 1. Draw BD, forming △BCD (Asmt. 11). 2. Construct a line through C ∥ BD (§179). 3. Produce AB to meet this line at G (Asmt. 13). 4. Draw DG (Asmt. 11). 5. In like manner, construct EM and EN (Statements 1-4).

Then △NME = ABCDEF.

Proof: 1. CG ∥ DB (Const.). 2. ⟁ BGD and BCD have = altitudes (§193). 3. △BGD = △BCD (§241). 4. In like manner, △EGD = △EGM and △ENA = △EFA (Statements 1-3). 5. ABCDEF = △BCD + ABDE + △EFA (Asmt. 9) = △BGD + ABDE + △ENA (Asmt. 6) = AGDE + △ENA (Asmt. 9) = △AGE + △EGM + △ENA (Asmt. 9, 6) = △NME (Asmt. 9).

5. Given the pentagon ABCDE.

To construct a quad. = ABCDE.



Construction: 1. Draw diagonal BD, forming △BCD (Asmt. 11). 2. Construct a line through C ∥ DB (§179). 3. Produce AB to meet this line at F (Asmt. 13). 4. Draw DF (Asmt. 11).

Then AFDE is the required quad.

Proof: 1. CF ∥ DB (Const.). 2. $\triangle$BFD and BCD have = altitudes (§193). 3. $\triangle BFD = \triangle BCD$ (§241). 4. ABDE + $\triangle$BFD = ABDE + $\triangle$BCD (Asmt. 1). 5. ∴ AFDE = ABCDE (Asmt. 9, 6).

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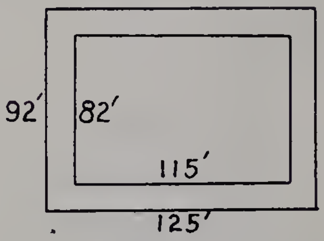
1. a. $18.2 \times 16 = 291.2$.
   b. $4\frac{2}{3} \times 7\frac{5}{8} = 35\frac{7}{12}$.
   c. $4.18 \times 9.32 = 38.9576$.
   d. $(x + 6)(4x - 3) = 4x^2 + 21x - 18$.
   e. $4xy(2x - 3y) = 8x^2y - 12xy^2$.
   f. $(x^2 + x + 1)(x^2 - x + 1) = x^4 + x^2 + 1$.

2. $(a^2 - ab - 2b^2) \div (a - 2b) = a + b$.

3. a. $360.43 \div 13.3 = 27.1$.
   b. 10 A. = 1600 sq.rd. $1600 \div 15 = 106\frac{2}{3}$ (rd.).
   c. $(4x^3 - 8x^2 + 4x) \div (4x) = x^2 - 2x + 1$.
   d. $(x^2 + x - 30) \div (x - 5) = x + 6$.

4. $92 \times 125 = 11{,}500$.
   $82 \times 115 = 9430$.
   $11{,}500 - 9430 = 2070$ (sq.ft.).

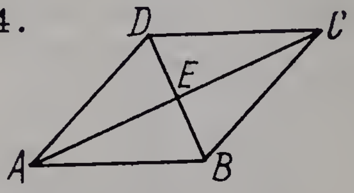


5. Area of trapezoid $= \frac{27}{2}(34 + 62) = 1296$. Let s = a side of the square. Then $s^2 = 1296$; $s = 36$ (ft.).

6. Planning the Proof: 1. We can compare areas of $\triangle$ by §§95, 96, 156, 216, 239-241, and Asmt. 2. We shall use Asmt. 7.
Proof: 1. $\triangle ABD = \frac{1}{2}$▱ABCD and $\triangle ABC = \frac{1}{2}$▱ABCD (§ 243). 2. ∴ $\triangle ABD = \triangle ABC$ (Asmt. 7).

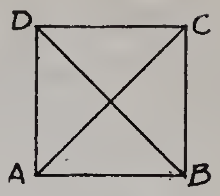
7. By §242, $\frac{1}{2} \times 18 \times 24 = 216$ (sq.ft.).

8. AB = BC = CD = AD = 13 and BD = 10. Then EB = 5. $AE = \sqrt{13^2 - 5^2} = 12$ (§250, Asmt. 2, 8). AC = 24. Area of ABCD $= \frac{1}{2}(AC \times BD)$ (§242). ABCD = $\frac{1}{2}(24 \times 10) = 120$ (sq.in.).



9. Let d = the other diagonal. Then $\frac{1}{2} \times 16d = 384$ (§242). $8d = 384$; $d = 48$ (in.).

10. Given square ABCD with diagonals AC and BD.
To prove that the area of ABCD $= \frac{1}{2}\overline{AC}^2$.



Planning the Proof: 1. We can find areas of polygons by §§ 228, 233, 235, 238, 242, 243, 244, 254, 257, and Asmt. 2. We shall use § 242, Asmt. 6.

Proof: 1. ABCD is a square (Given). 2. ABCD is a rect. and a rhombus (§185). 3. Area of ABCD = $\frac{1}{2}AC \times BD$ (§242). 4. But BD = AC (Ex. 8, page 206). 5. ∴ Area ABCD = $\frac{1}{2}\overline{AC}^2$ (Asmt. 6).

11. Let d = a diagonal. Then, from Ex. 10, above, area = $\frac{1}{2}d^2 = 60$; whence $d^2 = 120$; $d = 2\sqrt{30}$, or 10.95 (in.).

12. By §254, $A = \frac{s^2}{4}\sqrt{3}$.

a. $A = \frac{64}{4}\sqrt{3} = 16\sqrt{3}$ (sq.in.), or 27.712 sq.in.

b. $A = \frac{100}{4}\sqrt{3} = 25\sqrt{3}$ (sq.in.), or 43.30 sq.in.

c. $A = \frac{196}{4}\sqrt{3} = 49\sqrt{3}$ (sq.in.), or 84.868 sq.in.

d. $A = \frac{(3\sqrt{3})^2}{4}\sqrt{3} = \frac{27}{4}\sqrt{3}$, or 15.59.

e. $A = \frac{1.732^2}{4}\sqrt{3} = \frac{3}{4}\sqrt{3}$, or 1.299.

f. $A = \frac{(\frac{1}{2}\sqrt{3})^2}{4}\sqrt{3} = \frac{3}{16}\sqrt{3}$, or .325.

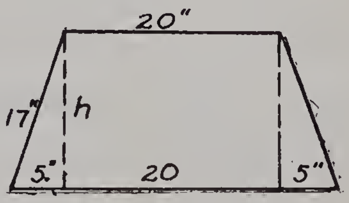
13. Let 2x and 5x represent the sides. Then $10x^2 = 1440$; $x^2 = 144$; $x = 12$. Then $2x = 24$ and $5x = 60$. 60 rd. by 24 rd. Ans.

14. Let 3x and 5x represent the sides. Then $15x^2 = 240$; $x^2 = 16$; $x = 4$. Then $3x = 12$ and $5x = 20$. Let d = length of diagonal in feet. Then $d = \sqrt{12^2 + 20^2} = \sqrt{544} \doteq 4\sqrt{34}$, or 23.32 (ft.).

15. Let b = the base of the △. Then $\frac{1}{2}b(x + 4) = x^2 - x - 20$; whence $\frac{1}{2}b = x - 5$, and $b = 2x - 10$.

16. $h = \sqrt{17^2 - 5^2} = 2\sqrt{66}$.

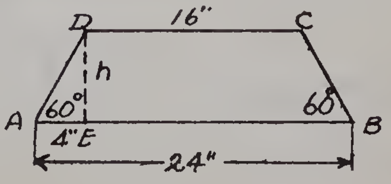
$$S = \frac{1}{2} \times 2\sqrt{66}(20 + 30)$$
$$= 50\sqrt{66} = 50 \times 8.124$$
$$= 406.2 \text{ (sq.in.)}.$$



17. $AE = \frac{1}{2}(24 - 16) = 4$.

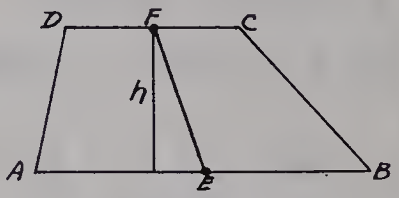
By §136, $AD = 2 \times 4 = 8$.

$$h = \sqrt{8^2 - 4^2} = 4\sqrt{3}.$$
$$S = \frac{1}{2} \times 4\sqrt{3}\ (16 + 24) = 80\sqrt{3}$$
$$= 80 \times 1.732 = 138.56 \text{ (sq.in.)}.$$



18. $S = \frac{1}{2} \times \frac{64}{3}(40 + 56) = 1024$. Let s = the number of inches in the side of the square. Then $s^2 = 1024$. Solving, s = 32 (in.).

19. Given trapezoid ABCD with altitude h and EF a line segment joining the midpoints of bases AB and DC.

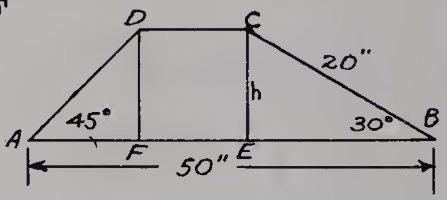


To prove that AEFD and EBCF are = trapezoids.

Planning the Proof: 1. We can prove trapezoids = by Asmt. 2. We shall use Asmt

Proof: 1. AEFD and EBCF are trapezoids (their bases are part of the bases of the given trapezoid (§206). 2. Trapezoids AEFD and EBCF have the same altitude h (§206). 3. Trapezoid AEFD = $\frac{1}{2}h(AE + DF)$ and trapezoid EBCF = $\frac{1}{2}h(EB + FC)$ (§244). 4. But EB = AE and FC = DF (Given). 5. ∴ Trapezoid AEFD = trapezoid EBCF (Asmt. 6, 7).

20. $h = \frac{1}{2}$ of 20 in. = 10 in. (§136). h = DF = 10 in. Since ∠A = 45°, rt. △AFD is isos. Hence AF = DF = 10 in. $EB = \sqrt{20^2 - 10^2} = \sqrt{300} = 10\sqrt{3}$ (in.). $DC = FE = 50 - 10 - 10\sqrt{3} = 40 - 10\sqrt{3}$ (in.).



$S = \frac{1}{2} \times 10(50 + 40 - 10\sqrt{3}) = 450 - 50\sqrt{3} = 450 - 50 \times 1.732 =$ 363.4 (sq.in.).

21. Proof: 1. Place trapezoid T upside down in the position shown by construction lines (Asmt. 16). 2. The bases are ∥ (§206). 3. The two bases of the complete figure are st. lines (§162, Asmt. 25, §158a). 4. The bases are = (Asmt. 1). 5. The complete quadrilateral is a ▱ (§223c). 6. The area of the ▱ = $h(b + b')$ (§235). 7. The ▱ = 2T (Asmt. 9, 6). 8. ∴ $2T = h(b + b')$ (Asmt. 6). 9. ∴ $T = \frac{1}{2}h(b + b')$ (Asmt. 4).

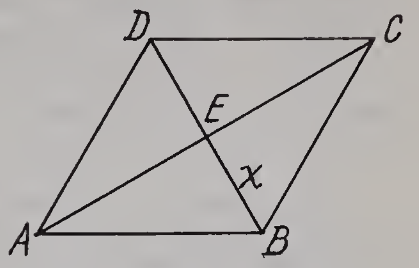
22. Let x = the number of inches in the shorter base. Then x + 12 = the number of inches in the longer base. $\frac{1}{2} \times 15(x + x + 12) = 390$. Solving, x = 20 and x + 12 = 32.

20 in., 32 in. Ans.

23. Let s = the number of inches in each leg. Then $\frac{1}{2}s^2 = 72$; $s^2 = 144$; s = 12 (in.). Let h = the number of inches in the hypotenuse. Then $d^2 = 12^2 + 12^2 = 288$; $d = 12\sqrt{2}$, or 16.97 (in.).

12 in., 12 in., $12\sqrt{2}$ in. or 16.97 in. Ans.

24. $AB = \frac{1}{4}(120) = 30.$ $AE = \frac{1}{2} \times 48 = 24.$
$x^2 + 24^2 = 30^2.$ $x^2 + 576 = 900.$
$x^2 = 324;$ $x = 18.$
$BD = 36.$
Area of $ABCD = \frac{1}{2} \times 48 \times 36 = 864$ (§242).
864 sq.in. Ans.



25. Let s = the number of inches in a side of the square. Then $s^2 = 121 + 135 = 256;$ whence $s = 16.$

26. Let s = one of the equal legs. Then $\frac{1}{2}s^2 = 288$ (§238); $s^2 = 576;$ $s = 24.$ Let x = the number of inches in the hypotenuse. Then $x^2 = 24^2 + 24^2 = 576 + 576 = 1152.$ $x = 24\sqrt{2},$ or 33.94 (in.).

27. Let b = the number of inches in the other base. Then $\frac{8.5}{2}(b + 4) = 51$ (§244); $8.5(b + 4) = 102;$ $8.5b + 34 = 102;$ $8.5b = 68;$ $b = 8$ (in.).

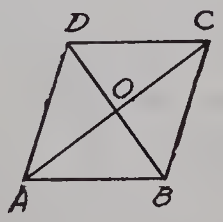
28. Let s = a side of the triangle and h = the altitude. Then $\frac{s^2}{4}\sqrt{3} = 12.25\sqrt{3}$ (§254); $s^2 = 49;$ $s = 7.$ Then $h = \frac{7}{2}\sqrt{3}$ (§254), or $h = 6.06.$

29. Let $3x$ = the width of the rectangle and $5x$ = the length of the rectangle. Then $(3x)(5x) = 2535,$ $15x^2 = 2535,$ $x^2 = 169,$ and $x = 13.$ Then $3x = 13$ and $5x = 65.$ The width is 39 and the length 65.

30. Let $3x$ = the width of the lot and $8x$ = the length of the lot. Then $(3x)(8x) = 9600,$ $24x^2 = 9600,$ $x^2 = 400,$ and $x = 20.$ Then $3x = 60$ and $8x = 160.$ The width is 60 ft. and the length 160 ft.

31. Let $3x$ and $4x$ = respectively the number of inches in the diagonals. Then, by §242, $\frac{1}{2} \times 3x \times 4x = 150.$ Solving, $x = 5,$ $3x = 15,$ and $4x = 20.$

$\triangle AOB$ is a rt. $\triangle$ (§220a). $OA = \frac{1}{2} \times 20 = 10$ and $OB = \frac{1}{2} \times 15 = 7.5$ (§217c). Then $AB = \sqrt{10^2 + 7.5^2} = \sqrt{156.25} = 12.5$ (§250). $AB = BC = CD = DA = 12.5$ (in.) (§194).



32. By §257, $s = \frac{1}{2}(18 + 20 + 24) = 31,$ and $A = \sqrt{31(31 - 18)(31 - 20)(31 - 24)} = \sqrt{31031} = 176.15+.$ From § 238, $h = \frac{2 \times \text{area}}{b}.$ Then the altitude upon side of 24 in. is $\frac{2 \times 176.2}{24},$ or 14.68+ in. 14.7 in. Ans.

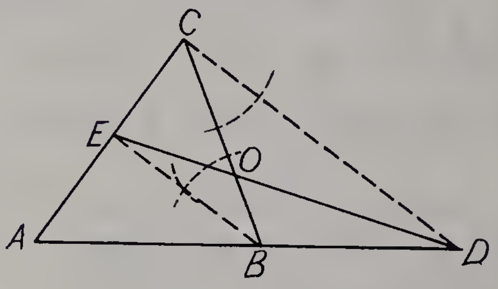
33. By §257, $s = \frac{1}{2}(35 + 84 + 91) = 105$, and $A = \sqrt{105 \times 70 \times 21 \times 14}$ $= 1470$. Let h = the altitude to side 91. Then $\frac{91}{2}h = 1470$ (§238); whence $91h = 2940$; $h = 32.3$.

34. Let h = the altitude to the side 16. Then h = 6 (§136). By §235, $S = bh = 6 \times 16 = 96$ (sq.ft.).

35. (1) Area large square $= a^2 + 2ab + b^2$ and area small square $= a^2 - 2ab + b^2$ (§233). The area of shaded surface $= a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) = 4ab$.

(2) The width of the shaded surface $= \frac{a + b - (a - b)}{2} = \frac{a + b - a + b}{2} = b$.

36. The altitude = 432 ÷ 18 = 24 (ft.). The diagonal = $\sqrt{24^2 + 18^2} = \sqrt{900} = 30$ (ft.).



37. To transform △ABC into a triangle with base AD.

Construction: 1. Draw DC (§9). 2. Construct BE ∥ DC (§179). 3. Draw ED (§9).

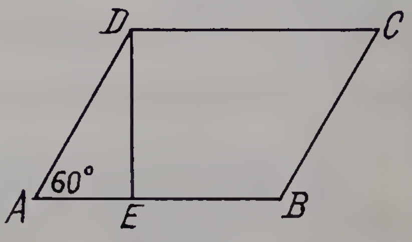
Then ADE is the required triangle.

Proof: 1. △BOE = △BOE (Iden.). 2. △BOD = △COE (Ex. 9, p. 246). 3. △ADE = △ABC (Asmt. 6, 9).

38. Let ℓ = length of rectangle. Then $\ell^2 + 9^2 = 25^2$; $\ell^2 + 81 = 625$; $\ell^2 = 544$, and $\ell = 23.324$ (ft.). Area $= 9 \times 23.324 = 209.92$ (sq.ft.).

39. 1 acre = 160 sq.rd. 11.25 acres = 1800 sq.rd. Let w = the width of the field. Then 2w = the length. By Asmt. 40, $S = bh = 2w^2 = 1800$ (sq.rd.). Then $w^2 = 900$; $w = 30$, and $2w = 60$. Perimeter = 30 + 60 + 30 + 60 = 180 (rd.).

40. Let AD = x and AB = x + 4. Then $AE = \frac{x}{2}$ (§136). $\overline{DE}^2 = x^2 - \frac{x^2}{4}$ (§250, Asmt. 2), and $DE = \frac{x}{2}\sqrt{3}$ (Asmt. 8). Area of ABCD $= \frac{x}{2}\sqrt{3}(x + 4)$ (§235). But $\frac{x}{2}\sqrt{3}(x + 4) = 126\sqrt{3}$. Then $x(x + 4) = 252$; $x^2 + 4x = 252$. $x^2 + 4x + 4 = 256$; $x + 2 = \pm 16$; $x = 14$, disregarding the minus value; $x + 4 = 18$. 14 ft. and 18 ft. Ans.



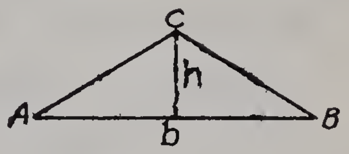
41. Referring to the figure for Ex. 40, let AD = 12 and AB = 16.

Then AE = 6 (§136), and $\overline{DE}^2 = 12^2 - 6^2$ (250, Asmt. 2); DE = $6\sqrt{3}$. Area of ABCD = 16 × $6\sqrt{3} = 96\sqrt{3}$, or 166.27 (sq.ft.).

42. AC = BC = $\frac{p - b}{2}$.

$$h = \sqrt{\left(\frac{p - b}{2}\right)^2 - \left(\frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{p^2 - 2pb}.$$

$$\text{Area} = \frac{1}{2}b \times \frac{1}{2}\sqrt{p^2 - 2pb} = \frac{b}{4}\sqrt{p^2 - 2pb}.$$



43. Let s = a side of the equilateral △. Then, by §254, $h = \frac{s}{2}\sqrt{3}$. Solving for s, $s = \frac{2}{3}\sqrt{3}h$. Area = $\frac{s^2}{4}\sqrt{3}$ (§254) = $\frac{(\frac{2}{3}\sqrt{3}h)^2}{4}\sqrt{3} = \frac{h^2}{3}\sqrt{3}$.

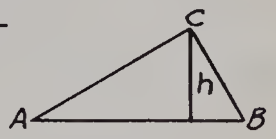
44. From §254, when a side is s, the altitude is $\frac{s}{2}\sqrt{3}$. The difference between a side and the altitude is $s - \frac{s}{2}\sqrt{3}$, or $\frac{s}{2}(2 - \sqrt{3})$. Then $\frac{s}{2}(2 - \sqrt{3}) = 2(2 - \sqrt{3})$. Solving, s = 4. Area $= \frac{s^2}{4}\sqrt{3} = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3}$ (sq.ft.), or 6.928 sq.ft.

45. If the shortest side is 15, the hypotenuse is 30 (§136). The longer leg = $\sqrt{30^2 - 15^2} = \sqrt{675} = 15\sqrt{3}$. Area = $\frac{1}{2} \times 15 \times 15\sqrt{3} = \frac{225}{2}\sqrt{3}$. $\frac{225}{2} \times 1.732 = 194.85$.

46. Let x = the length of the shorter leg. Then 2x = the length of the hypotenuse (§136), and $\sqrt{(2x)^2 - x^2} = \sqrt{3x^2} = x\sqrt{3}$ = the length of the longer leg. The area is $\frac{x^2}{2}\sqrt{3}$, or $\frac{1.732}{2}x^2$. Then $\frac{1.732}{2}x^2 = 17.32$. Solving, $x = 2\sqrt{5}$, or 4.472, $2x = 4\sqrt{5}$, or 8.944, and $x\sqrt{3} = 2\sqrt{15}$ or 7.746.

47. Given rt. △ABC with altitude h upon hypotenuse AB.

To prove that AC × BC = h × AB.



Proof: 1. △ABC = $\frac{1}{2}$h × AB (§238). 2. Considering BC as the base, AC is the altitude (§90). 3. ∴ △ABC = $\frac{1}{2}$AC × BC (§238). 4. ∴ $\frac{1}{2}$AC × BC = $\frac{1}{2}$h × AB (Asmt. 7). 5. ∴ AC × BC = h × AB (Asmt. 3).

48. Area of square BF = $\overline{BC}^2 = 24$. BC = $\sqrt{24}$. CD = $\frac{BC}{2}\sqrt{3}$ (§254). CD = $\frac{\sqrt{24} \cdot \sqrt{3}}{2} = \frac{\sqrt{72}}{2}$. Area of square DG = $\overline{CD}^2 = \left(\frac{\sqrt{72}}{2}\right)^2$ = 18 (sq.in.).

49. Given rt. △DEG with ∠DGE the rt. ∠ and DB the square on DE.

To prove that the square DB = the square with side GE + the square with side DG.

Planning the Proof: 1. We can compare areas of polygons by §§95, 96, 156, 216, 231, 232, 236, 237, 239, 240, 241, 243. 2. We shall use §§ 233, 238.

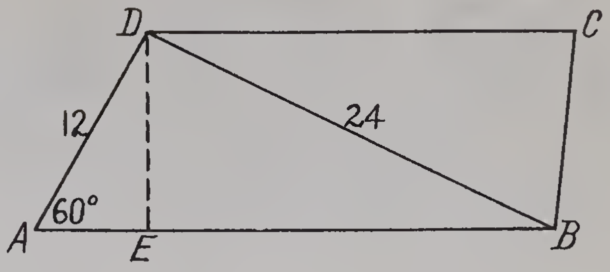
Proof: 1. Extend DG through G (Asmt. 13). 2. From A, draw AH ⊥ DG extended (§178a). 3. From B, draw BC ⊥ AH extended (Asmt. 13, §178a). 4. EG ⊥ DH (Given). 5. AC ⊥ DH (Const.). 6. ∴ EG ∥ AC and DH ∥ BF (§160d). 7. ∴ ∠EDG = ∠DAH (§158b). 8. DEBA is a square (Given). 9. ∴ DE = AD (§185). 10. Rt. △DEG ≅ rt. △DHA (§156a). 11. Similarly, rt. △DHA ≅ rt. △ACB ≅ rt. △BFE ≅ rt. △DGE (Statements 5–10). 12. ∴ AH = BC = FE = DG and DH = AC = BF = EG (§97a). 13. ∴ FG = GH = HC = CF (Asmt. 2). 14. ∴ GFCH is a square (§185). 15. Square DB = △DEG + △EFB + △ABC + △AHD + square GFCH (Asmt. 9). 16. Square DB = 4△DEG + square GFCH (Asmt. 6). 17. Area of square DB = $\overline{DE}^2$ and area of square GC = $\overline{GF}^2$ (§233). 18. Area △DEG = $\frac{1}{2} \times DG \times GE$ (§238). 19. ∴ $\overline{DE}^2 = 4 \times \frac{1}{2} \times DG \times GE + \overline{GF}^2$ (Asmt. 6). 20. $\overline{DE}^2 = 2 \times DG \times GE + (GE - FE)^2$ (Asmt. 6). 21. $\overline{DE}^2 = 2 \times DG \times GE + \overline{GE}^2 - 2 \times GE \times FE + \overline{FE}^2$ (Asmt. 6). 22. $\overline{DE}^2 = 2 \times DG \times GE + \overline{GE}^2 - 2 \times GE \times DG + \overline{FE}^2$ (Asmt. 6). 23. $\overline{DE}^2 = \overline{GE}^2 + \overline{FE}^2$ (Asmt. 6). 24. $\overline{FE}^2 = \overline{DG}^2$ (Asmt. 8). 25. $\overline{DE}^2 = \overline{GE}^2 + \overline{DG}^2$ (Asmt. 6). 26. ∴ The square DB = the square with side GE + the square with side DG (Asmt. 6).

50. EB = 64 − 44 = 20. CE = 16. By §257, in △EBC, $s = \frac{1}{2}(a + b + c) = \frac{1}{2}(18 + 20 + 16) = 27$. Area of △EBC = $\sqrt{s(s - a)(s - b)(s - c)} = \sqrt{27 \times 9 \times 7 \times 11} = 9\sqrt{231} = 9 \times 15.199 = 136.791$. Let h = the altitude of the trapezoid and △. Then, by §238, $\frac{1}{2}h \times EB = 136.791$, or $\frac{1}{2}h \times 20 = 136.791$. Solving, h = 13.6791. Area of ▱AECD = 44 × 13.6791 = 601.8804. Area of ABCD = 136.8 + 601.9 = 738.7.

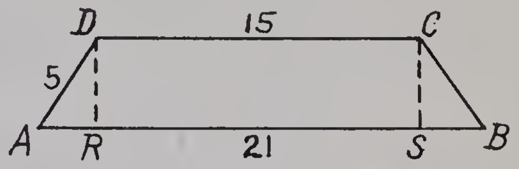
51. Let d = the number of inches in the shorter diagonal and D = the number of inches in the longer diagonal. Then D + d = 69 and D − d = 21. Solving, d = 24 and D = 45.
Area = $\frac{1}{2} \times 24 \times 45 = 540$ (sq.in.).

52. Let d = the number of inches in the shorter diagonal. Then d + 7 = the number of inches in the longer diagonal. Then $\frac{1}{2}d(d + 7) = 165$. Solving, d = 15, and d + 7 = 22.

15 in., 22 in. Ans.



53. AD = 12 and $\angle A = 60°$. Then AE = 6 (§136). $\overline{DE}^2 = 12^2 - 6^2 = 108$; DE = $6\sqrt{3}$. $\overline{EB}^2 = 24^2 - \overline{DE}^2 = 576 - 108 = 468$; EB = 21.634. Then DC = 21.634 and AB = 27.634. Area of ABCD = $\frac{DE}{2}(AB + DC) = 3\sqrt{3}(27.634 + 21.634) = 256.0$ (sq.in.).

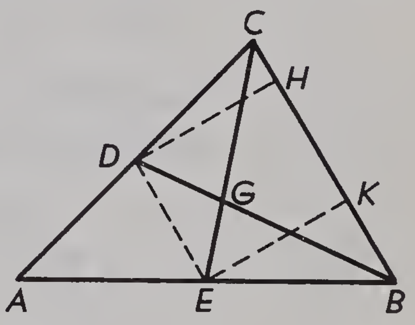


54. AR = $\frac{1}{2}(21 - 15) = 3$. DR = $\sqrt{5^2 - 3^2} = 4$. Area of ABCD = $\frac{4}{2}(15 + 21) = 72$ (sq.in.).

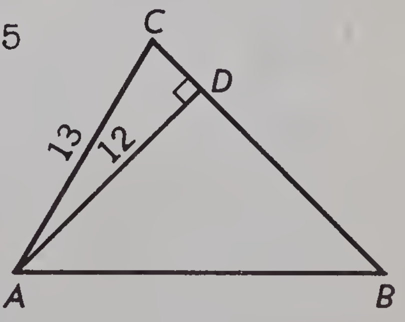
55. S = bh = b × 16 = 480 (§228); b = 30. Let d = the number of feet in the diagonal. Then $d^2 = 16^2 + 30^2$ (§250). Solving, d = 34 (ft.).

56. 1. △AED = △BEC (Ex. 9, page 246). 2. △ABC = 126 and △ABE = 81 (Given). 3. ∴ △BEC = 45 (Asmt. 2). 4. ∴ △AED = 45 (Asmt. 6).

57. Planning the Proof: 1. We can prove two figures = by Asmt. 2. We shall use Asmt. 6 and 2.

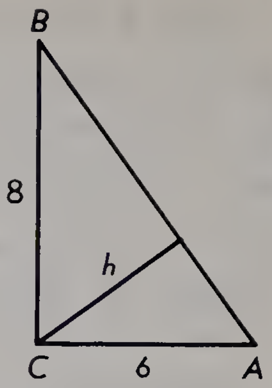


Proof: 1. BD and CE are medians (Given). 2. D and E are midpoints of AC and AB (§89). 3. Draw DE (Asmt. 11). 4. DE ∥ BC (§204). 5. Construct EK ⊥ BC and DH ⊥ BC (§178a). 6. EK = DH (§193). 7. Area △EBC = area △BDC (Ex. 7, p. 244). 8. Area △BGC = area △BGC (Iden.). 9. Area △EBC - area △BGC = area △BDC - area △BGC (Asmt. 2). 10. Area △EBG = area △DGC (Asmt. 6). 11. Area △EBC = area △AEC (Ex. 7, p. 244). 12. Area △BGC = area quad. AEGD (Asmt. 2).



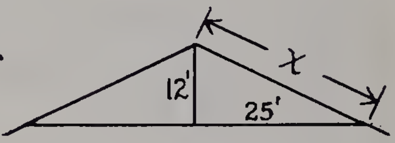
58. $\overline{CD}^2 + 12^2 = 13^2$. ∴ $\overline{CD}^2 = 169 - 144 = 25$ and CD = 5. Let DB = y. Then $\overline{AB}^2 = y^2 + 12^2 = y^2 + 144$, and AB = $\sqrt{y^2 + 144}$. $\sqrt{y^2 + 144} + 13 + 5 + y = 38$ and $\sqrt{y^2 + 144} = 20 - y$. ∴ $y^2 + 144 = 400 - 40y + y^2$, 40y = 256, y = 6.4, and BC = 6.4 + 5 = 11.4. ∴ Area △ABC = $\frac{1}{2} \times 12 \times 11.4 = 68.4$.

59. AB = $\sqrt{6^2 + 8^2} = 10$. Let h = altitude to AB from C. Then the area of ABC = $\frac{1}{2} \times h \times 10 = 5h$. But the area of $\triangle ABC = \frac{1}{2} \times 6 \times 8 = 24$. Then $5h = 24$, and $h = 4.8$. The altitude is 4.8 ft.



PAGE 273 (Applications)

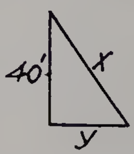
1. (1) The area to be painted = $2 \times 50 \times 20 + 2 \times 60 \times 20 + 2 \times \frac{1}{2} \times 50 \times 12 = 5000$ (sq.ft.). The cost of the paint for one coat = $\frac{5000}{400} \times \$4.50 = \$56.25$. Cost of labor in applying it = $2 \times \$56.25 = \$112.50$. Cost of 2 coats = $2 \times \$56.25 + 2 \times \$112.50 = \$337.50$.



(2) $x = \sqrt{12^2 + 25^2} = 27.7$ (ft.), or 28 ft., approx. Rounding off 18 in. to 2 ft. (since the other measurements have been made only to the nearest foot), we have $x + 2 = 30$ (ft.). Length of roof = $60 + 4 = 64$ (ft.). Area of roof = $2 \times 64 \times 30 = 3840$ (sq.ft.), or 3800 sq.ft., approx. $38 \times \$6 = \$228$.

2. $x = \sqrt{12^2 + 25^2} = 27.7$ (ft.), or 28 ft.; $x + 1 = 28 + 1 = 29$ (ft.). Length = $60 + 2 = 62$ (ft.). Area of $\frac{1}{2}$ the roof = $62 \times 29 = 1800$ (sq.ft.). Area of whole roof = $2 \times 1800 = 3600$ (sq.ft.). Neglecting labor, the cost of the roofing would be $\frac{3600}{100} \times \$17 = \$612$. $612. Ans.

3. $x = 90 - 40 = 50$ (ft.). $y = \sqrt{50^2 - 40^2} = 30$ (ft.).



4. Construction: 1. Draw a line from A to E, the midpoint of BC (Asmt. 11). 2. Draw DE (Asmt. 11). 3. Through A construct AF $\parallel$ DE (§179). 4. Draw DF (Asmt. 11).

Then DF is the required line.

Proof: 1. EC = EB (Const.). 2. $\triangle AEB = \frac{1}{2}\triangle ABC$ (§240). 3. △ ADE and DEF have DE as a base and equal altitudes (§193). 4. $\therefore \triangle ADE = \triangle DEF$ (§241). 5. $\triangle EDB = \triangle EDB$ (Iden.). 6. From 4 and 5, $\triangle AEB = \triangle DBF$ (Asmt. 1, 9, 6). 7. From 2 and 6, $\triangle DBF = \frac{1}{2}\triangle ABC$ (Asmt. 6). 8. $\therefore$ ADFC = $\frac{1}{2}\triangle ABC$ (Asmt. 2). 9. $\therefore \triangle DBF$ = quad. ADFC (Asmt. 6).

5. He first drew AC and bisected it. Then he drew ED and EB. Next he drew BD. Then he drew EF through E $\parallel$ BD. Last he drew DF.

Proof: 1. △AED = △ECD and △AEB = △BEC (§241). 2. △AED + △AEB = △ECD + △BEC (Asmt. 1). 3. Quad. ABED = quad. BEDC (Asmt. 9, 6). 4. Polygon AFOED + △OFB = quad. BODC + △EOD (Asmt. 9, 6). 5. △EOD = △OFB (Ex. 9, page 246). 6. ∴ polygon AFOED + △EOD = quad. BODC + △OFB (Asmt. 6). 7. ∴ △AFD = quad. BFDC (Asmt. 9, 6).

6. Proof: 1. Draw AD intersecting TE in O (Asmt. 11). 2. △ABD = △ADC (§241). 3. △AOE + quad. EBDO = polygon AOTDC + △ODT (Asmt. 9, 6). 4. △AOE = △ODT (Ex. 9, page 246). 5. △ODT + quad. EBDO = polygon AOTDC + △AOE (Asmt. 6). 6. Quad. EBDT = polygon AETDC (Asmt. 9, 6).

PAGE 274 (Space Geometry)

1. Rectangular parallelepipeds: chalk box, match box, bookcase, shoe box. Prisms: The examples just named are particular types. Many-sided towers or pillars are also examples. 2. 6; 5. 3. 12; 12. 4. 8; 10.

(For Exs. 5 and 6, use the figure at the top of page 274 in the text.)

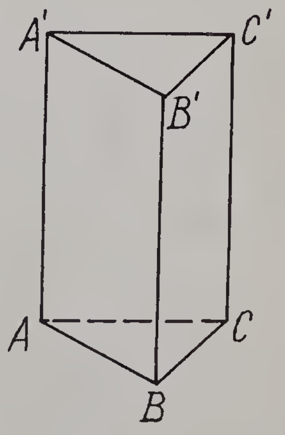
5. Proof: 1. AA' ∥ BB' (§264). 2. AB ∥ A'B' (§153). 3. ∴ ABB'A' is a ▱ (§185). 4. Likewise BCC'B', CDD'C', EDD'E', and AEE'A' are ▱s (Statements 1-3).

6. Proof: 1. ABB'A' is a ▱ (Ex. 5). 2. ∴ AA' = BB' (§217a). 3. Likewise BB' = CC' = DD' = EE' (Statements 1, 2, Asmt. 7).

7. Given rt. prism ABC-A'B'C' with ABC and A'B'C' the bases.

To prove that ABB'A', BCC'B', and ACC'A' are rectangles.

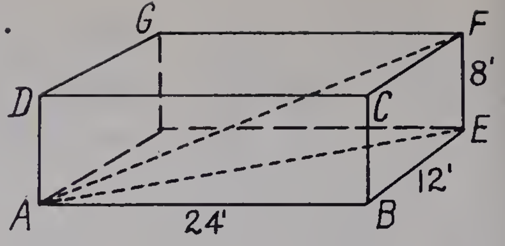
Proof: 1. AC' is a rt. prism (Given). 2. ABB'A' is a ▱ (Ex. 5). 3. AA' ⊥ plane ABC (§264). 4. ∠ A'AB is a rt. ∠ (§147). 5. ∴ ABB'A' is a rect. (§185). 6. Likewise, BCC'B' and CC'A'A are rects.



8. Given triangular prism ABC-A'B'C' with ABC and A'B'C' the bases.

To prove that △ABC ≅ △A'B'C'.

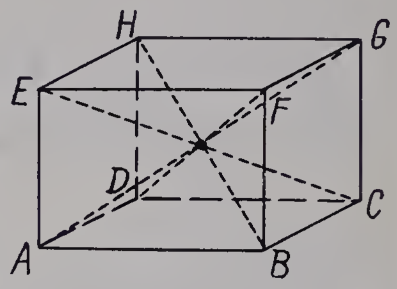
Proof: 1. ABB'A' is a ▱ (Ex. 5). 2. ∴ AB = A'B' (§217a). 3. Likewise, BC = B'C' and AC = A'C' (Statements 1, 2). 4. ∴ △ABC ≅ △A'B'C' (§95c).



9. $\overline{AF}^2 = \overline{AE}^2 + \overline{EF}^2$ and $\overline{AE}^2 = \overline{AB}^2 + \overline{BE}^2$. Then $AF^2 = \overline{AB}^2 + \overline{BE}^2 + \overline{EF}^2$; $AF^2 = 24^2 + 12^2 + 8^2 = 784$; AF = 28 (ft.).

10. Given rectangular parallelepiped AG with diagonals AG, BH, EC, and DF.

To prove that $\overline{AB}^2 + \overline{BC}^2 + \overline{DC}^2 + \overline{AD}^2 + \overline{AE}^2 + \overline{BF}^2 + \overline{CG}^2 + \overline{DH}^2 + \overline{EF}^2 + \overline{FG}^2 + \overline{HG}^2 + \overline{EH}^2 = \overline{AG}^2 + \overline{BH}^2 + \overline{CE}^2 + \overline{DF}^2$.



Proof: 1. Draw AC (Asmt. 11). 2. ABCD is a rect. (§264). 3. $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$ (§250). 4. CG ⊥ plane ABCD (§264). 5. CG ⊥ BC, AC, and DC (§147). 6. $\overline{AG}^2 = \overline{AC}^2 + \overline{CG}^2$ (§250). 7. $\overline{AG}^2 = \overline{AB}^2 + \overline{BC}^2 + \overline{CG}^2$ (Asmt. 6). 8. In like manner, it can be shown that $\overline{BH}^2 = \overline{EH}^2 + \overline{EF}^2 + \overline{BF}^2$, $\overline{CE}^2 = \overline{EA}^2 + \overline{AD}^2 + \overline{DC}^2$, and $\overline{DF}^2 = \overline{FG}^2 + \overline{GH}^2 + \overline{HD}^2$ (Statements 1-7). 9. Then $AG^2 + BH^2 + \overline{CE}^2 + \overline{DF}^2 = \overline{AB}^2 + \overline{BC}^2 + \overline{CG}^2 + \overline{EH}^2 + \overline{EF}^2 + \overline{BF}^2 + \overline{EA}^2 + \overline{AD}^2 + \overline{DC}^2 + \overline{FG}^2 + \overline{GH}^2 + \overline{HD}^2$ (Asmt. 1).

Page 275 (Space Geometry)

1. (1) Area of each face = $6^2 = 36$ (sq.in.). Area of cube = 6 x 36 sq.in. = 216 sq.in.

(2) Volume = 36 x 6 = 216 (cu.in.).

2. (1) The area of each lateral face = 4^2, or 16 (sq.in.). The lateral area = 3 x 16 sq.in. = 48 sq.in.

(2) The area of a base = $\frac{s^2}{4}\sqrt{3} = 4\sqrt{3}$. The volume = base x altitude = $4\sqrt{3} \times 4 = 16\sqrt{3}$, or 27.7 cu.in.

3. (1) The third side of the base = $\sqrt{8^2 + 6^2} = 10$ (in.). The lateral area = 8 x 12 + 6 x 12 + 10 x 12 = 288 (sq.in.). The area of both bases = $2 \times \frac{1}{2} \times 8 \times 6 = 48$ (sq.in.). The total area = 288 sq.in. + 48 sq.in. = 336 sq.in.

(2) The volume = area of base x altitude = 24 x 12 = 288 (cu.in.).

4. The dimensions are 8 yd., 12 yd., and 4 yd.

(1) Area of floor = 8 x 12 = 96 (sq.yd.). The volume = 96 x 4 = 384 (cu.yd.).

(2) The lateral area = 2 x 8 x 4 + 2 x 12 x 4 = 160 (sq.yd.).

5. (1) The total area = $2 \times 5 \times 10 + 2 \times 10 \times 16 + 2 \times 5 \times 16 = 580$ (sq.in.).

(2) The volume = $16 \times 10 \times 5 = 800$ (cu.in.).

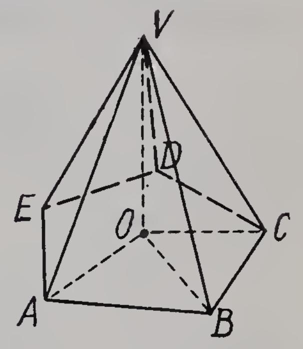
6. The lateral area = $6 \times \frac{1}{2} \times 2 = 6$ (sq.ft.).

7. The volume, in cu.ft. = $140 \times 50 \times 6 = 42{,}000$.
42,000 cu.ft. = 72,576,000 cu.in.
$72{,}576{,}000 \div 231 = 314{,}181.8$ (gal.).

PAGE 276 (Space Geometry)

1. Given regular pyramid V-ABCDE.

To prove that VA = VB = VC = VD = VE.



Proof: 1. From V draw VO $\perp$ plane ABC (§178a). 2. Draw OA and OB (Asmt. 11). 2. VO $\perp$ OA and OB (§147). 3. OA = OB (§268). 4. VO = VO (Iden.). 5. $\therefore$ Rt. $\triangle$AOV $\cong$ rt. $\triangle$BOV (§96d). 6. $\therefore$ AV = BV (§97a). 7. In like manner, it can be proved that AV = CV = DV = EV (Statements 2-6).

2. Given regular pyramid V-ABCDE.

To prove that $\triangle$s ABV, BCV, CDV, DEV, and EAV are $\cong$ isos. $\triangle$s.

Proof: 1. From V, draw VO $\perp$ plane ABC (§178a). 2. Draw OA, OB, and OC (Asmt. 11). 3. VO $\perp$ OA, OB, and OC (§147). 4. OA = OB = OC (§268). 5. VO = VO (Iden.). 6. $\therefore$ Rt. $\triangle$AOV $\cong$ rt. $\triangle$BOV $\cong$ rt. $\triangle$COV (§96d). 7. $\therefore$ AV = BV = CV (§97a). 8. $\therefore$ $\triangle$s AVB and BVC are isos. (§56). 9. AB = BC (§268). 10. $\therefore$ $\triangle$AVB $\cong$ $\triangle$BVC (§95c). 11. In like manner, it can be proved that $\triangle$s CVD, DVE, and EVA are isos. and are $\cong$ $\triangle$s AVB and BVC (Statements 2-10).

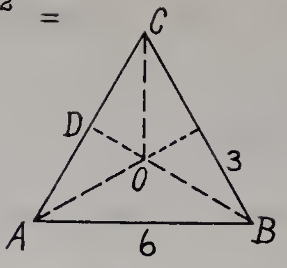
3. 6; 12; 16.

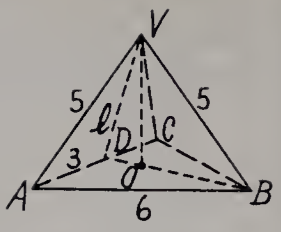
4. (1) The slant height bisects each side of the base. Let x = the number of inches in the slant height. Then $x^2 + 8^2 = 17^2$; whence $x^2 + 64 = 289$; $x^2 = 225$, and $x = 15$ (in.).

(2) Let h = the altitude. Then we have $h^2 + (\frac{16}{2})^2 = 15^2$; $h^2 = 15^2 - 8^2 = 161$; $h = 12.7$ (in.).

5. (1) Let ℓ = the number of inches in the slant height. Then $\ell^2 + 3^2 = 5^2$; $\ell^2 + 9 = 25$; $\ell^2 = 16$; $\ell = 4$ (in.).

(2) Let $\triangle$ABC represent the base of the pyramid, with O the foot of the altitude and OB = OA. Let x = BO, and y = OD.





Then $x^2 - y^2 = 9$ and $(x + y)^2 = 6^2 - 3^2 =$ 27 (§250). Then $\frac{x + y}{x - y} = 3$ (Asmt. 4). Solving, $x = 2y$. Then $BO = \frac{2}{3}$ of BD. But $BD = \frac{s}{2}\sqrt{3} = 3\sqrt{3}$ (§254). $\therefore BO = \frac{2}{3}(3\sqrt{3}) = 2\sqrt{3}$. $\therefore VO = \sqrt{5^2 - (2\sqrt{3})^2} = \sqrt{13}$, or 3.6 (in.).

6. Area of each lateral face $= \frac{1}{2} \times 5 \times 7 = 17.5$ (sq.in.). Then lateral area of the pyramid $= 5 \times 17.5 = 87.5$ (sq.in.).

7. The slant height $= \sqrt{7^2 - 3^3} = 2\sqrt{10}$ (in.). Area of each lateral face $= \frac{1}{2} \times 6 \times 2\sqrt{10} = 6\sqrt{10}$ (sq.in.). Lateral area $= 6 \times 6\sqrt{10} = 36\sqrt{10}$, or 113.83 (sq.in.).

8. The area of each face $= \frac{s^2}{4}\sqrt{3} = \frac{64}{4}\sqrt{3} = 16\sqrt{3}$ (sq.in.). The area of the pyramid $= 4 \times 16\sqrt{3}$ sq.in. $= 64\sqrt{3}$ sq.in., or 110.85 sq.in.

9. The slant height $= \sqrt{13^2 - 5^2} = 12$ (in.). The lateral area $= 6(\frac{1}{2} \times 10 \times 12) = 360$ (sq.in.). The base consists of 6 ≅ equilateral $\triangle$. The area of the base $= 6(\frac{s^2}{4}\sqrt{3}) = 6 \times \frac{100}{4}\sqrt{3} = 150\sqrt{3}$, or 259.8 (sq.in.).

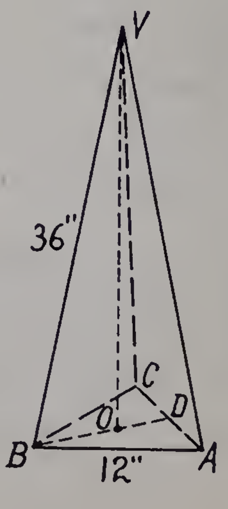
The total area = 360 sq.in. + 259.8 sq.in. = 619.8 sq.in.

PAGE 277 (First Set)

1. $V = \frac{1}{3}bh = \frac{1}{3} \times 2^2 \times 3 = 4$ (cu.ft.).

2. $V = \frac{1}{3}bh$. $h = \sqrt{26^2 - 10^2} = 24$ (ft.).
$V = \frac{1}{3} \times 20^2 \times 24 = 3200$ (cu.ft.).

3. $V = \frac{1}{3}bh$. $b = \frac{s^2}{4}\sqrt{3} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3}$.
$V = \frac{1}{3} \times 16\sqrt{3} \times 20 = 184.75$ (cu.in.).



4. (1) $BD = \frac{s}{2}\sqrt{3} = \frac{12}{2}\sqrt{3} = 6\sqrt{3}$.
$BO = \frac{2}{3}(BD)$ (Ex. 5, page 276).
$BO = \frac{2}{3} \times 6\sqrt{3} = 4\sqrt{3}$.
$VO = \sqrt{36^2 - (4\sqrt{3})^2} = 35.33$ (in.).
$V = \frac{1}{3}bh$.
$b = \frac{s^2}{4}\sqrt{3} = \frac{144}{4}\sqrt{3} = 36\sqrt{3} = 62.35$ (sq.in.).
$V = \frac{1}{3} \times 62.35 \times 35.33 = 734.3$ (cu.in.).

(2) The slant height $= \sqrt{36^2 - 6^2} = 6\sqrt{35} = 35.496$. The lateral area $= 3(\frac{1}{2} \times 12 \times 35.496) = 638.93$.

Total area = 62.35 sq.in. + 638.93 sq.in. = 701.28 sq.in.

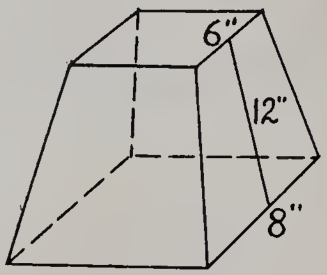
5. The volume of the prism is 3 times the volume of the pyramid.

6. Because only regular pyramids have congruent lateral faces.

PAGE 277 (Second Set)

1. (a) Proof: 1. Plane ABC $\parallel$ plane DEF (§270). 2. $\therefore$ AB $\parallel$ DE, BC $\parallel$ EF, and AC $\parallel$ DF (§153).

(b) Proof: 1. AB $\parallel$ DE, BC $\parallel$ EF, and AC $\parallel$ DF (Ex. 1a, above). 2. AD, BE, and CF are not $\parallel$ (§§270 and 268). 3. $\therefore$ ABED, BCFE, and ACFD are trapezoids (§206).



2. The lateral faces of the frustum are trapezoids (Ex. 1b). Area of each face $= \frac{1}{2} \times 12(6 + 8) = 84$. Lateral area $= 4 \times 84 = 336$.

336 sq.in. Ans.

3. (a) Proof: 1. AB $\parallel$ DC (§264). 2. ABCD is a plane (§145f). 3. Plane BC $\parallel$ plane AD (§264). 4. AD $\parallel$ BC (§153). 5. $\therefore$ ABCD is a ▱ (§185). 6. AB $\perp$ two edges of the base at their point of intersection (Definition of a cube, §264). 7. AB $\perp$ plane BC (§148). 8. AB $\perp$ BC (§147). 9. ABCD is a rectangle (§185). 10. AC = BD (Ex. 8, page 206).

(b) Proof: 1. ABCD is a ▱ (Step 5 of Ex. 3a, above). 2. $\therefore$ AC and BD bisect each other (§195).

PAGE 279

1. Overdetermined. 2. Overdetermined. 3. Underdetermined. 4. Determined. 5. Underdetermined. 6. Determined. 7. Overdetermined.

PAGES 279-280 (Review Questions)

1. a. $S = bh$. b. $S = bh$. c. $S = bh$ or $S = \frac{1}{2}dd'$. d. $S = s^2$ or $S = \frac{1}{2}d^2$. e. $S = \frac{1}{2}bh$ or $S = \sqrt{s(s-a)(s-b)(s-c)}$. f. $S = \frac{s^2}{4}\sqrt{3}$. g. $S = \frac{1}{2}h(b + b')$. h. $a^2 + b^2 = c^2$. i. $h = \frac{s}{2}\sqrt{3}$. 2. (1) No. (2) Yes. 3. The ratio of their altitudes. 4. Not necessarily. 5. Right triangle. 6. Not necessarily; not necessarily; yes.

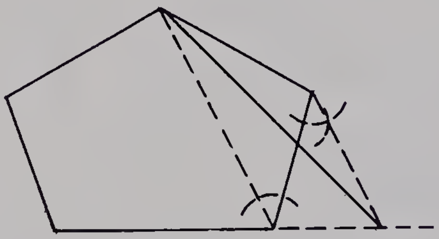
7. (1) A Greek mathematician who proved many theorems of geometry. The Pythagorean Theorem derives its name from him. (2) Euclid compiled and wrote the first geometry. (3) Archimedes did much work in plane and solid geometry and in physics. 8. Yes. 9. Yes. 10. $\frac{1}{4}$. 11. Quadrupled. 12. The areas are to each other as their altitudes. 13. Divide the area by one half the base. 14. (1) Take one fourth of the perimeter. (2) Divide the diagonal by $\sqrt{2}$. 15. Subtract the square of the one leg from the square of the hypotenuse and find the square root of the difference. 16. § 255. 17. Divide the area by one half the sum of the two bases.

PAGES 281-282 (Test 17)

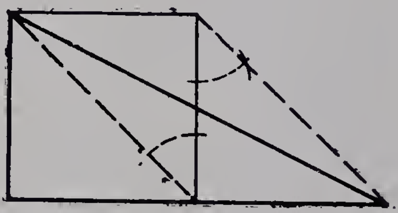
1. F. 2. F. 3. F. 4. T. 5. T. 6. F. 7. T.
8. F. 9. F. 10. T. 11. F. 12. T. 13. F. 14. F.
15. F. 16. T. 17. T.

PAGE 282 (Test 18)

1. This construction is like one on page 268, Ex. 5.



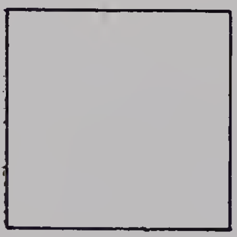
(Ex. 1)

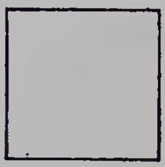


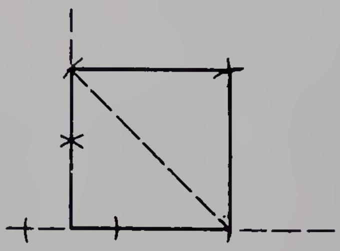
(Ex. 2)

2. This construction depends on §263.

3. This construction is like the one in Ex. 3, page 267.







PAGES 282-283 (Test 19)

1. Area = 11 x 8 = 88 (sq.ft.).

2. Area = $\frac{1}{2}$ x 6 x 8 = 24 (sq.ft.).

3. Other leg = $\sqrt{13^2 - 5^2}$ = 12. Area = $\frac{1}{2}$ x 12 x 5 = 30 (sq.ft.).

4. Area = $\frac{s^2}{4}\sqrt{3} = 25\sqrt{3}$, or 43.3 (sq.ft.).

5. Altitude = $\frac{2 \times 66}{14 + 8} = 6$ (ft.).

6. Area = $\frac{1}{2} \times 12 \times 12 = 72$ (sq.ft.).

7. Altitude = $550 \div 22 = 25$. A diagonal = $\sqrt{25^2 + 22^2} = 33.3$.

8. Area of trapezoid = $\frac{1}{2} \times 9(32 + 18) = 225$.
Side of square = $\sqrt{225} = 15$.

9. Altitude = $\frac{1}{2} \times 12 = 6$ (§136). Area = $12 \times 6 = 72$ (§235).

10. Let x = one base. Then $2x$ = the other.
Then $\frac{8}{2}(x + 2x) = 144$; $x = 12$; and $2x = 24$.
12 in., 24 in. Ans.

11. $s = \frac{1}{2}(6 + 8 + 12) = 13$.
Area = $\sqrt{13 \times 7 \times 5 \times 1} = \sqrt{455} = 21.33+$. 21.3. Ans.

12. Area = $\frac{1}{2} \times 10 \times 16 = 80$.

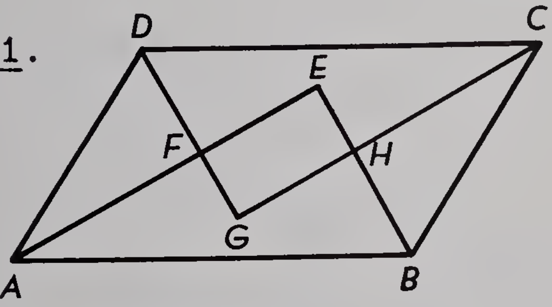
13. $h = \frac{s}{2}\sqrt{3} = \frac{12}{2}\sqrt{3} = 6\sqrt{3} = 6 \times 1.732 = 10.392$.

14. $h = \sqrt{5^2 - 3^2} = 4$. Area = $\frac{1}{2} \times 4(12 + 18) = 60$.

15. $320 = h \times 5h$. $5h^2 = 320$. $h = 8$.

PAGE 283 (Test 20)

1.



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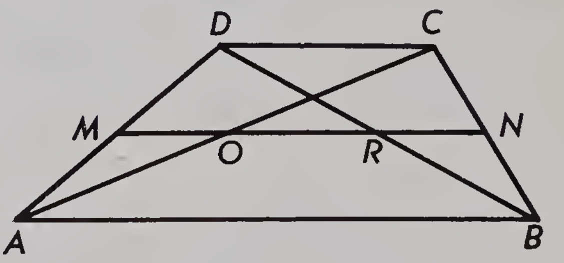

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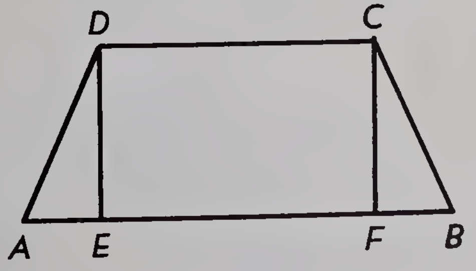

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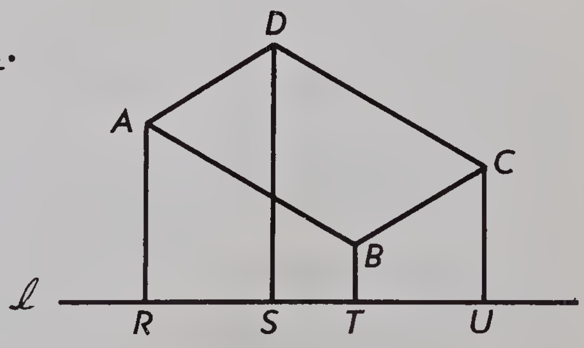

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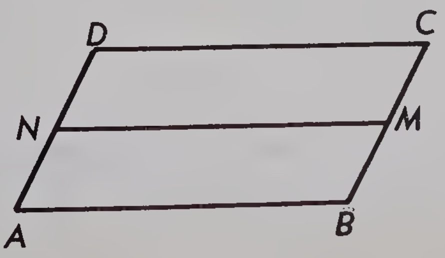

Chapter 9. Circles, Angles and Arcs

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1. The minor arcs $\widehat{m}$, $\widehat{p}$. $\widehat{q}$, $\widehat{r}$, $\widehat{KJ}$, $\widehat{KH}$, and $\widehat{HJ}$. The major arc is $\widehat{n}$.

2. ∠CDE; ∠CED; $\widehat{KJH}$.

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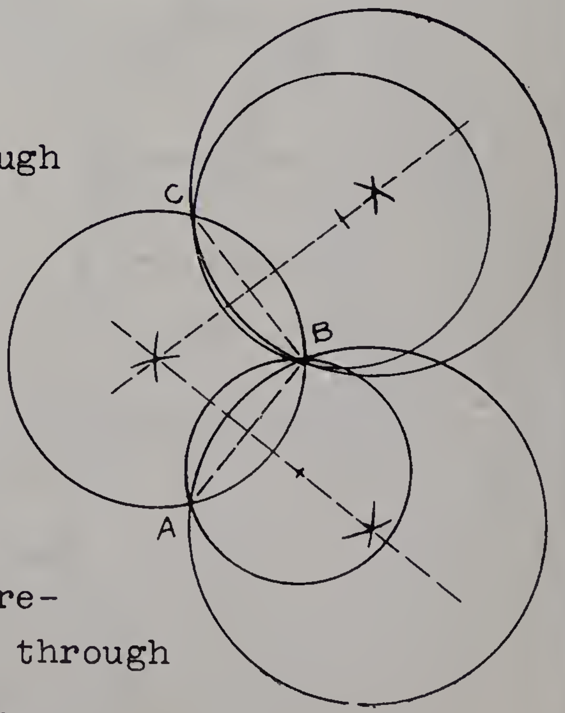
1. Given AB and CD, two diameters of ⊙O.
To prove that $\widehat{AC} = \widehat{DB}$.
Planning the Proof: 1. We can prove arcs = by §§ 281c, 282, 283. 2. We shall use § 282.
Proof: 1. ∠AOC = ∠BOD (§ 50d). 2. $\widehat{AC} = \widehat{DB}$ (§ 282).

2. There are 60° in each angle (§ 284).

3. Planning the Proof: 1. We can prove ∠s = by §§ 50, 98, 158, 219, 284. 2. We shall use § 284.
Proof: 1. $\widehat{AC} = \widehat{BC}$ (Given). 2. ∠AOC = ∠BOC (§ 284). 3. Then CO bisects ∠AOB (§ 22).

4. Use figure of Ex. 3.
Given ⊙O, and CO bisecting ∠AOB.
To prove that $\widehat{AC} = \widehat{CB}$.
Planning the Proof: 1. We can prove arcs = by §§ 281c, 282, 283. 2. We shall use § 282.
Proof: 1. CO bisects ∠AOB (Given). 2. ∠AOC = ∠BOC (§ 22). 3. $\widehat{AC} = \widehat{CB}$ (§ 282).

5. (1) Yes.
(2) The center of a ⊙ passing through A and B is on the ⊥ bisector of line segment AB and the center of a ⊙ passing through B and C is on the ⊥ bisector of BC. If a ⊙ passes through all three points, its center is the intersection of the two ⊥ bisectors. When the 3 points lie in a st. line, these ⊥s do not intersect (§ 160d). Therefore, there is no ⊙ that can be passed through A, B, and C when they lie in a st. line.



6. Planning the Proof: 1. We can prove: (a) arcs = by §§ 281c, 282, 283; (b) that a line is a diameter by § 277. 2. We shall use: (a) § 283; (b) § 277.

Proof: 1. $\overset{\frown}{AC} = \overset{\frown}{BD}$ (Given). 2. $\overset{\frown}{ACB} = \overset{\frown}{ADB}$ (§283). 3. $\overset{\frown}{BC} = \overset{\frown}{AD}$ (Asmt. 2). 4. ∠BOC = ∠AOD (§284). 5. ∠AOD is supp. ∠BOD (Asmt. 24). 6. ∴ ∠BOC is supp. ∠BOD (Asmt. 6). 7. ∴ COD is a st. line (Asmt. 25). 8. ∴ COD is a diameter (§277).

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1. (1) Arcs equal, §§ 281e, 282, 283, 286. (2) Chords equal, § 287. (3) Central angles equal, § 284.

2. (1) The arcs are equal. (2) §286.

3. 1. ABCDEF is an equilateral hexagon inscribed in ⊙ O (Given). 2. ACE is a triangle inscribed in ⊙ O (Given). 3. $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EF} = \overset{\frown}{AF}$ (§ 286). 4. $\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{CD} + \overset{\frown}{DE} = \overset{\frown}{EF} + \overset{\frown}{AF}$ (Asmt. 1). 5. $\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{AC}$, $\overset{\frown}{CD} + \overset{\frown}{DE} = \overset{\frown}{CE}$, $\overset{\frown}{EF} + \overset{\frown}{FA} = \overset{\frown}{EA}$ (Asmt. 9. 6. $\overset{\frown}{AC} = \overset{\frown}{CE} = \overset{\frown}{EA}$ (Asmt. 6). 7. Chord AC = chord CE = chord EA (§ 287). 8. △ABC ≅ △CDE ≅ △AEF (§ 95c). 9. ∠ABC = ∠CDE = ∠AFE (§98b). 10. ∠BAC = ∠DCE = ∠FEA = ∠BCA = ∠CED = ∠EAF (§§98b, 98a).

4. Planning the Proof: 1. We can prove that a quadrilateral is a ▱ by § 223. 2. We shall use § 223b.

Proof: 1. $\overset{\frown}{AB} = \overset{\frown}{DC}$ and $\overset{\frown}{AD} = \overset{\frown}{BC}$ (Given). 2. AB = DC and AD = BC (§287). 3. ∴ ABCD is a ▱ (§ 223b).

5. (1) $\frac{1}{3}$ of 360° = 120°. (2) $\frac{1}{4}$ of 360° = 90°.
(3) $\frac{1}{6}$ of 360° = 60°. (4) $\frac{1}{8}$ of 360° = 45°.

6. (1) No. (2) Yes.

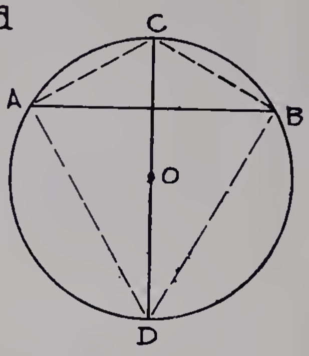
7. The midpoint of an arc bisects the arc, and the center of an arc is the center of the circle of which the arc is a part.

8. Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 281, 287. 2. We shall use § 287.

Proof: 1. Diameter AB bisects $\overset{\frown}{CD}$ (Given). 2. $\overset{\frown}{DB} = \overset{\frown}{BC}$ (§ 22). 3. $\overset{\frown}{ADB} = \overset{\frown}{ACB}$ (§ 283). 4. $\overset{\frown}{AD} = \overset{\frown}{AC}$ (Asmt. 2). 5. Chord AD = chord AC (§ 287).

9. Planning the Proof: 1. We can prove arcs = by §§ 281c, 282, 283, 286. 2. We shall use § 286, Asmt. 2.

Proof: 1. AC = AD (Given). 2. $\widehat{AC} = \widehat{AD}$ (§286). 3. AB is a diameter (Given). 4. $\widehat{ACB} = \widehat{ADB}$ (§283). 5. $\widehat{BD} = \widehat{BC}$ (Asmt. 2).



10. Given ⊙O, chord AB, chord CD, $\widehat{AC} = \widehat{BC}$, and $\widehat{AD} = \widehat{BD}$.

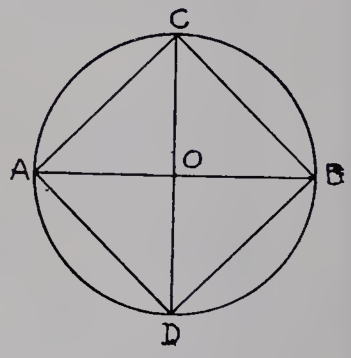
To prove that CD is the ⊥ bisector of AB.

Planning the Proof: 1. We can prove that a line is the ⊥ bisector of a line segment by §140. 2. We shall use §140.

Proof: 1. Draw AC, BC, AD, and BD (Asmt. 11). 2. $\widehat{AC} = \widehat{BC}$ and $\widehat{AD} = \widehat{BD}$ (Given). 3. Chord AC = chord BC and chord AD = chord BD (§287). 4. ∴ CD is the ⊥ bisector of AB (§140).

11. Planning the Proof: 1. We can prove arcs = by §§ 281c, 282, 283, 286. 2. We shall use §282.

Proof: 1. AC ∥ OD (Given). 2. ∴ $\angle x = \angle y$ (§158a). 3. AO = OC (Asmt. 26). 4. ∴ $\angle x = \angle m$ (§98a). 5. $\angle m = \angle n$ (§158a). 6. ∴ $\angle n = \angle y$ (Asmt. 7). 7. ∴ $\widehat{CD} = \widehat{BD}$ (§282).



12. Given AB and CD, two ⊥ diameters of ⊙O, and chords AD, DB, BC, and CA.

To prove that ADBC is a square.

Planning the Proof: 1. We can prove that a quadrilateral is a square by §185. 2. We shall use §185.

Proof: 1. AB and CD are two ⊥ diameters (Given). 2. $\angle AOC = \angle BOC = \angle BOD = \angle AOD$ (§14). 3. ∴ $\widehat{AC} = \widehat{BC} = \widehat{BD} = \widehat{AD}$ (§282). 4. ∴ AC = BC = BD = AD (§287). 5. ∴ ACBD is a ▱ (§223). 6. In rt. △AOC, AO = OC (Asmt. 6). 7. ∴ $\angle OAC = \angle OCA$ (§98a). 8. But $\angle OAC + \angle OCA = 90°$ (§161). 9. ∴ $2\angle OCA = 90°$ (Asmt. 6). 10. ∴ $\angle OCA = 45°$ (Asmt. 4). 11. In like manner, $\angle OCB = 45°$ (Statements 6-11). 12. Then $\angle ACB = 90°$ (Asmt. 1). 13. ∴ ADBC is a square (§185).

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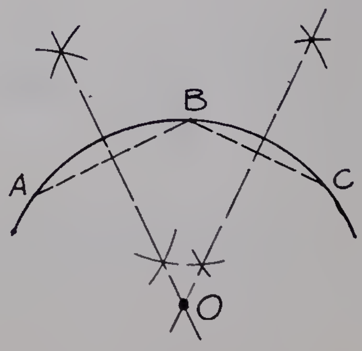


1. Given $\widehat{ABC}$ an arc of a ⊙.

To find the center of the ⊙.

Construction: 1. Draw chords AB and BC (Asmt. 11). 2. Construct the ⊥ bisectors of AB and BC (§177b).

Then the ⊥ bisectors intersect in a point O which is the center of the ⊙ .

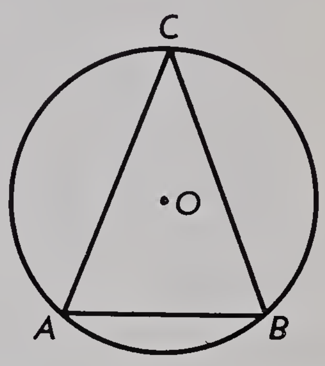
Proof: 1. Each ⊥ bisector passes through O, the center of the ⊙ (§ 292). 2. The ⊥ bisectors intersect in only one point (Asmt. 12).

2. Not necessarily.

3. §§ 51, 159, 220, 291.

4. Planning the Proof: 1. We can prove ∠s = by §§50, 98, 158, 219, 284. 2. We shall use § 98a.

Proof: 1. In ⊙ O, $\widehat{AC} = \widehat{BC}$ (Given). 2. Chord AC = chord BC (§ 287). 3. ∠ A = ∠B (§ 98a).



5. Use figure for Ex. 4.

Planning the Proof: 1. We can prove a △ isos. by § 56. 2. We shall use §56.

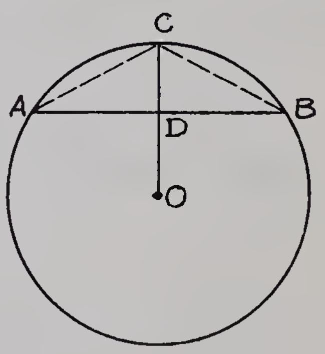
Proof: 1. In △ABC, ∠A = ∠B (Given). 2. AC = BC (§157a). 3. △ABC is isos. (§56).

6. Given ⊙O, D the midpoint of chord AB, C the midpoint of $\widehat{AB}$, and line CD.

To prove that CD ⊥ AB.

Planning the Proof: 1. We can prove two lines ⊥ to each other by §§ 51, 159, 220, 291. 2. We shall use § 159b.

Proof: 1. Draw AC and BC (Asmt. 11). 2. AD = DB (Given). 3. $\widehat{AC} = \widehat{BC}$ (Given). 4. AC = BC (§ 282). 5. ∴ CD ⊥ AB (§159b).

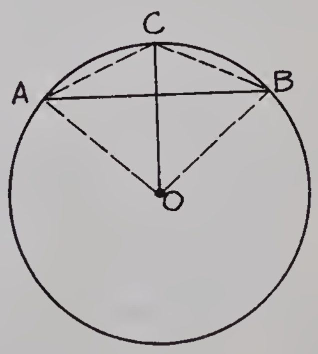


7. Given ⊙O with radius OC bisecting $\widehat{AB}$, and chord AB.

To prove that OC is the ⊥ bisector of AB.

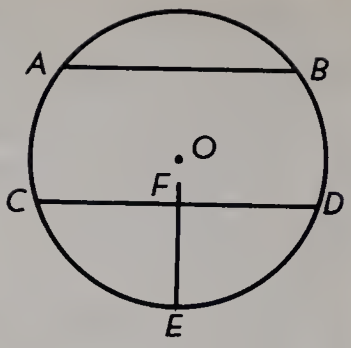
Planning the Proof: 1. We can prove that a line is the ⊥ bisector of a line segment by § 140. 2. We shall use § 140.

Proof: 1. Draw OA, OB, CA, and CB (Asmt. 11). 2. OA = OB (Asmt. 26). 3. $\widehat{AC} = \widehat{BC}$ (Given). 4. AC = BC (§ 287). 5. ∴ OC is the ⊥ bisector of AB (§ 140).



8. Given AB and CD two ∥ chords of ⊙O, and EF the ⊥ bisector of chord CD.

Prove that EF bisects chord AB.

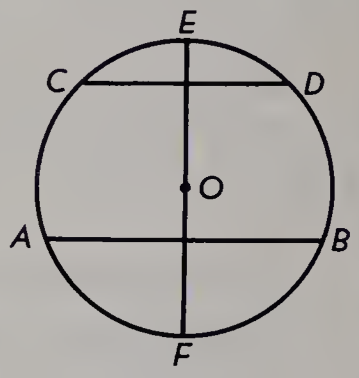


Planning the Proof: 1. We can prove that a line segment is bisected by §§22, 140, 290. 2. We shall use § 290.

Proof: 1. EF is the ⊥ bisector of CD (Given). 2. EF passes through O (§292). 3. AB ∥ CD (Given). 4. EF ⊥ AB (§159a). 5. EF bisects AB (§ 290).

9. Given ⊙O, with diameter EF bisecting chords AB and CD.

To prove that AB ∥ CD.

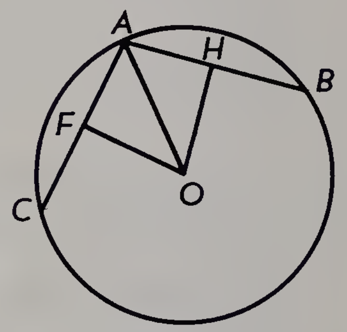


Planning the Proof: 1. We can prove lines ∥ by §§160, 221. 2. We shall use §160d.

Proof: 1. Diameter EF bisects AB and CD (Given). 2. EF ⊥ AB and EF ⊥ CD (§291). 3. AB ∥ CD (§ 160d).

10. Given ⊙O, chord AB = chord AC, F the midpoint of AC, H the midpoint of AB, and line segments OF, OA, and OH.

To prove that ∠OAB = ∠OAC.

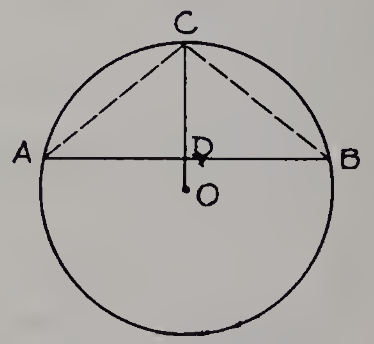


Planning the Proof: 1. We can prove ∠s = by §§50, 98, 158, 219, 284. 2. We shall use §98b.

Proof: 1. AB = AC (Given). 2. F is the midpoint of AC and H the midpoint of AB (Given). 3. AF = AH (Asmt. 9, 6, 4 and § 22). 4. OF ⊥ AC and OH ⊥ AB (§ 291). 5. ∠OFA and ∠OHA are rt. ∠s (§ 14). 6. △AOF and △AOH are rt. △s (§57). 7. AO = AO (Iden.). 8. △OFA ≅ △OHA (§156b). 9. ∠OAC = ∠OAB (§ 98b).

11. Given ⊙O with C the midpoint of $\overset{\frown}{AB}$, D the midpoint of chord AB, and line CD.

To prove that CD passes through O.



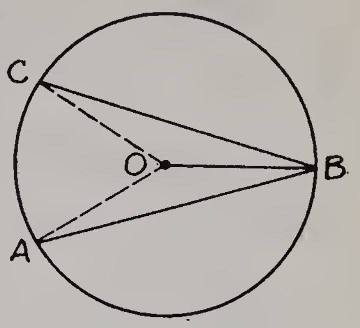
Planning the Proof: 1. We can prove that a line passes through a point by §§139, 141, 292. 2. We shall use § 292.

Proof: 1. Draw AC and BC (Asmt. 11). 2. AD = DB (Given). 3. $\overset{\frown}{AC} = \overset{\frown}{BC}$ (Given). 4. AC = BC (§ 287). 5. CD is the ⊥ bisector of AB (§ 140). 6. ∴ CD passes through O (§ 292).

12. Given $\odot O$ with radius OB, chords AB and CB, and $\angle ABO = \angle OBC$.

To prove $AB = CB$.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 281a, 287, 290. 2. We shall use § 97a.

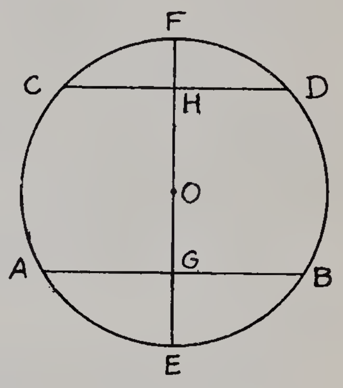


Proof: 1. Draw OA and OC (Asmt. 11). 2. $OA = OC = OB$ (Asmt. 26). 3. $\angle A = \angle OBA$ and $\angle C = \angle OBC$ (§ 98a). 4. But $\angle ABO = \angle OBC$ (Given). 5. $\therefore \angle A = \angle C$ (Asmt. 7). 6. From 4 and 5, $\angle AOB = \angle COB$ (§ 158c). 7. $\therefore \triangle AOB \cong \triangle COB$ (§ 95a). 8. $\therefore AB = CB$ (§ 97a).

13. Given $\odot O$ with chord $AB \parallel$ chord CD, and diameter EF bisecting CD in H.

To prove that EF bisects AB.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 281a, 287, 290. 2. We shall use § 290.



Proof: 1. $CH = HD$ (Given). 2. $\therefore OH \perp CD$ (§ 291). 3. $CD \parallel AB$ (Given). 4. $\therefore OE \perp AB$ (§ 159a). 5. $\therefore AG = GB$ (§ 290). 6. $\therefore$ EF bisects AB (§ 22).

14. Given $\odot O$, A equidistant from C and D, and B equidistant from C and D.

To prove that AB is a diameter.

Planning the Proof: 1. We can prove that a line segment is a diameter by §§ 277, 285. 2. We shall use § 277.

Proof: 1. Draw chord CD (Asmt. 11). 2. A is equidistant from C and D (Given). 3. B is equidistant from C and D (Given). 4. $\therefore$ AB is the $\perp$ bisector of CD (§ 140). 5. $\therefore$ AB passes through the center of the circle (§ 292). 6. $\therefore$ AB is a diameter (§ 277).

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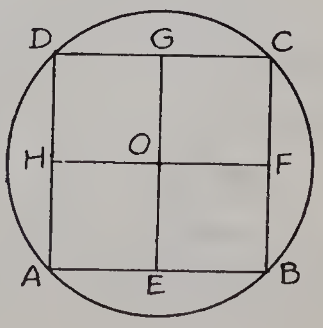
1. (1) The chords are equidistant from the center. (2) § 293.

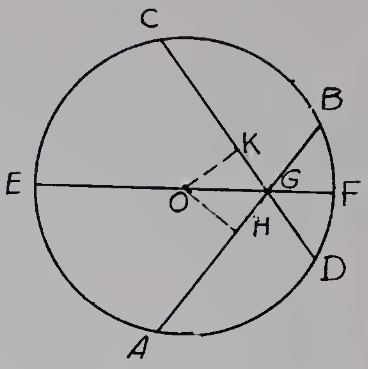
2. Given ABCD, a square inscribed in $\odot O$, with OE, OF, OG, and OH the distances from O to the sides.

To prove that $OE = OF = OG = OH$.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 281a, 287, 290, 293. 2. We shall use § 293.



Proof: 1. AB = BC = CD = AD (Given, §§185, 194). 2. OE = OF = OG = OH (§293).

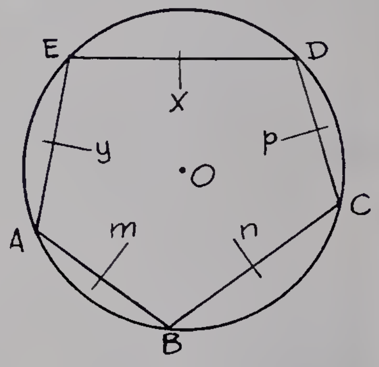


3. Given the = chords AB and CD of ⊙O intersecting at G on diameter EF.

To prove that ∠OGH = ∠OGK.

Planning the Proof: 1. We can prove ∠s = by §§ 50, 98, 158, 219, 284. 2. We shall use § 98b.

Proof: 1. Draw OH ⊥ AB and OK ⊥ CD (§178a). 2. AB = CD (Given). 3. ∴ OH = OK (§ 293). 4. OG = OG (Iden.). 5. △OHG and △OKG are rt. △s (§§14, 57). 6. ∴ rt. △OHG ≅ rt. △OKG (§156b). 7. ∴ ∠OGH = ∠OGK (§98b).



4. Given ⊙O, the inscribed polygon ABCDE, and m, n, p, x, and y, the ⊥ bisectors of the sides of ABCDE.

To prove that m, n, p, x, and y meet in a point.

Planning the Proof: 1. We can prove that a line passes through a point by §§ 139, 141, 292. 2. We shall use § 292.

Proof: 1. m is the ⊥ bisector of AB, n is the ⊥ bisector of BC, etc. (Given). 2. Then m, n, p, x, and y pass through the point O, the center of the circle (§292).

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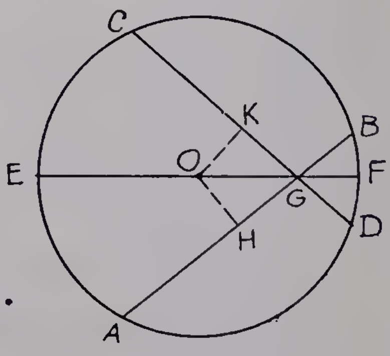
1. The contrapositive of Theorem 43 is "If two chords in a circle or in equal circles are unequal, the chords are not equally distant from the center." Theorem 43 has been proved true. Then the contrapositive of Theorem 43 is true (§134, Asmt. 38).

2. Given chords AB and CD of ⊙ O, intersecting the diameter EF at G, making ∠AGE = ∠CGE.



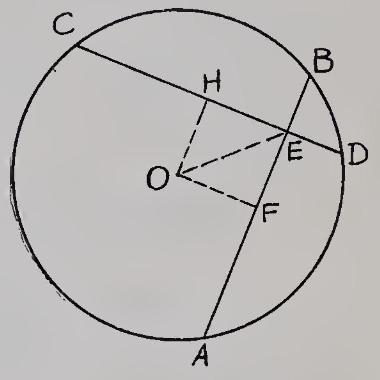
To prove that AB = CD.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 281a, 287, 290, 293, 294. 2. We shall use § 294.

Proof: 1. Draw OH ⊥ AB and OK ⊥ CD (§178a). 2. ∠ AGE = ∠EGC (Given). 3. OG = OG (Given). 4. ∠ OKG and ∠OHG are rt. ∠s (§14). 5. △OKG and △OHG are rt. $\triangle$s (§57). 6. $\triangle$OKG $\cong$ $\triangle$OHG (§156a). 7. OH = OK (§97a). 8. AB = AC (§294).

3. Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 281a, 287, 290, 293, 294. 2. We shall use §97a, Asmt. 1.

Proof: 1. Draw OE $\perp$ AP and OF $\perp$ CP (§178a). 2. AB = CD (Given). 3. Then OE = OF (§293). 4. OP = OP (Iden.). 5. $\angle$OEP and $\angle$OFP are rt. $\angle$s (§14). 6. $\triangle$OEP and $\triangle$OFP are rt. $\triangle$s (§57). 7. $\triangle$OEP $\cong$ $\triangle$OFP (§156b). 8. ∴ EP = FP (§97a). 9. AE = $\frac{1}{2}$AB = CF = $\frac{1}{2}$CD (§290). 10. ∴ AE = CF (Asmt. 4). 11. From 8 and 10, AP = CP (Asmt. 1, 9, 6).



4. Given $\odot$O with AB and CD two = chords intersecting at E.

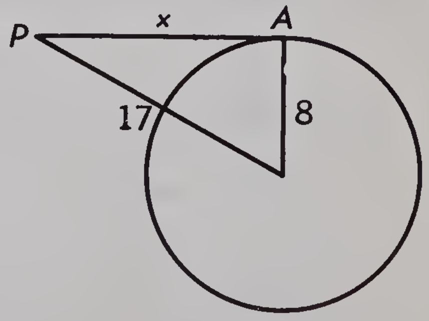
To prove that AE = CE and EB = ED.

Planning the Proof: 1. We can prove line segments = by (same as in Ex. 2). 2. We shall use §97a, Asmt.

Proof: 1. Draw OE (Asmt. 11). 2. Draw OF $\perp$ AB and OH $\perp$ CD (§178a). 3. AB = CD (Given). 4. OF = OH (§293). 5. OE = OE (Iden.). 6. $\angle$OFE and $\angle$OHE are rt. $\angle$s (§14). 7. $\triangle$OFE and $\triangle$OHE are rt. $\triangle$s (§57). 8. $\triangle$OEF $\cong$ $\triangle$OEH (§156b). 9. FE = HE (§97a). 10. AF = $\frac{1}{2}$AB and CH = $\frac{1}{2}$CD (§290). 11. ∴ AF = CH (Asmt. 4). 12. From 9 and 11, AE = CE (Asmt. 1, 9, 6). 13. From 12 and 3, EB = ED (Asmt. 2).

PAGE 299

1. If a point is perpendicular to a radius of a circle at its point on the circle, it is tangent to the circle.



2. Let x = the number of inches in PA.

$x^2 + 8^2 = 17^2$.

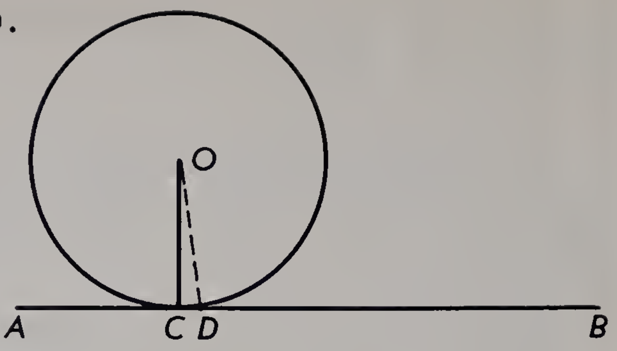
$x^2 + 64 = 289$.

$x^2 = 225$.

$x = \pm 15$. 15 inches. Ans.

PAGE 300

Given $\odot$ O, radius OC $\perp$ tangent AB, and D the point of contact of AB and $\odot$O.



To prove that OC passes through D.

Planning the Proof: 1. We can prove that a line passes through a point by §§139, 141, 292, 296. 2. We shall use the coincidence method of proof.

Proof: 1. $OC \perp AB$ and AB is tangent to ⊙O at D (Given). 2. Draw OD (Asmt. 11). 3. $OD \perp AB$ (§295). 4. OD and OC coincide and OC passes through D (Asmt. 20).

PAGES 301-304

1. a. $\overline{TS}^2 = 49 - 25$. $\overline{TS}^2 = 24$. $TS = 2\sqrt{6}$. $RS = 4\sqrt{6}$, or 9.796.

b. TS = 10. $\overline{OS}^2 = 100 + 144$. $\overline{OS}^2 = 244$. $OS = 2\sqrt{61}$, or 15.62.

c. TS = 6 and RS = 12.

2. a. $PA = \sqrt{13^2 - 5^2} = 12$. c. $PO = \sqrt{5.6^2 + 4.2^2} = 7$.

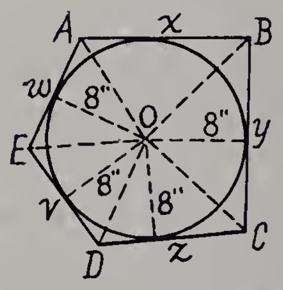
b. $AO = \sqrt{34^2 - 30^2} = 16$.

3. a. 120°. b. 60°. c. 60°. d. 10 in.

4. $x^2 + 5x = 2x + 10$. $x^2 + 3x - 10 = 0$. $(x + 5)(x - 2) = 0$. $x - 2 = 0$, $x = 2$. $x^2 + 5x = 14$ and $2x + 10 = 14$.

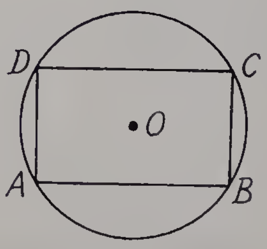
14 in. each. Ans.

5. Let x = number of inches from the center to the chord. Then $x^2 + 36 = 100$. $x^2 = 64$. $x = 8$ (in.).



6. △AOB = 4x; △BOC = 4y; △COD = 4z; △DOE = 4v; △AOE = 4w. Adding, polygon ABCDE = 4(x + y + z + v + w). But x + y + z + v + w = 62. ∴ polygon ABCDE = 4 · 62 = 248 (sq.in.).

7. PB = PC = 14" (§300).



8. Given rect. ABCD inscribed in ⊙O.

To prove that AB and CD are equidistant from O.

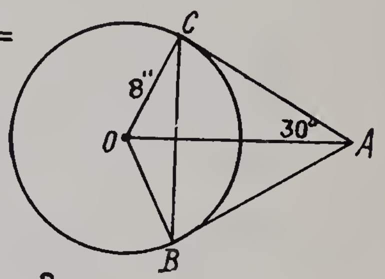
Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 281a, 287, 290, 293, 294, 300, 301. 2. We shall use §293.

Proof: 1. ABCD is a rectangle inscribed in ⊙O (Given). 2. AB = CD (§217a). 3. AB and CD are chords (§§288, 277). 4. ∴ AB and CD are equidistant from O (§293).

9. Planning the Proof: 1. Same as in Ex. 8. 2. We shall use Asmt., § 300.

Proof: 1. AB, BC, and AC are tangents to ⊙O at points D, F, and E respectively (Given). 2. AC = BC (Given). 3. AC = AE + EC and BC = BF + FC (Asmt. 9). 4. Then AE + EC = BF + FC (Asmt. 6). 5. EC = FC (§300). 6. AE = BF (Asmt. 2). 7. AE = AD and BF = DB (§300). 8. AD = DB (Asmt. 7).

10. ∠BAC = 60°. Then ∠OAC = 30°. ∠OCA = 90°. ∠AOC = 60°. OC = 8. OA = 16 (§136). $\overline{AC}^2 = 16^2 - 8^2 = 192$. AC = $8\sqrt{3}$ in., or 13.9 in.



11. Let r = the radius. Then $r^2 = (r - 1)^2 + 5^2$. $r^2 = r^2 - 2r + 1 + 25$. 2r = 26. r = 13 (in.).

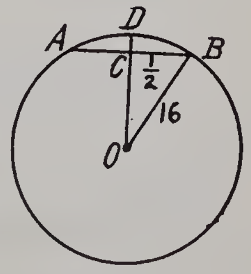
12. $51^2 = 24^2 + 45^2$. ∴ ABC is a rt. △ (§255). CD = $\frac{1}{2}$AB = 25.5 (§205). Then CD = 25.5 in.

13. The dotted line = AB. Then AB = $\sqrt{26^2 - 10^2}$ = 24 (in.).

14. Planning the Proof: 1. (Same as in Ex. 8). 2. We shall use §217d, Asmt. 2.

Proof: 1. AB is a diameter, CE ⊥ EF, and DF ⊥ EF (Given). 2. Draw a ⊥ from O to EF (§178a). 3. This line bisects EF (§290). 4. This line, CE, and DF are ∥ (§160d). 5. CO = OD (§217d). 6. AO = OB (Asmt. 26). 7. AC = DB (Asmt. 2).

15. The wheels are made flat by a difference of hardness in parts of the wheel, or by sliding on the rail.

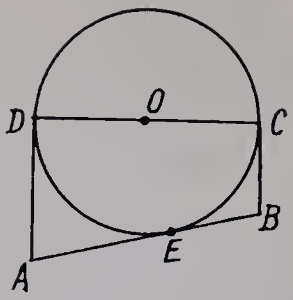


Let x = DC. Then CO = 16 - x. $(16 - x)^2 + (\frac{1}{2})^2 = 16^2$; $256 - 32x + x^2 + \frac{1}{4} = 256$; $x^2 - 32x + \frac{1}{4} = 0$. By formula,

$$x = \frac{32 \pm \sqrt{1024 - 4(1)(\frac{1}{4})}}{2} = \frac{32 \pm 31.984}{2} =$$

.008, discarding + value. .008 of an inch must be ground off.

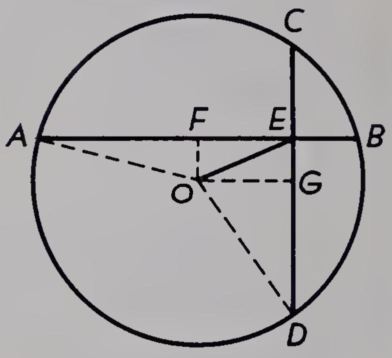
16. Planning the Proof: 1. We can find areas by § 273. 2. We shall use § 273e.



Proof: 1. $AD \perp DC$ and $BC \perp DC$ (§295). 2. $AD \parallel BC$ (§160d). 3. ABCD is a trapezoid (§206). 4. ABCD = $\frac{CD}{2}(AD + BC)$ (§273e). 5. AD = AE and BC = EB (§300). 6. ABCD = $\frac{CD}{2}(AE + EB)$ (Asmt. 6). 7. ABCD = $\frac{1}{2}CD \times AB$ (Asmt. 6).

17. OB = 12 in. The distance of BC from the center of the circle is 6 in. (§136).

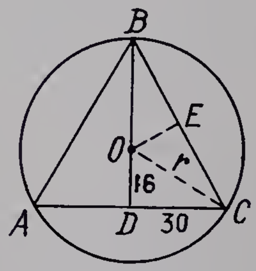
18. AB + DC = AD + BC (§300, Asmt. 9, 6, 1). 34 = 16 + BC. BC = 18. Ans.



19. AB = 32. CD = 24. Draw $OG \perp CD$ and $OF \perp AB$. Draw AO and OD. AF = 16. AO = 20. $OF = \sqrt{20^2 - 16^2} = 12$. GD = 12. OD = 20. $OG = \sqrt{20^2 - 12^2} = 16$. OF = EG. $OE = \sqrt{12^2 + 16^2} = 20$. Since OE = 20, E is a point on the circle and AB and CD meet at right angles on the circle. 20 in. Ans.

20. $OB = \sqrt{20^2 + 15^2} = 25$. The diameter is 50.

21. Case I. When the center of the circle is inside the $\triangle$. $r^2 = 16^2 + 30^2$. Solving, r = 34. BD = 16 + 34 = 50. $BC = \sqrt{50^2 + 30^2} = 58.31$. $OE = \sqrt{34^2 - 29.155^2} = 17.49$ (in.).

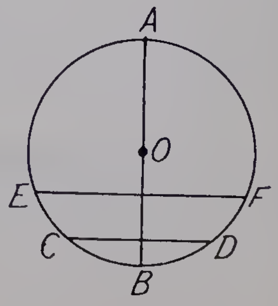


Case II. When the center of the circle is outside the triangle. $r^2 = 16^2 + 30^2$. Solving, r = 34. BD = 34 - 16 = 18. $BC = \sqrt{18^2 + 30^2} = 34.99$. $OE = \sqrt{34^2 - 17.49^2} = 29.15$ (in.).

PAGE 305

1. Planning the Proof: 1. We can prove $\triangle$s = by §§50, 98, 158, 219, 284, 300. 2. We shall use §98a.

Proof: 1. PA and PB are tangents to ⊙O from P (Given). 2. PA = PB (§300). 3. $\angle PAB = \angle PBA$ (§98a).



2. Given ⊙O, diameter AB bisecting chord EF, and $EF \parallel$ chord CD.

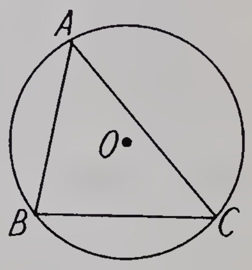
To prove that AB bisects CD.

Planning the Proof: 1. We can prove that a line segment is bisected by §§22, 140, 290, 301. 2. We shall use §290.

Proof: 1. AB bisects EF (Given). 2. $AB \perp EF$ (§291). 3. $EF \parallel CD$ (Given). 4. $AB \perp CD$ (§159a). 5. AB bisects CD (§290).

3. Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 281a, 287, 290, 293, 294, 300, 301. 2. We shall use § 97a.

Proof: 1. Draw $DE \perp AB$ and $DF \perp AC$ (§172). 2. AD = AD (Iden.). 3. $\angle DAF = \angle DAE$ (§300). 4. $\angle AED$ and $\angle AFD$ are rt. $\angle$s (§14). 5. $\triangle DFA$ and $\triangle DEA$ are rt. $\triangle$s (§57). 6. $\triangle DEA \cong \triangle DFA$ (§156a). 7. DE = DF (§97a).

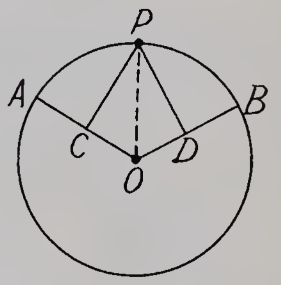


4. Given $\triangle ABC$ inscribed in $\odot O$ and $\angle A = \angle C$.

To prove that $\overset{\frown}{AB} = \overset{\frown}{BC}$.

Planning the Proof: We can prove arcs = by §§281c, 282, 283, 286, 290. 2. We shall use §286.

Proof: 1. $\angle A = \angle C$ (Given). 2. AB = BC (§157a). 3. $\overset{\frown}{AB} = \overset{\frown}{BC}$ (§286).

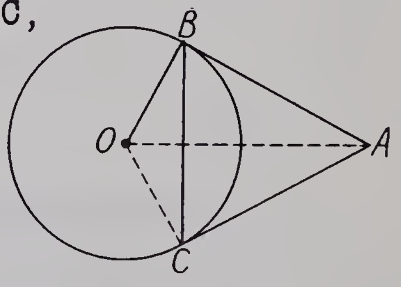


5. Given point P on $\odot O$ and PC = PD.

To prove $\overset{\frown}{AP} = \overset{\frown}{PB}$.

Planning the Proof: 1. We can prove arcs equal by §§281c, 282, 283, 286, 290. 2. We shall use § 282.

Proof: 1. PC = PD (Given). 2. Draw PO (Asmt. 11). 3. PO = PO (Iden.). 4. $\therefore$ $\angle PCO$ and $\angle PDO$ are rt. $\angle$s (§14). 5. $\triangle PCO = \triangle PDO$ are rt. $\triangle$s (§57). 6. $\triangle POC \cong \triangle POD$ (§156b). 7. $\angle AOP = \angle BOP$ (§98b). 8. $\overset{\frown}{AP} = \overset{\frown}{PB}$ (§282).



6. Given tangents AB and AC to $\odot O$, chord BC, and radius OB.

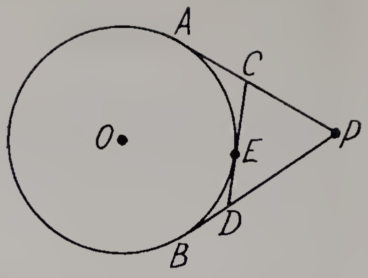
To prove that $\angle A = 2\angle OBC$.

Planning the Proof: 1. We can prove $\angle$s = by §§50, 98, 158, 219, 284, 300. 2. We shall use §50e.

Proof: 1. Draw OA and OC (Asmt. 11). 2. $OB \perp AB$ (§295). 3. OB = OC (Asmt. 26). 4. AB = AC (§300). 5. $OA \perp BC$ (§159b). 6. $\angle OBC = \angle OAB$ (§158b). 7. $\angle CAB = 2\angle OAB$ (§300). 8. $\angle CAB = 2\angle OBC$ (Asmt. 6).

7. Given tangents PA and PB to $\odot O$, tangent CD at E intersecting PA in C and PB in D.

To prove that PC + CD + PD = PA + PB.

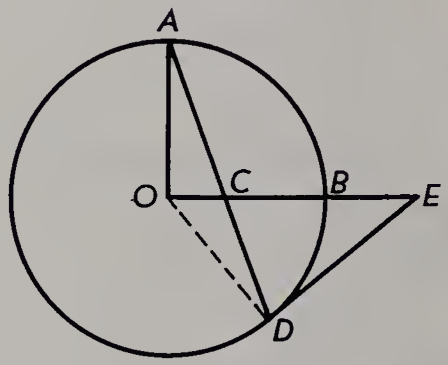


Planning the Proof: 1. We can prove sum of segments = by §97c. 2. We shall use §97c.

Proof: 1. AC = CE and BD = DE (§300). 2. PC = PC and PD = PD (Iden.). 3. AC + BD + PC + PD = CE + DE + PC + PD (Asmt. 1). 4. PA + PB = DC + PC + PD (Asmt. 9, 6).

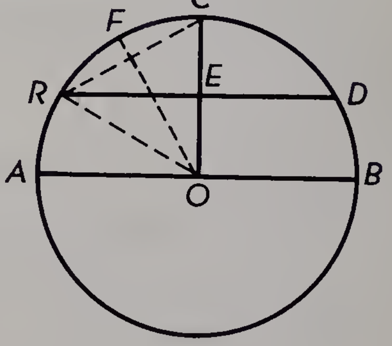
8. Planning the Proof: 1. We can prove lines ∥ by §§160, 221. 2. We shall use §160b.

Proof: 1. Draw OB (Asmt. 11). 2. PA = PB (§300). 3. ∠OPA = ∠OPB (§300). 4. OP = OP (Iden.). 5. △ AOP ≅ △BOP (§95a). 6. ∠AOP = ∠BOP (§98b). 7. OB = OC (Asmt. 26). 8. ∠OCB = ∠OBC (§98a). 9. ∠AOB = ∠OCB + ∠OBC (§123). 10. ∠AOB = 2∠OCB (Asmt. 6). 11. ∠AOB = ∠AOP + ∠BOP (Asmt. 9). 12. ∠AOB = 2∠AOP (Asmt. 6). 13. 2∠AOP = 2∠OCB (Asmt. 7). 14. ∠AOP = ∠OCB (Asmt. 4). 15. OP ∥ CB (§160b).



9. Planning the Proof: 1. We can prove a triangle isosceles by §56. 2. We shall use §56.

Proof: 1. Draw OD (Asmt. 11). 2. AO ⊥OE (Given). 3. ∠ AOC is a rt. ∠ (§14). 4. △ AOC is a rt. △ (§57). 5. ∠OCA is comp. ∠ A (§124). 6. ∠OCA = ∠DCE (§50d). 7. ∠DCE is comp. ∠A (Asmt. 6). 8. DE is tangent ⊙O at D (Given). 9. DE ⊥ OD (§295). 10. ∠ ODE is a rt. ∠ (§14). 11. ∠CDE is comp. ∠CDO (§ 20). 12. OA = OD (Asmt. 26). 13. ∠ A = ∠CDO (§98a). 14. From 7, 11, and 13, ∠DCE = ∠CDE (§50b). 15. CE = CD (§157a). 16. △ DCE is isos. (§56).



10. Planning the Proof: 1. We can prove arcs equal by §§281c, 282, 283, 286, 290. 2. We shall use § 282, Asmt.

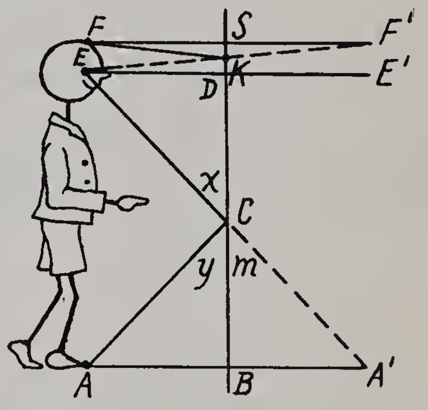
Proof: 1. CO ⊥ AB (Given). 2. RD is the ⊥ bisector of CO (Given). 3. Draw RO and RC (Asmt. 11). 4. CE = EO (§22). 5. ∠OER = ∠CER (§14). 6. RE = RE (Iden.). 7. △ REO ≅ △REC (§95a). 8. RC = RO (§ 97a). 9. RO = CO (Asmt. 26). 10. RC = RO = CO (Asmt. 6). 11. ∠ROC + ∠ORC + ∠OCR = 180° (§120). 12. ∠ROC = ∠ORC = ∠OCR (§84).

13. $3\angle ROC = 180°$ (Asmt. 6). 14. $\angle ROC = 60°$ (Asmt. 4). 15. $\angle AOC$ = a rt. $\angle$ or 90° (§14). 16. $\angle AOR = 30°$ (Asmt. 2). 17. Draw OF bisecting $\angle ROC$ (Asmt. 18). 18. $\angle ROF = \angle FOC = 30°$ (Asmt. 4). 19. $\overset{\frown}{AR} = \overset{\frown}{RF} = \overset{\frown}{FC}$ (§282). 20. $\overset{\frown}{RC} = \overset{\frown}{RF} + \overset{\frown}{FC}$ (Asmt. 9). 21. $\overset{\frown}{RC} = 2\overset{\frown}{AR}$ (Asmt. 6). 22. $\overset{\frown}{CD} = \overset{\frown}{RC}$ (§290). 23. $\overset{\frown}{CD} = 2\overset{\frown}{AR}$ (Asmt. 6).

PAGES 306-307

1. (1) As far behind the mirror as the object is in front of it. (2) The image has the same size and shape as the object.

2. (1) The rays of light from a point when reflected by the mirror seem to meet at the image of the point. (2) The two eyes and the object form the vertices of a triangle. When the object is close to the eyes, the angle between the rays to the eyes is larger than when the object is farther away. With one eye one does not have this triangle.

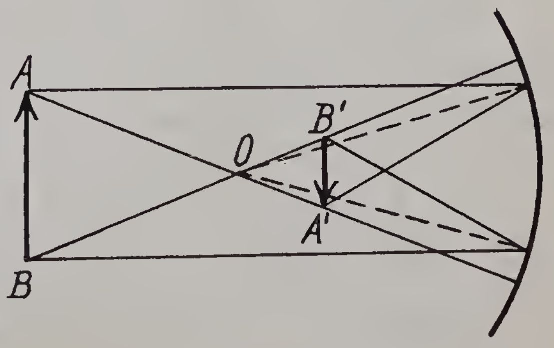


3. 1. From Ex. 1, p. 168, $\angle x = \angle y = \angle m$. 2. EDBA is a rect. (§§160d, 185). 3. Then AB = ED (§217a). 4. $\angle ABD = \angle EDC$ (§50a). 5. $\angle A = \angle E$ (§158c). 6. $\triangle ABC \cong \triangle CDE$ (§95b). 7. CD = BC or $DC = \frac{1}{2}DB$ (§97a). 8. BD = AE (§217a). 9. $DC = \frac{1}{2}AE$ (Asmt. 6). 10. In the same manner it can be shown that $DK = \frac{1}{2}EF$ (Statements 1-9). 11. $\therefore CK = \frac{1}{2}AF$ (Asmt. 1). 12. The mirror must be at least as long as CK.

4. The image is direct since the image of the right hand is to the right of the image of the left hand. The image of the right hand appears to be a left hand.

5. The inside of the glass of water is as far behind the plate glass as the candle is in front of the plate glass.

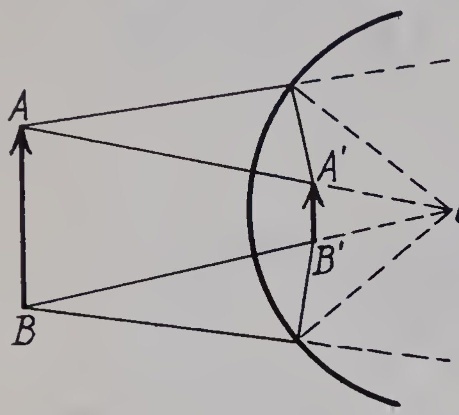
The magician often uses mirrors for producing illusions on the stage.



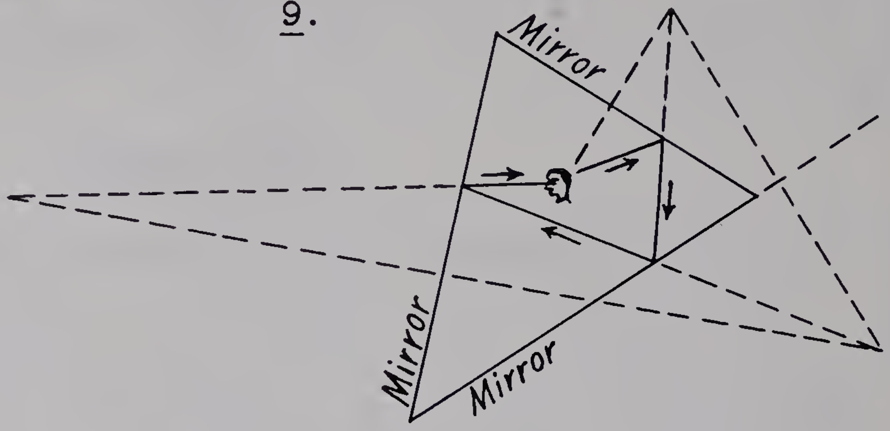
6. The ray from B through O is reflected back through O because a radius forms equal angles with the tangent at the point of tangency: §50b.

The image is real and inverted.

8. The image is virtual or unreal since it is only the apparent meeting of the reflected rays. The image is upright. (See diagram at left below.



9.



10. The images are smaller and one can see more at a glance in them.

PAGES 309-310

1. (a) No common tangents. (b) One common external tangent. (c) Two common external tangents. (d) One common internal tangent and two common external tangents. (e) Two common internal tangents and two common external tangents.

2. (1) None. (2) One. (3) Two.

3. Planning the Proof: 1. We can prove: (a) △s = by §§ 50, 98, 158, 219, 284, 300; (b) lines ∥ by §§ 160, 221. 2. We shall use: (a) § 98a, Asmt. 6; (b) § 160b.

Proof: 1. OA = OC (Asmt. 26). 2. ∴ ∠x = ∠y (§98a). 3. ∠m = ∠x + ∠y (§123). 4. ∴ ∠m = 2∠x (Asmt. 6). 5. Draw OD bisecting ∠BOC (§166). 6. Then 2∠BOD = 2∠x (Asmt. 6). 7. ∠BOD = ∠x (Asmt. 4). 8. ∴ OD ∥ AC (§160b).

4. Planning the Proof: 1. We can prove: (a) △s = by §§ 50, 98, 158, 219, 284, 300; (b) that a line is the ⊥ bisector by §140. 2. We shall use: (a) § 98a; (b) § 140.

Proof: 1. PA = PB (§300). 2. ∴ ∠r = ∠s (§ 98a). 3. Draw OA and OB (Asmt. 11). 4. OA = OB (Asmt. 26). 5. From 1 and 4, PO is the ⊥ bisector of AB (§ 140).

5. Planning the Proof: 1. We can prove △s = by §§ 50, 98, 158, 219, 284, 300. 2. We shall use § 158a.

Proof: 1. AB is tangent to ⊙ O at A and to ⊙ O' at B (Given).

2. Draw OA and O'B (Asmt. 11). 3. OA $\perp$ AB and O'B $\perp$ AB (§295). 4. OA $\parallel$ O'B (§160d). 5. $\angle$O = $\angle$O' (§158a).

6. Given $\odot$O and $\odot$O' tangent externally at C, with PC the common internal tangent and PA and PB tangents drawn from P, any point in PA.

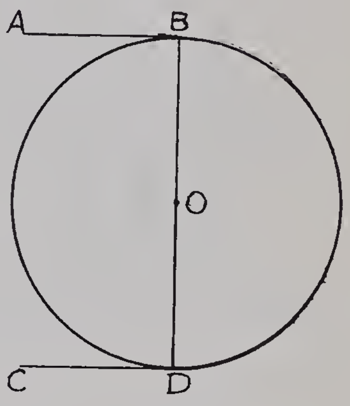
To prove that PA = PB.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 281a, 287, 290, 293, 294, 300. 2. We shall use §300, Asmt. 7.

Proof: 1. PA = PC and PB = PC (§300). 2. ∴ PA = PB (Asmt. 7).

7. Given $\odot$O, diameter BOD, tangent AB, and tangent CD.

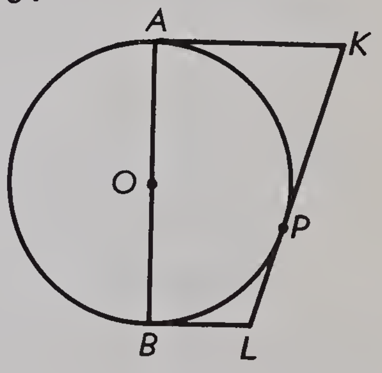
To prove that AB $\parallel$ CD.



Planning the Proof: 1. We can prove lines $\parallel$ by §§160, 221. 2. We shall use §160d.

Proof: 1. AB is tan. $\odot$O at B and CD is tan. $\odot$O at D (Given). 2. AB $\perp$ BD and CD $\perp$ BD (§295). 3. AB $\parallel$ CD (§160d).

8. Planning the Proof: 1. We can find areas of polygons by §273. 2. We shall use §273, b, e.



Proof: 1. AB is a diameter of $\odot$O (Given). 2. BL, LK, and AK are tangents to $\odot$O (Given). 3. BL $\perp$ AB and AK $\perp$ AB (§295). 4. BL $\parallel$ AK (§160d). 5. BL = LP and AK = KP (§300). 6. If LK $\parallel$ BA, BLKA is a ▱ (§185). Area BLKA = BL x AB (§273b). 7. KL = KP + PL (Asmt. 9). 8. KL = 2BL (Asmt. 6). 9. BL = $\frac{1}{2}$KL (Asmt. 4). 10. Area BLKA = $\frac{1}{2}$AB x KL (Asmt. 6). 11. If LK is not $\parallel$ BA, BLKA is a trap. (§206). 12. Area BLKA = $\frac{1}{2}$AB(BL + AK) (§273e). 13. Area BLKA = $\frac{1}{2}$AB(LP + PK) (Asmt. 6). 14. Area BLKA = $\frac{1}{2}$AB x LK (Asmt. 9, 6).

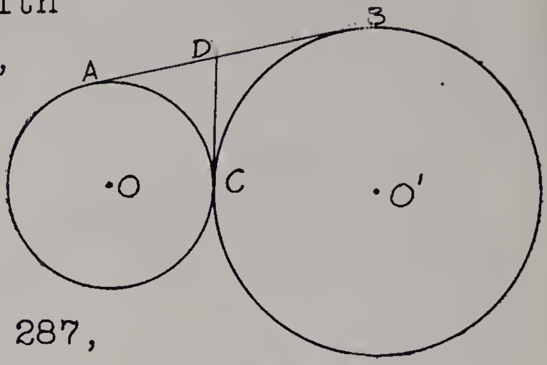
9. Planning the Proof: 1. We can prove a line segment = the sum of two line segments by §§97c, 207. 2. We shall use Asmt. 1, 6, 9.

Proof: 1. AE = AC and EB = BD (§300). 2. AE + EB = AC + BD (Asmt. 1). 3. ∴ AB = AC + BD (Asmt. 9, 6).

10. Planning the Proof: (Same as in Ex. 9).

Proof: 1. AE = AH, EB = BF, DG = DH, and GC = CF (§300).

2. AE + EB + DG + GC = AH + DH + BF + FC (Asmt. 1). 3. AB + DC = AD + BC (Asmt. 9, 6).

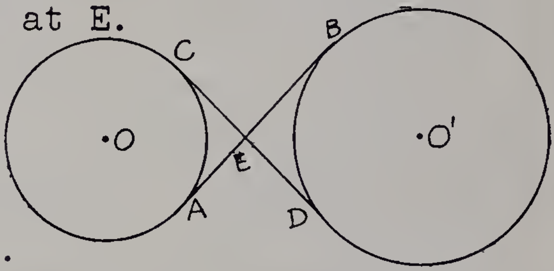


11. Given ⊙O tangent to ⊙O' at C, with DC their common internal tangent, and AB, one common external tangent, intersecting DC at D.

To prove that AD = DB.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 281a, 287, 290, 293, 294, 300. 2. We shall use §300, Asmt. 6, § 22.

Proof: 1. AD = CD and BD = CD (§300). 2. DA = DB (Asmt. 7). 3. DC bisects AB (§22).

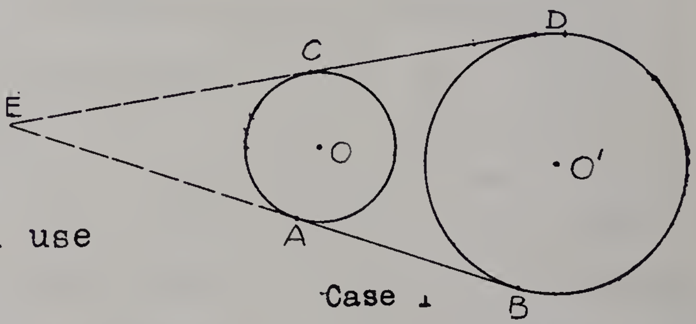


12. Given ⊙O and ⊙O', each without the other, and common internal tangents AB and CD intersecting at E.

To prove that AB = CD.

Planning the Proof: 1. We can prove line segments = by (Same as in Ex. 11). 2. We shall use §300, Asmt. 1.

Proof: 1. EC = EA and ED = EB (§300). 2. ∴ EC + ED = EA + EB (Asmt. 1). 3. ∴ AB = CD (Asmt. 9, 6).



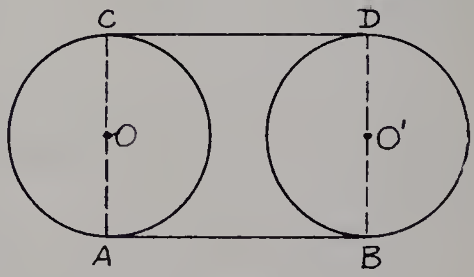
Case I

13. Given ⊙O and ⊙O', each without the other, and AB and CD their common external tangents.

To prove that AB = CD.

Planning the Proof: 1. We can prove line segments = by (Same as in Ex. 11). 2. We shall use §300, Asmt. 9, 2.

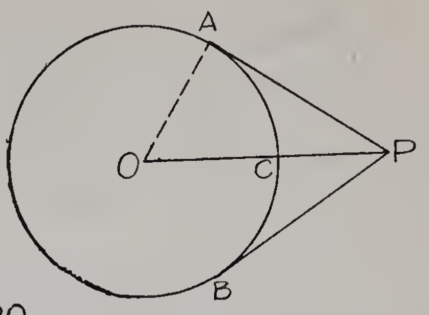
Proof: Case I. AB not ∥ CD. 1. Extend AB and CD (Asmt. 13). 2. AB meets CD in some point E (Asmt. 12). 3. EB = ED and EA = EC (§300). 4. Then AB = CD (Asmt. 9, 2).



Case II

Case II. AB ∥ CD. 1. Draw OA, OC, O'B, and O'D (Asmt. 11). 2. OA ⊥ AB, O'B ⊥ AB, OC ⊥ CD, and O'D ⊥ CD (§295). 3. OA ⊥ CD and O'B ⊥ CD (§159a). 4. ∴ OA coincides with OC and O'B coincides with O'D (§94). 5. AC ∥ BD (§160d). 6. ∴ ABDC is a ▱ (§223a). 7. ∴ AB = CD (§217a).

14. Given ⊙O, PA and PB, tangents to O from P, ∠BPA = 60°, and PO intersecting ⊙O in C.



To prove that OC = CP.

Planning the Proof: 1. We can prove line segments = by (Same as in Ex. 11). 2. We shall use Asmt.

Proof: 1. Draw AO (Asmt. 11). 2. OA ⊥ AP (§295). 3. ∠APB = 60° (Given). 4. ∠APO = 30° (§300, Asmt. 9, 6, 4). 5. ∴ AO = $\frac{1}{2}$OP (§136). 6. OA = OC (Asmt. 26). 7. ∴ OC = $\frac{1}{2}$OP (Asmt. 6). 8. OC = $\frac{1}{2}$(OC + CP) (Asmt. 6). 9. 2 OC = OC + CP (Asmt. 3). 10. ∴ OC = CP (Asmt. 2).

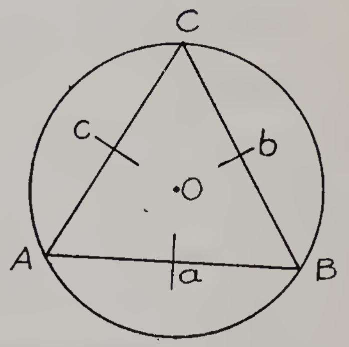
15. Planning the Proof: 1. We can prove the value of the sum of line segments by §§97c, 207. 2. We shall use §300, Asmt.

Proof: 1. AB, AC, and EF are tangents to ⊙O (Given). 2. Perimeter of △AFE = AE + AF + EF (§55). 3. EF = FG + GE (Asmt. 9). 4. Perimeter of △AFE = AE + AF + FG + GE (Asmt. 6). 5. FG = FB and GE = EC (§300). 6. Perimeter of △AFE = AE + AF + FB + EC (Asmt. 6). 7. AE + EC = AC and AF + FB = AB (Asmt. 9). 8. Perimeter of △AFE = AC + AB (Asmt. 6). 9. AC + AB is constant (Given). 10. Perimeter of △AFE is constant (Asmt. 6).

PAGES 310-312

1. Use the figure for Ex. 14, above. 1. OC = 5 in. 2. OC = $\frac{1}{2}$OP (Ex. 14, above). 3 ∴ PO = 10 in.

2. Given ⊙O and the inscribed △ABC, with a, b, and c, the ⊥ bisectors of AB, BC, and AC respectively.

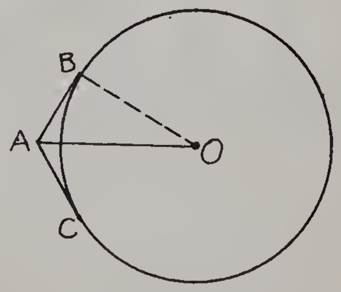


To prove that a, b, and c meet in a point.

Planning the Proof: 1. We can prove that a line passes through a point by §§139, 141, 292, 296. 2. We shall use §292.

Proof: 1. a is the ⊥ bisector of AB, b is the ⊥ bisector of BC, and c is the ⊥ bisector of AC (Given). 2. ∴ a, b, and c pass through O (§292).

3. Given ⊙O, AB and AC, tangents to O from A, the line AO, and ∠BAC = 120°.



To prove that AO = AB + AC.

Planning the Proof: 1. We can prove that a line segment is equal to the sum of two other line segments by §§97c, 207. 2. We shall use Asmt.

Proof: 1. Draw OB (Asmt. 11). 2. $AB \perp OB$ (§295). 3. $\angle BAC = 120°$ (Given). 4. $\therefore \angle BAO = 60°$ (§300, Asmt. 9, 6, 4). 5. $\therefore \angle O = 30°$ (§124). 6. $\therefore AB = \frac{1}{2}AO$ (§136). 7. $AC = AB$ (§300). 8. $\therefore AC = \frac{1}{2}AO$ (Asmt. 6). 9. $\therefore AB + AC = AO$ (Asmt. 1).

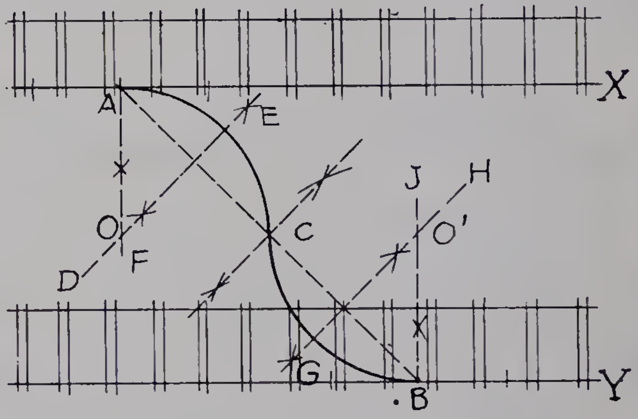
4. 1. $PF \parallel OB$ (Given). 2. $\therefore \angle r = \angle s$ (§158a). 3. $PE \perp PO$ (§295). 4. $\angle x$ is comp. of $\angle r$ (§20). 5. $CO \perp OB$ (Given). 6. $\therefore \angle y$ is comp. of $\angle s$ (§20). 7. $\therefore \angle x = \angle y$ (§50b).

5. Draw 2 chords. Construct the $\perp$ bisectors of the chords. These $\perp$ bisectors pass through the center of the circle. Then the radius can be easily measured.

6. (1) $EO \perp AD$ (§295). (2) $OF \perp BC$ (§295). (3) $BE = BF$ (§300). (4) $\angle O + \angle OEB + \angle EBF + \angle BFO = 360°$ (§210). $\angle E + \angle F = 180°$ (Asmt. 1). $\therefore \angle O + \angle EBF = 180°$, or $\angle O$ is supp. to $\angle EBF$ (Asmt. 2). But $\angle x$ is supp. to $\angle EBF$ (§53a). $\therefore \angle x = \angle O$ (§50c). (5) The radius should be large because the curve would then be more gradual. (6) Yes, when $\angle O$ is greater than 90° (§210).

7. a. They are tangent externally.
   b. The points O, B, and O' are in a st. line (§295, Asmt. 20).

8. Construction: 1. Draw AB (Asmt. 11). 2. Bisect AB and let C be its midpoint (§177b). 3. Construct DE the $\perp$ bisector of AC (§177b). 4. At A, construct $AF \perp AX$, intersecting ED in O (§178b). 5. With O as a center and OA as a radius, draw $\widehat{AC}$ of a ⊙ (Asmt. 15). 6. Construct GH the $\perp$ bisector of BC (§177b). 7. At B, construct $BJ \perp BY$, intersecting GH in O' (§178b). 8. With O' as a center and O'B as a radius, draw $\widehat{BC}$ of a ⊙ (Asmt. 15). Then AECB is the required curve.



9, 10, 11. The drawings in the exercises show the methods of construction.

PAGE 313 (Top)

1. Circle.　　2. Ellipse.

PAGE 313 (Middle)

1. Circle. 2. Ellipse.

PAGES 313-314

1. Proof: 1. Draw OO' $\perp$ m (§149a). 2. Draw OA, OB, O'A, and O'B (Asmt. 11). 3. OO' $\perp$ AO' and BO' (§147). 4. OA = OB (§306). 5. OO' = OO' (Iden.). 6. Rt. $\triangle$AOO' $\cong$ rt. $\triangle$OBO' (§156b). 7. AO' = BO' (§97a). 8. In the same manner, it may be proved that any other point of intersection has the same distance from O' as A (Statements 3-7). 9. $\therefore$ the intersection is a $\odot$ (§276).

2. The arc AEB of the smaller circle is the longer.

3. Yes; §298. 4. Two.

5. Given the two circles with equal radii and on sphere O.
To prove that their planes are equidistant from O.
Proof: 1. Draw OA and OB $\perp$ the planes of the two circles (§149a). 2. OA $\perp$ CA and OB $\perp$ EB (§147). 3. CO = EO (§306). 4. From the proof of Ex. 1, A and B are the centers of the Ⓢ. 5. CA = EB (Given). 6. Rt. $\triangle$CAO $\cong$ rt. $\triangle$EBO (§156b). 7. AO = BO (§97a). 8. $\therefore$ the two planes are equidistant from O.

6. An infinite number. 7. One. 8. A great circle; small circles; because the arc of a great circle is the shorter.

PAGE 315

1. (a) and (b) are true.
2. (a) is true and (b) is false.
3. (a) is true and (b) is false.
4. (a) is true and (b) is false.
5. Both (a) and (b) are false.
6. Dick assumed the converse of "An automobile will not run without gasoline" to be true when he decided that the gas tank was empty.
7. Lucy made her conclusion by assuming the converse of "If the temperature is 212°, water boils" to be true.
8. The conclusion is false because the converse is not always true, even if the original statement is true.

PAGES 315-316 (Review Questions)

1. Two radii (§278); two chords (§278). 2. (1) an infinite number (§277). (2) One (§277). 3. (1) Two (§277). (2) An infinite number (§277). 4. One; none. 5. 4 inches. 6. Diameter. 7. (1) One. (2) Two. 8. The center of the circle (§292). 9. (1) Radii, diameters. (2) Chords. 10. (1) Tangent. (2) Chord or secant. 11. On the circle. 12. Point of tangency, or point of contact. 13. The length of the radius. 14. 12 inches (§296). 15. A secant. 16. (1) Yes. (2) Yes. (3) Not necessarily. 17. Yes, unless the chord is a diameter in which case it may, or may not, be perpendicular (§291). 18. The common internal tangent is a tangent to both ⓢ which intersects the line segment joining the centers,—but the common external tangent is a tangent to both ⓢ and it does not intersect the line segment joining the centers (§303). 19. inscribed.

PAGES 317-318 (Test 21)

1. tangent to. 2. diameter. 3. the center of. 4. diameter. 5. equal. 6. an external point. 7. common chord. 8. no. 9. one. 10. chord.

PAGE 318 (Test 22)

1. a. §178a. b. §290. c. §290. d. Asmt. 2.

2. a. §283. b. Given. c. §286. d. Asmt. 2. e. § 287. f. Iden. g. §95c. h. §98b i. §160b.

PAGE 319 (Test 23)

1. Chord. 2. Four. 3. 60°. 4. Isosceles. 5. Two chords. 6. Equidistant from the center.

PAGE 319 (Test 24)

1. (1) Given $\odot O$ with CD bisecting $\overset{\frown}{AB}$ and chord AB.
(2) To prove that CD $\perp$ AB.

2. (1) Given $\odot O$ with radii OA and OB, C the midpoint of $\overset{\frown}{AB}$, CD $\perp$ OA, and CE $\perp$ OB.
(2) To prove that CD = CE.

Chapter 10. Measurement of Angles and Arcs

PAGE 322

1. Yes. 2. Yes. 3. (1) Yes. (2) Yes. (3) No; to have more than two points in common, the circles would have to coincide, and then the number of points in common would be infinite. 4. The chord and the radii drawn to its extremities form an equilateral △. Hence the central ∠ contains 60°. 5. Chords. 6. Its central ∠ increases in the same ratio. 7. Yes. 8. No. 9. $\overset{\frown}{BC}$. 10. ∠ BOC. 11. (1) Yes. (2) Yes. 12. (1) Yes. (2) Yes.

13-14. These are to be original constructions by the student.

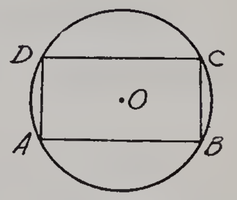
PAGE 325

1. (1) $\angle B = \frac{1}{2}$ of 140° = 70°. (2) $\angle C = \frac{1}{2}$ of 100° = 50°.
(3) $\angle A = 180° - 70° - 50° = 60°$.

2. (1) $\overset{\frown}{BC} = 2 \times 40° = 80°$. (2) $\overset{\frown}{AB} = 2 \times 46° = 92°$.
(3) $\overset{\frown}{AC} = 360° - 80° - 92° = 188°$.

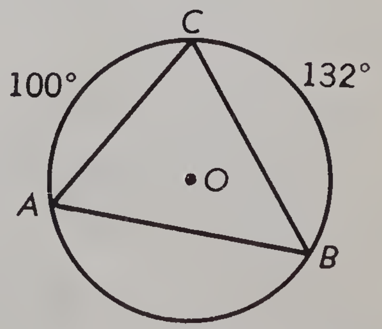
3. (1) $\angle B = \frac{1}{2}$ of 168° = 84°. (2) $\angle C = \frac{1}{2}$ of 114° = 57°.
(3) $\angle A = 180° - 84° - 57° = 39°$.

4. Let x = the number of degrees in $\overset{\frown}{AB}$, and in $\overset{\frown}{BC}$. 2x = the number of degrees in $\overset{\frown}{AC}$. $x + x + 2x = 360$. $4x = 360$. $x = 90$ and $2x = 180$. $\overset{\frown}{AB} = \overset{\frown}{BC} = 90°$ and $\overset{\frown}{AC} = 180°$. (1) $\angle A = \frac{1}{2}$ of 90° = 45°. (2) $\angle C = \frac{1}{2}$ of 90° = 45°. (3) $\angle B = \frac{1}{2}$ of 180° = 90°.



5. Given ▱ABCD inscribed in ⊙O.
To prove that ABCD is a rect.
Planning the Proof: 1. We can prove that a ▱ is a rect. by §185. 2. We shall use §185.
Proof: 1. ∠A + ∠C = 180° (§321). 2. ∠A = ∠C (§219).
3. ∴ 2∠A = 180° (Asmt. 6). 4. ∴ ∠A = 90° (Asmt. 4).
5. ∴ ABCD is a rect. (§185).

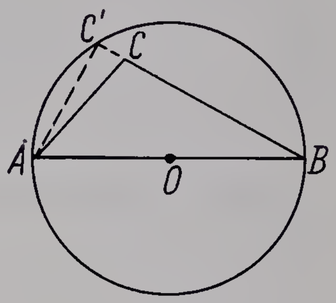
6. In the figure $\overset{\frown}{AB} = 360° - 100° - 132° = 128°$.
(1) $\angle A = \frac{1}{2}$ of 132° = 66°.
(2) $\angle B = \frac{1}{2}$ of 100° = 50°.
(3) $\angle C = \frac{1}{2}$ of 128° = 64°.



66°, 50°, 64°. Ans.

7. ∠BOC = 60°. Then $\widehat{BC}$ = 60°, and $\widehat{AC}$ = 120°. ∠B = $\frac{1}{2}$(120°) = 60°. △BOC is equilateral. ∴ BC = 9 in.
△ ABC is a rt. △ . ∴ AC = $\sqrt{18^2 - 9^2} = 9\sqrt{3}$, or 15.6 (in.).

8. The ▱ is a rect. (Ex. 5). Let 8x and 15x = the sides. Then $(8x)^2 + (15x)^2 = 34^2$. $64x^2 + 225x^2 = 1156$; $289x^2 = 1156$; $x^2 = 4$; x = 2. Then 8x = 16 and 15x = 30. Area = 16 x 30 = 480 (sq.in.).



9. Given rt. ∠ ACB.
To prove that ∠ACB can be inscribed in a semicircle.
Planning the Proof: 1. We can prove that an angle is inscribed by §278. 2. We shall use §278.
Proof: 1. Draw AB (Asmt. 11). 2. With AB as a diameter, draw ⊙O (§177b, Asmt. 15). 3. C is either on ⊙O, or not on it. 4. Suppose C is not on the circle and that BC cuts the circle in C'. Draw AC' (Asmt. 11). 5. ∠ AC'B = a rt. ∠ (§319). 6. Then AC ⊥ BC and AC' ⊥ BC (Given, Statement 5). 7. This is impossible. ∴ C is on the circle (§94). 8. Then ∠C is inscribed in a semicircle (§278).

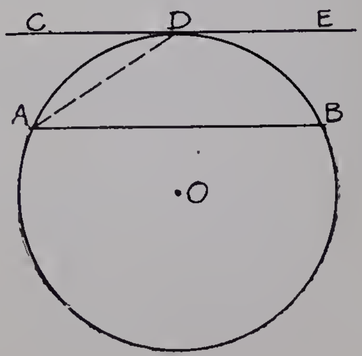
PAGES 326-327

1. (1) ∠x = $\frac{1}{2}$ of 84° = 42°. (2) ∠x = $\frac{1}{2}$(180° - 68°) = 56°.

2. (1) $\widehat{CD}$ = 2 x 25° = 50°. (2) $\widehat{CD}$ = 2 x 47° = 94°.

3. (1) The minor arc = 2 x 43° = 86°.
(2) The major arc = 360° - 86° = 274°.

4. The third angle of the △ = 180° - 37° - 75° = 68°. One arc = 2 x 37° = 74°, another arc = 2 x 75° = 150°, and the third arc = 2 x 68° = 136°.



5. Given ⊙O with chord AB, $\widehat{AD} = \widehat{DB}$, and line CE tangent to the ⊙ at D.
To prove that CE ∥ AB.
Planning the Proof: 1. We can prove lines ∥ by §§160, 221. 2. We shall use §160b.
Proof: 1. Draw AD (Asmt. 11). 2. ∠ADC $\overset{\circ}{=} \frac{1}{2}\widehat{AD}$ (§322). 3. ∠DAB $\overset{\circ}{=} \frac{1}{2}\widehat{DB}$ (§318).

4. $\overset{\frown}{AD} = \overset{\frown}{DB}$ (Given). 5. $\frac{1}{2}\overset{\frown}{AD} = \frac{1}{2}\overset{\frown}{DB}$ (Asmt. 4). 6. ∴ $\angle ADC = \angle DAB$ (Asmt. 7). 7. ∴ $CE \parallel AB$ (§160b).

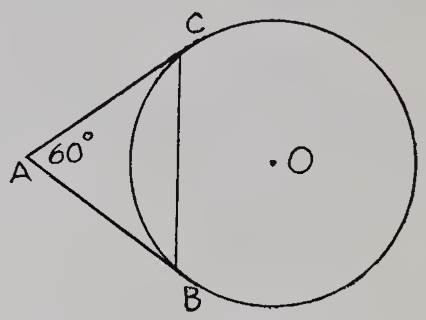
6. $\angle A = \frac{1}{2}$ of 144° = 72° = $\angle B$. $\angle C = 180° - (2 \times 72°) = 180° - 144° = 36°$.

7. Planning the Proof: 1. We can prove arcs = by §§312, 313, 317. 2. We shall use §322, Asmt.

Proof: 1. $\angle ABD \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BC}$ and $\angle ABD \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BD}$ (§322). 2. $\frac{1}{2}\overset{\frown}{BC} \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BD}$ (Asmt. 6). 3. $\overset{\frown}{BC} \overset{\circ}{=} \overset{\frown}{BD}$ (Asmt. 3).

8. Given ⊙O with tangents AB and AC, chord BC, and $\angle A = 60°$.

To prove that △ABC is equilateral.



Planning the Proof: 1. We can prove △s equilateral by §§56, 129. 2. We shall use §129.

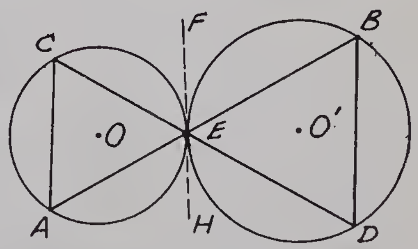
Proof: 1. $\angle A = 60°$ (Given). 2. $\angle B \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BC}$ and $\angle C \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BC}$ (§322). 3. ∴ $\angle B = \angle C$ (Asmt. 7). 4. $\angle B + \angle C + 60° = 180°$ (§120). 5. $2\angle B + 60° = 180°$ (Asmt. 6). 6. $2\angle B = 120°$ (Asmt. 2). 7. ∴ $\angle B = 60°$ (Asmt. 4). 8. ∴ $\angle C = 60°$ (Asmt. 6). 9. ∴ △ABC is equilateral (§129).

9. Planning the Proof: 1. We can prove lines ∥ by §§160, 221. 2. We shall use §160b.

Proof: 1. BC is a common internal tangent of ⊙O and ⊙O' (Given). 2. Draw OO' (Asmt. 11). 3. OO' passes through A (§303). 4. BD is tangent to ⊙O and CE is tangent to ⊙O' (Given). 5. $\angle BAD = \angle CAE$ (§50d). 6. $\angle BAD \overset{\circ}{=} \frac{1}{2}\overset{\frown}{DA}$ and $\angle CAE \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AE}$ (§322). 7. $\frac{1}{2}\overset{\frown}{DA} \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AE}$ (Asmt. 7). 8. $\angle BDA \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AD}$ and $\angle CEA \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AE}$ (§322). 9. $\angle BDA = \angle CEA$ (Asmt. 7). 10. $DB \parallel CE$ (§160b).

10. Given ⊙O and ⊙O' tangent externally at E, line segments AB and DC through E, and chords AC and BD.

To prove that $AC \parallel DB$.



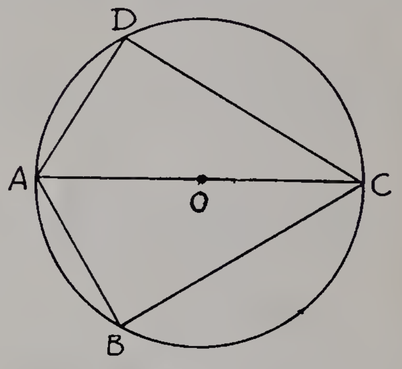
Planning the Proof: (Same as in Ex. 9).

Proof: 1. AE and CE are chords of ⊙O and ED and EB are chords of ⊙O' (§277). 2. Let FH be the common internal tangent of ⊙s O and O' through E (§§302, 303). 3. $\angle C \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AE}$ and $\angle D \overset{\circ}{=} \frac{1}{2}\overset{\frown}{EB}$ (§318). 4. $\angle AEH \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AE}$ and $\angle BEF \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BE}$ (§322). 5. ∴ $\angle C = \angle AEH$ and $\angle D = \angle BEF$ (Asmt. 7). 6. $\angle AEH = \angle BEF$ (§50d). 7. ∴ $\angle C = \angle D$ (Asmt. 7). 8. ∴ $AC \parallel DB$ (§160b).

11. Planning the Proof: (Same as in Ex. 9).

Proof: 1. Both $\circledS$ are tangent to a common line AB at B (§302). 2. $\angle D \overset{\circ}{=} \frac{1}{2}\overset{\frown}{EB}$ and $\angle GFB \overset{\circ}{=} \frac{1}{2}\overset{\frown}{GB}$ (§318). 3. $\angle ABE \overset{\circ}{=} \frac{1}{2}\overset{\frown}{EB}$ and $\angle ABE \overset{\circ}{=} \frac{1}{2}\overset{\frown}{GB}$ (§322). 4. $\therefore \frac{1}{2}\overset{\frown}{EB} \overset{\circ}{=} \frac{1}{2}\overset{\frown}{GB}$ (Asmt. 7). 5. $\therefore \angle D = \angle GFB$ (Asmt. 7). 6. $\therefore$ DE $\parallel$ FG (§160b).

12. Given quadrilateral ABCD inscribed in $\odot O$ with AB = AD, CB = CD, and diagonal AC.



To prove that AC is a diameter of $\odot O$.

Planning the Proof: 1. We can prove that a line is a diameter by §§277, 285, 292, 296. 2. We shall use §285.

Proof: 1. AB = AD and CB = CD (Given). 2. $\overset{\frown}{AB} = \overset{\frown}{AD}$ and $\overset{\frown}{CB} = \overset{\frown}{CD}$ (§312b). 3. $\therefore \overset{\frown}{ABC} = \overset{\frown}{ADC}$ (Asmt. 1, 9, 6). 4. $\therefore$ AC is a diameter of $\odot O$ (§285).

PAGES 328-329

1. (1) $\angle x = \frac{1}{2}(150° + 64°) = 107°$. (2) $\angle x = \frac{1}{2}(79° + 141°) = 110°$.

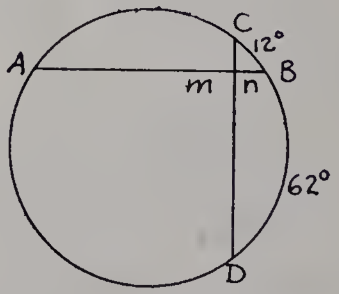
2. From §323, $\angle x \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{BC} + \overset{\frown}{AD})$. Substituting, $110° = \frac{1}{2}(145° + AD)$; whence $220° = 145° + AD$, and $AD = 75°$.

3. $\angle BED \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{DB} + \overset{\frown}{AC}) = \frac{1}{2}(55° + 62°) = 58.5°$. $\angle x = 180° - \angle BED = 180° - 58.5° = 121.5°$.

4. Planning the Proof: 1. We can prove $\triangle$s = by §§ 50, 98, 158, 219, 310, 320. 2. We shall use §320.

Proof: 1. Both $\angle C$ and $\angle B$ are inscribed in $\overset{\frown}{ACBD}$ (§ 279). 2. $\angle C = \angle B$ (§320). 3. $\angle A$ and $\angle D$ are inscribed in $\overset{\frown}{CADB}$ (§ 279). 4. $\angle A = \angle D$ (§320).

5. $\angle m \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{AD} + 12)$ and $\angle n \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{AC} + 62)$. Since $\angle m = \angle n$, $\frac{1}{2}(\overset{\frown}{AD} + 12°) = \frac{1}{2}(\overset{\frown}{AC} + 62°)$. Simplifying, $\overset{\frown}{AD} - \overset{\frown}{AC} = 50°$. But $\overset{\frown}{AD} + \overset{\frown}{AC} = 360° - 12° - 62° = 286°$. Adding these two equations, we have $2\overset{\frown}{AD} = 336°$; whence $\overset{\frown}{AD} = 168°$. Substituting, $168° + \overset{\frown}{AC} = 286°$; whence $\overset{\frown}{AC} = 118°$. 168°, 118°. Ans.

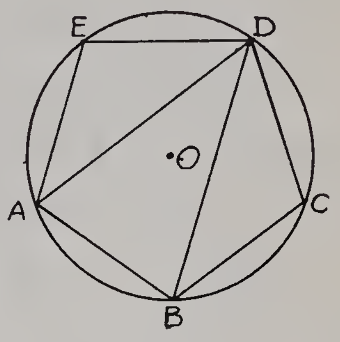


6. $\overset{\frown}{BC} \overset{\circ}{=} 2\angle D = 88°$, $\overset{\frown}{CD} + 88° + 102° = 360°$. $\overset{\frown}{CD} = 170°$. $\angle BCD = \frac{1}{2}(\overset{\frown}{BD}) = \frac{1}{2}(102°) = 51°$. $\angle ACB = \frac{1}{2}\overset{\frown}{BC} = \frac{1}{2}(88°) = 44°$.

$\angle D + \angle A + \angle ACB + \angle BCD = 180°$ (§120). $44° + \angle A + 44° + 51° = 180°$. $\angle A = 41°$.

7. Given the regular pentagon ABCDE inscribed in ⊙O with DA and DB diagonals drawn from D.

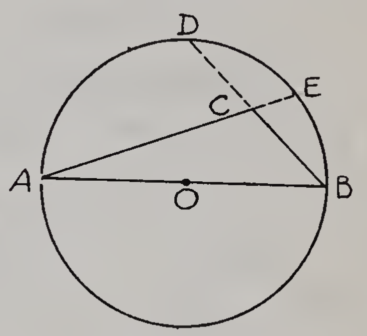
To prove that $\angle EDA = \angle ADB = \angle BDC$.



Planning the Proof: 1. (Same as in Ex. 4). 2. We shall use Asmt. 7.

Proof: 1. $\angle EDA \overset{\circ}{=} \frac{1}{2}\overset{\frown}{EA}$, $\angle ADB \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AB}$, and $\angle BDC \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BC}$ (§318). 2. ABCDE is a regular pentagon (Given). 3. Then $EA = AB = BC$ (§184). 4. $\overset{\frown}{EA} = \overset{\frown}{AB} = \overset{\frown}{BC}$ (§312b). 5. $\frac{1}{2}\overset{\frown}{EA} = \frac{1}{2}\overset{\frown}{AB} = \frac{1}{2}\overset{\frown}{BC}$ (Asmt. 4). 6. $\therefore \angle EDA = \angle ADB = \angle BDC$ (Asmt. 7).

8. Given ⊙O, with a diameter AB, and lines CA and CB drawn from any point C inside the ⊙ to A and B respectively.

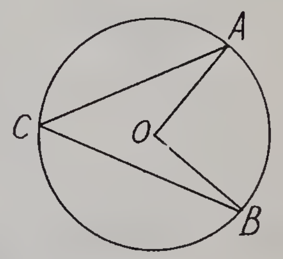


To prove that $\angle ACB$ is obtuse.

Planning the Proof: 1. We can prove that an angle is obtuse by §16. 2. We shall use §16.

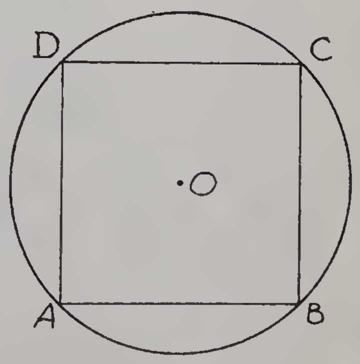
Proof: 1. Produce AC and BC through C to meet the ⊙ in E and D respectively (Asmt. 13). 2. $\angle ACB \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{AB} + \overset{\frown}{DE})$ (§323). 3. $\overset{\frown}{AB} = 180°$ (§283). 4. Hence $\angle ACB \overset{\circ}{=} 90° + \frac{1}{2}\overset{\frown}{DE}$ (Asmt. 6). 5. $\therefore \angle ACB$ is more than 90° (Asmt. 9). 6. $\angle ACB + \angle ACD = \angle BCD$ (Asmt. 9). 7. $\angle ACB$ is less than $\angle BCD$ (Asmt. 9). 8. $\angle BCD$ is a st. $\angle$ (§16). 9. $\therefore \angle ACB$ is less than a st. $\angle$ (Asmt. 6). 10. $\therefore \angle ACB$ is obtuse (§16).

9. $\angle O + \angle C = 135°$. $\angle O \overset{\circ}{=} \overset{\frown}{AB}$. $\angle C \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AB}$. $2\angle C \overset{\circ}{=} AB$. $\angle O = 2\angle C$. $2\angle C + \angle C = 135°$. $3\angle C = 135°$. $\angle C = 45°$. $\angle O = 90°$.



10. Given a rhombus ABCD inscribed in ⊙O.

To prove that ABCD is a square.

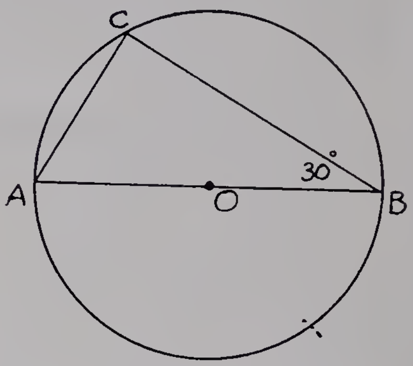


Planning the Proof: 1. We can prove that a quad. is a square by §185. 2. We shall use §185.

Proof: 1. ABCD is a rhombus (Given). 2. $\therefore AB = BC = CD = DA$ (§194). 3. $\angle A + \angle C = 180°$ (§321). 4. But $\angle A = \angle C$ (§219).

5. ∴ 2∠A = 180° (Asmt. 6). 6. ∴ ∠A = 90° (Asmt. 4).
7. ∴ ABCD is a square (§185).

11. Given △ABC inscribed in ⊙O with AB a diameter and ∠B = 30°.

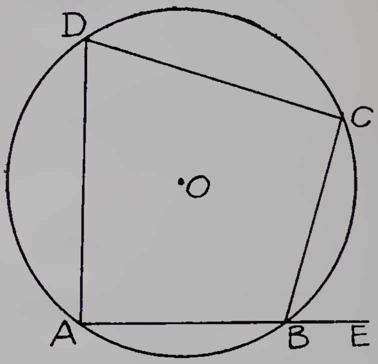


To prove that AC = AO.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309.
2. We shall use Asmt. 6.

Proof: 1. AB is a diameter and ∠B = 30° (Given). 2. ∠C = 90° (§319). 3. ∴ ABC is a rt. △ (§57). 4. ∴ AC = $\frac{1}{2}$AB (§136). 5. ∴ AC = AO (Asmt. 6).

12. Given ABCD, a quad. inscribed in ⊙O with ∠EBC an ext. ∠ of ABCD.



To prove that ∠EBC = ∠D.

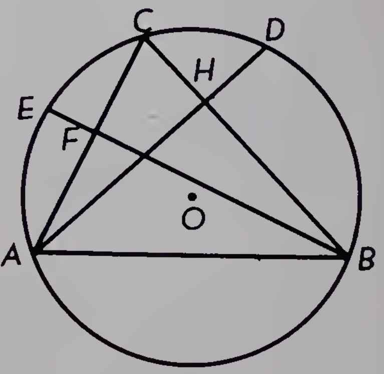
Planning the Proof: 1. We can prove ∠s = by §§50, 98, 158, 219, 310, 320. 2. We shall use §50c.

Proof: 1. ∠EBC is the supplement of ∠ABC (§21). 2. ∠D is the supplement of ∠ABC (§321).
3. ∴ ∠EBC = ∠D (§50c).

13. Planning the Proof: 1. We can prove ∠s are rt. ∠s by §§51, 159, 220, 311, 319. 2. We shall use §319.

Proof: 1. Draw DC, the common internal tangent (§303).
2. AD = CD and BD = CD (§309b). 3. ∴ AD = BD (Asmt. 7). 4. A ⊙ with D as a center and a radius = DA will pass through A, C, and B (§281b). 5. AB is a diameter of this ⊙ (§277).
6. ∴ ∠ACB is inscribed in this semicircle (§279). 7. ∴ ∠ACB is a rt. ∠ (§319).

14. Planning the Proof: 1. We can prove arcs = by §§312, 313, 317. 2. We shall use §323, Asmt.



Proof: 1. AD ⊥ BC and BE ⊥ AC (Given).
2. ∠BFC and ∠AHC are rt. ∠s (§14).
3. ∠BFC = ∠AHC (§50a). 4. ∠BFC $\overset{\circ}{=}$ $\frac{1}{2}(\widehat{AE} + \widehat{BC})$ and ∠AHC $\overset{\circ}{=}$ $\frac{1}{2}(\widehat{AC} + \widehat{BD})$ (§323).
5. $\frac{1}{2}(\widehat{AE} + \widehat{BC}) \overset{\circ}{=} \frac{1}{2}(\widehat{AC} + \widehat{BD})$ (Asmt. 7).
6. $\widehat{AE} + \widehat{BC} \overset{\circ}{=} \widehat{AC} + \widehat{BD}$ (Asmt. 3). 7. $\widehat{BC} = \widehat{BD} + \widehat{DC}$ and $\widehat{AC} = \widehat{AE} + \widehat{EC}$ (Asmt. 9).
8. $\widehat{AE} + \widehat{BD} + \widehat{DC} \overset{\circ}{=} \widehat{AE} + \widehat{EC} + \widehat{BD}$ (Asmt. 6). 9. $\widehat{DC} = \widehat{EC}$ (Asmt. 2).

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1. $\angle x = \frac{1}{2}(112° - 54°) = 29°$.

2. $63° = \frac{1}{2}(230° - \widehat{x})$; $126° = 230° - \widehat{x}$; $\widehat{x} = 104°$.

3. $\widehat{GH} = 360° - 60° - 70° - 80° = 150°$.

   (1) $\angle A = \frac{1}{2}(210° - 150°) = 30°$

   (2) $\angle B = \frac{1}{2}(280° - 80°) = 100°$

   (3) $\angle C = \frac{1}{2}(290° - 70°) = 110°$

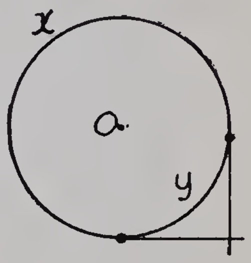
   (4) $\angle D = \frac{1}{2}(300° - 60°) = 120°$

4. a. $\frac{1}{2}(100° - \widehat{BD}) = 30°$. $100° - \widehat{BD} = 60°$. $\widehat{BD} = 40°$.

   b. $\angle C = \frac{1}{2}(40° + 70°) = 55°$.

   c. $\widehat{BC} = 360° - 40° - 70° - 100° = 150°$.

   d. $\angle CBD = \frac{1}{2}(70° + 100°) = 85°$.

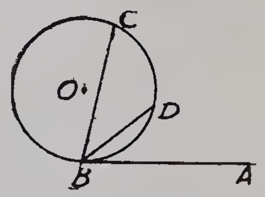


5. From §324, $\frac{1}{2}(\widehat{x} - \widehat{y}) = 90°$, and $\widehat{x} - \widehat{y} = 180°$. $\widehat{x} + \widehat{y} = 360°$. Adding, $2\widehat{x} = 540°$; whence $\widehat{x} = 270°$. Then $\widehat{y} = 90°$.

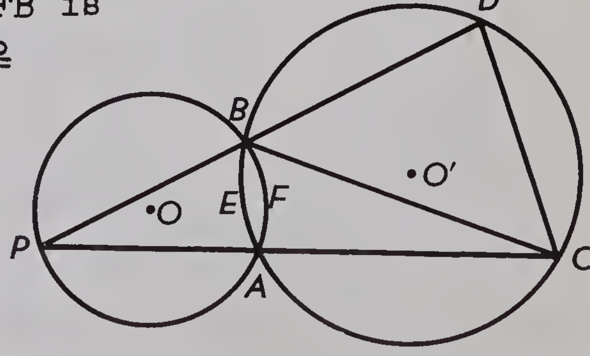
6. Given AB tangent to ⊙O at B, BC a chord, and $\angle ABD = \angle DBC$.

To prove that $\widehat{BD} = \widehat{DC}$.

Planning the Proof: 1. We can prove arcs = by §§312, 313, 317. 2. We shall use Asmt. 7, 3.



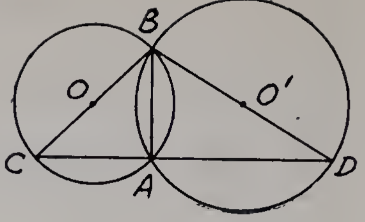
Proof: 1. $\angle ABD = \angle DBC$ (Given). 2. $\angle ABD \overset{\circ}{=} \frac{1}{2}\widehat{BD}$ (§322). 3. $\angle DBC \overset{\circ}{=} \frac{1}{2}\widehat{DC}$ (§318). 4. $\therefore \frac{1}{2}\widehat{BD} = \frac{1}{2}\widehat{DC}$ (Asmt. 7). 5. $\therefore \widehat{BD} = \widehat{DC}$ (Asmt. 3).



7. 1. $\angle P \overset{\circ}{=} \frac{1}{2}\widehat{AFB}$ (§318). Since $\widehat{AFB}$ is constant, $\angle P$ is constant. 2. $\angle PCB \overset{\circ}{=} \frac{1}{2}\widehat{AEB}$ (§318). Then $\angle PCB$ is constant. $\angle PBC = 180° - \angle P - \angle PCB$ (§120, Asmt. 2). Since $\angle P$ and $\angle PCB$ are constant, $\angle PBC$ is constant. 3. $\angle CBD = 180° - \angle PBC$. Then $\angle CBD$ is constant. 4. $\angle CBD \overset{\circ}{=} \frac{1}{2}\widehat{CD}$. $\widehat{CD} = 2\angle CBD$ (Asmt. 2). Then $\widehat{CD}$ is constant. If $\widehat{CD}$ is constant, chord CD is constant (§308b).

8. Given AB a common chord of Ⓢ O and O', BC a diameter of

⊙O, BD a diameter of ⊙O', AC a chord of ⊙O, and AD a chord of ⊙O'.

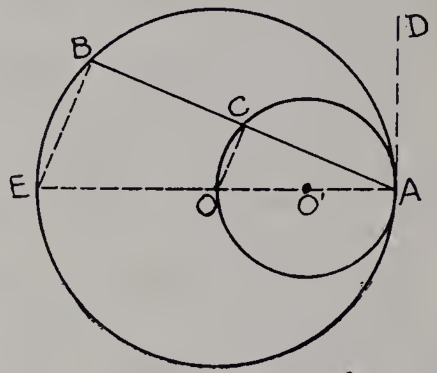


To prove that CAD is a st. line.

Planning the Proof: 1. We can prove that a line is straight by Asmt. 25. 2. We shall use Asmt. 25.

Proof: 1. ∠s CAB and DAB are inscribed in semicircles (§279). 2. ∴ ∠CAB and ∠DAB are rt. ∠s (§319). 3. ∴ ∠CAB is supp. to ∠DAB (§21). 4. ∴ CAD is a st. line (Asmt. 25).

9. Given Ⓢ O and O' tangent internally at A, ⊙O' passing through point O, and chord AB of ⊙O intersecting ⊙O' in C.

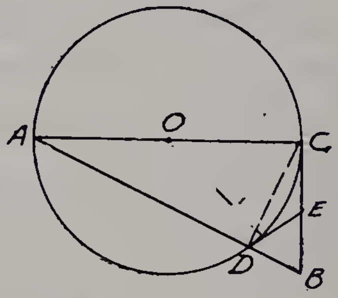


To prove that BC = CA.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 308, 309. 2. We shall use § 217e.

Proof: 1. Ⓢ O and O' are tangent to a common line, as AD, at A (§302). 2. Draw EA ⊥ AD at A (§178b). 3. EA passes through O and O'(§296). 4. ∴ EA is a diameter of ⊙O and OA is a diameter of ⊙O' (§277). 5. Draw EB and OC (Asmt. 11). 6. ∠ EBA and ∠OCA are rt. ∠s (§319). 7. ∴ EB ∥ OC (§160d). 8. EO = OA (Asmt. 26). 9. ∴ BC = CA (§217e).

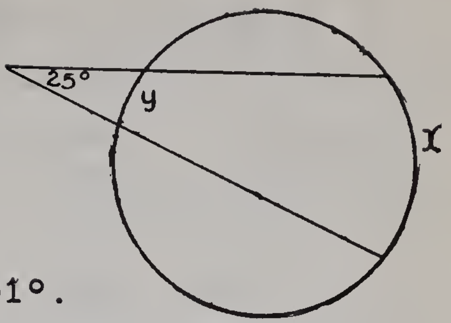
10. Given △ABC, right-angled at C, with leg AC a diameter of ⊙O, AB cutting ⊙O at D, and tangent DE intersecting BC in E.



To prove that BE = EC.

Planning the Proof: 1. (Same as in Ex. 9). 2. We shall use Asmt.

Proof: 1. Draw DC (Asmt. 11). 2. DE is tangent to ⊙O at D (Given). 3. BC ⊥ AC (Given). 4. ∠ C a rt. ∠ (§ 14). 5. BC is tangent to ⊙O at C (§ 298). 6. EC = ED (§309b). 7. ∴ ∠EDC = ∠ECD (§ 98a). 8. ∠ ADC = 90° or CD ⊥ AB (§319). 9. ∠ BDE is a complement of ∠EDC (§52a). 10. ∠B is a complement of ∠BCD (§ 161). 11. ∴ ∠B = ∠BDE (§50b). 12. ∴ BE = ED (§ 157). 13. ∴ from 6, BE = EC (Asmt. 7).



11. Let x = the number of degrees in the larger arc, and y = the number of degrees in the smaller arc. By §324, $\frac{1}{2}(\widehat{x} - \widehat{y}) = 25°$; whence $\widehat{x} - \widehat{y} = 50°$. Since $\widehat{x} + \widehat{y} = 152°$, by adding we have $2\widehat{x} = 202°$; whence $\widehat{x} = 101°$. Then $101° - \widehat{y} = 50°$; whence $\widehat{y} = 51°$.

12. $\angle E = \angle G$.

Proof: 1. $\angle E \overset{\circ}{=} \frac{1}{2}(\widehat{DC} - \widehat{AB})$ and $\angle G \overset{\circ}{=} \frac{1}{2}(\widehat{AD} - \widehat{BC})$ (§324). 2. $\angle E \overset{\circ}{=} \frac{1}{2}\widehat{DC} - \frac{1}{2}\widehat{AB}$ and $\angle G \overset{\circ}{=} \frac{1}{2}\widehat{AD} - \frac{1}{2}\widehat{BC}$ (Asmt. 6). 3. $\widehat{BCD} = 180°$ and $\widehat{BAD} = 180°$ (Given, §§ 283, 315). 4. $\widehat{DC} + \widehat{BC} = 180°$ and $\widehat{AD} + \widehat{AB} = 180°$ (Asmt. 9, 6). 5. $\widehat{DC} \overset{\circ}{=} 180° - \widehat{BC}$ and $\widehat{AD} \overset{\circ}{=} 180° - \widehat{AB}$ (Asmt. 2). 6. $\angle E \overset{\circ}{=} 90° - \frac{1}{2}\widehat{BC} - \frac{1}{2}\widehat{AB}$ and $\angle G \overset{\circ}{=} 90° - \frac{1}{2}\widehat{AB} - \frac{1}{2}\widehat{BC}$ (Asmt. 6). 7. $\angle E = \angle G$ (Asmt. 7).

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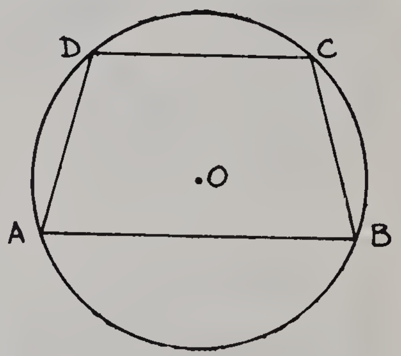
1. Given trapezoid ABCD inscribed in ⊙O with AB and DC its bases.



To prove that ABCD is isos.

Planning the Proof: 1. We can prove that a trapezoid is isos. by §206. 2. We shall use §206.

Proof: 1. AB and DC are the bases (Given). 2. AB ∥ DC (§206). 3. ∴ $\widehat{AD} = \widehat{BC}$ (§327). 4. AD = BC (§287). 5. ∴ ABCD is isos. (§206).

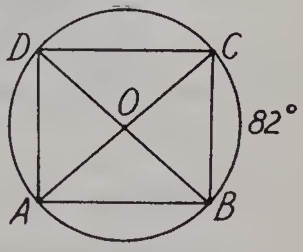
2. Use figure for Ex. 1.

Given trapezoid ABCD, inscribed in ⊙O, with bases AB and CD.

To prove that $\angle A = \angle B$ and $\angle D = \angle C$.

Planning the Proof: 1. We can prove ≜ = by §§50, 98, 158, 219, 310, 320. 2. We shall use Asmt. 7.

Proof: 1. AB ∥ CD (Given). 2. $\widehat{AD} = \widehat{BC}$ (§327). 3. $\angle B \overset{\circ}{=} \frac{1}{2}(\widehat{AD} + \widehat{DC})$ and $\angle A \overset{\circ}{=} \frac{1}{2}(\widehat{BC} + \widehat{DC})$ (§318). 4. $\widehat{AD} + \widehat{DC} = \widehat{BC} + \widehat{DC}$ (Asmt. 1). 5. $\frac{1}{2}(\widehat{AD} + \widehat{DC}) = \frac{1}{2}(\widehat{BC} + \widehat{DC})$ (Asmt. 4). 6. ∴ $\angle B = \angle A$ (Asmt. 7). 7. In like manner, $\angle D = \angle C$ (Statements 3-6).



3. $\widehat{AD} = 82°$, $\widehat{AB} = \widehat{DC} = 98°$. AC and BD are diameters, §285. $\angle BOC = \angle AOD = 82°$; $\angle AOB = \angle DOC = 98°$. $\angle OAB = \angle OBA = \angle OCD = \angle ODC = \frac{1}{2}(82°) = 41°$. $\angle OBC = \angle OCB = \angle OAD = \angle ODA = \frac{1}{2}(98°) = 49°$.

4. Planning the Proof: 1. We can prove arcs = by §§312, 313, 317, 327. 2. We shall use §§313, 327, Asmt. 7.

Proof: 1. Radius OB $\perp$ chord AD (Given). 2. $\widehat{AB} = \widehat{BC}$ (§313). 3. $\widehat{AB} \parallel \widehat{CD}$ (Given). 4. $\widehat{AC} = \widehat{BD}$ (§327). 5. $\widehat{AC} = \widehat{AB}$ (Asmt. 7).

5. If two arcs of a circle are unequal, they are not intercepted by parallel lines.

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Exercise

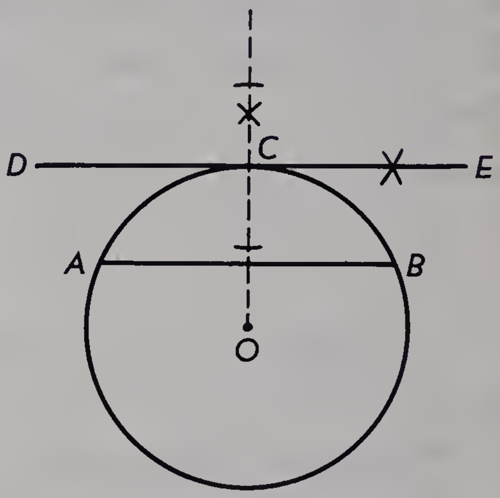
Given chord AB of ⊙O.

To construct a line ∥ AB and tangent to ⊙O.

Construction: 1. Construct radius OC $\perp$ AB (§178a). 2. Construct DE tangent to ⊙O at C (§329).

DE is the required line.

Proof: 1. AB $\perp$ OC and DE is tangent to ⊙O (Const.). 2. DE $\perp$ OC (§295). 3. DE ∥ AB (§160d).



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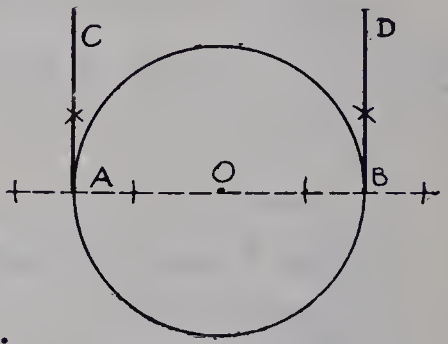
1. Given ⊙O.

To construct 2 ∥ tangents to ⊙O.

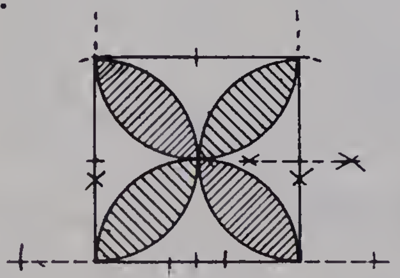
Construction: 1. Draw any diameter AB (Asmt. 11). 2. At A construct AC tangent to ⊙O (§329). 3. At B construct BD tangent to ⊙O (§329).

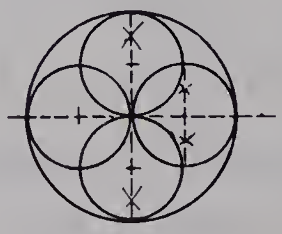
Then AC and BD are the required tangents.

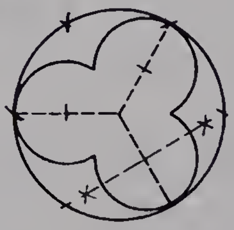
Proof: 1. AC and BD are tangents to ⊙O (Const.). 2. AC $\perp$ AB and BD $\perp$ AB (311b). 3. AC ∥ BD (§160d).



2.



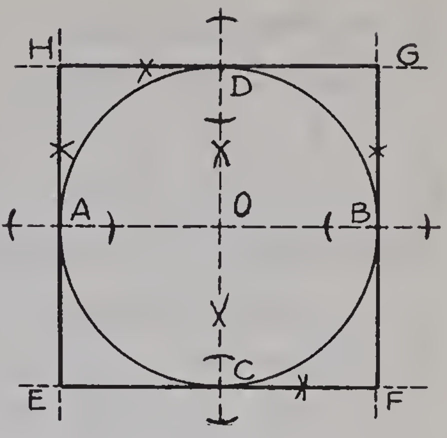




3. Given ⊙O.

To circumscribe a square about ⊙O.

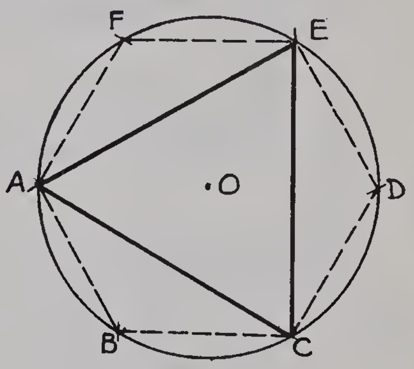
Construction: 1. Draw any diameter AB (Asmt. 11). 2. At O,



construct diameter CD ⊥ AB (§178b). 3. Construct tangents to ⊙O at A, C, B, and D, extending them to intersect at E, F, G, and H (§329, Asmt. 13).

Then EFGH is the required square.

Proof: 1. AB ⊥ CD (Const.). 2. The tangents at A and B are ⊥ AB and the tangents at C and D are ⊥ CD (§311b). 3. The tangents at A and B are ∥ to CD and to each other, and the tangents at C and D are ∥ to AB and to each other (§160d). 4. These tangents meet in points E, F, G, and H (§115). 5. EFGH is a ▱ (§185). 6. ∠E is a rt. ∠ (§159a). 7. ∴ EFGH is a rect. (§185). 8. EF = AB and EH = CD (§217b). 9. But AB = CD (Asmt. 9, 6, 26). 10. ∴ EF = EH (Asmt. 16). 11. ∴ EFGH is a square (§185). 12. EFGH is circumscribed about ⊙ O (§325).



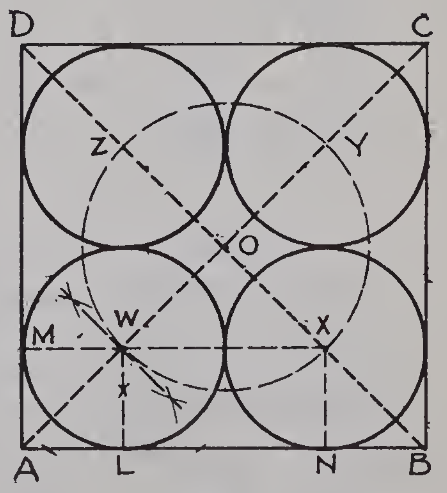
4. Given ⊙O.

To inscribe an equilateral △ in ⊙O.

Construction: 1. Inscribe ABCDEF, a regular hexagon, in ⊙O (§332). 2. Draw AC, CE, and AE (Asmt. 11).

Then ACE is the required △.

Proof: 1. AB = BC = CD = DE = EF = FA (Const.). 2. ∴ $\widehat{AB} = \widehat{BC} = \widehat{CD} = \widehat{DE} = \widehat{EF} = \widehat{FA}$ (§312b). 3. $\widehat{AC} = \widehat{CE} = \widehat{AE}$ (Asmt. 1). 4. ∴ $\widehat{AC} = \widehat{CE} = \widehat{AE}$ (§308b). 5. ∴ △ABC is equilateral (§56). 6. △ ABC is inscribed in ⊙O (§288).



5. Given the square ABCD.

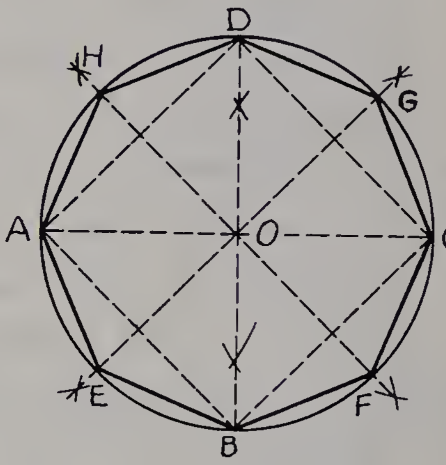
To construct 4 Ⓢ within ABCD, so that each ⊙ is tangent to 2 sides of ABCD and to 2 of the other Ⓢ.

Construction: 1. Draw AC and BD (Asmt. 11), intersecting in O (Asmt. 12). 2. Bisect OA (§177b), calling the point of bisection W. 3. With O as a center and a radius = OW, construct a ⊙ cutting OB at X, OC at Y, and OD at Z (Asmt. 15). 4. From W construct WL ⊥ AB (§178a). 5. With a radius = WL and W, X, Y, and Z respectively as centers draw Ⓢ W, X, Y, and Z.

Then ⓈW, X, Y, and Z are the required Ⓢ.

Proof: 1. ABCD is a square (Given). 2. ∠BAC = ∠CAD (Ex. 7, page 206, §185). 3. WL ⊥ AB (Const.). 4. Draw WM ⊥ AD (§178a).

5. AW = AW (Iden.). 6. $\therefore$ Rt. $\triangle$ ALW $\cong$ rt. $\triangle$ AMW (§156a). 7. $\therefore$ WL = WM (§97a). 8. WL is a radius of $\odot$ W (Const.). 9. $\therefore$ M lies on $\odot$W (Asmt. 42, §281). 10. $\therefore$ $\odot$W is tangent to AB at L and to AD at M (§298). 11. Similarly, ⓢX, Y, and Z are each tangent to 2 sides of ABCD. 12. WM $\parallel$ AL and AM $\parallel$ LW (§160d). 13. $\therefore$ AL = WM (§217b). 14. $\therefore$ AL = WL (Statement 7, Asmt. 7). 15. Draw XN $\perp$ AB (§178a). 16. OA = OB (§§185, 217c, Ex. 8). 17. $\angle$OBA = $\angle$OAB (§98a). 18. BX = AW (Const., Asmt. 2). 19. Rt. $\triangle$BNX $\cong$ rt. $\triangle$ALW (§156a). 20. $\therefore$ BN = AL (§97a). 21. Draw WX (Asmt. 11). 22. WX = $\frac{1}{2}$AB and WX $\parallel$ AB (§204). 23. WL $\parallel$ XN (§160d). 24. $\therefore$ LN = WX (§217a). 25. AB = AL + LN + BN (Asmt. 9) = WL + $\frac{1}{2}$AB + WL (Asmt. 6, 7). 26. $\frac{1}{2}$AB = 2WL (Asmt. 2). 27. $\therefore$ WX = 2WL (Asmt. 7). 28. $\therefore$ ⓢW and X are tangent (Asmt. 42, 20, §§311b, 302). 29. Similarly, $\odot$W is tangent to $\odot$Z and each of the other ⓢ is tangent to 2 others.



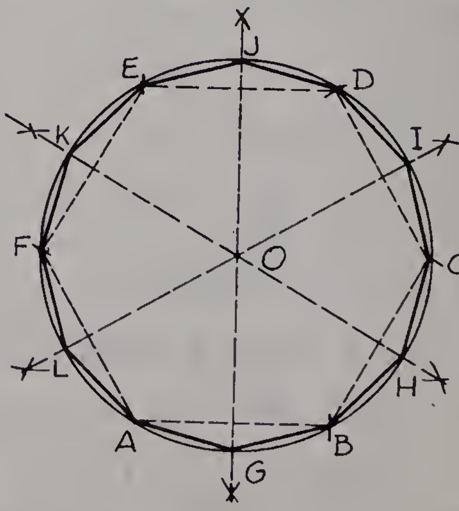
6. Given $\odot$O.

To inscribe an equilateral octagon in $\odot$O.

Construction: 1. Inscribe a square ABCD in $\odot$O (§331). 2. Bisect $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, and $\overset{\frown}{DA}$ and call the points of division E, F, G, and H respectively (§328). 3. Draw AE, EB, $\cdots$, HA (Asmt. 11).

Then AEB $\cdots$ H is the required octagon.

Proof: 1. AB = BC = CD = DA (Const., §194). 2. $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DA}$ (§312b). 3. $\therefore$ $\overset{\frown}{AE} = \overset{\frown}{EB} = \overset{\frown}{BF} = \cdots = \overset{\frown}{HA}$ (Const., Asmt. 4). 4. $\therefore$ AE = EB = BF = $\cdots$ = HA (§308b). 5. AEB $\cdots$ H is inscribed in $\odot$O (§288).



7. Given $\odot$O.

To inscribe an equilateral polygon of 12 sides in $\odot$O.

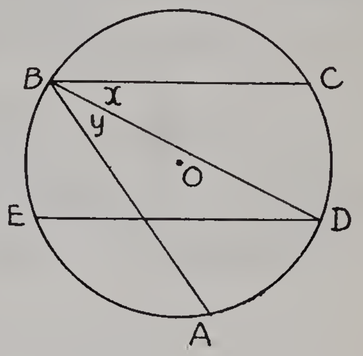
Construction: 1. Inscribe ABCDEF, a regular hexagon, in $\odot$O (§332). 2. Bisect $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\cdots$, $\overset{\frown}{FA}$, and call the points of division G, H, $\cdots$, L (§328). 3. Draw AG, GB, $\cdots$, LA.

Then AGB$\cdots$L is the required polygon.

Proof: 1. AB = BC = $\cdots$ = FA (Const.,

§184). 2. ∴ $\widehat{AB} = \widehat{BC} = \cdots = \widehat{FA}$ (§312b). 3. ∴ $\widehat{AG} = \widehat{GB} = \cdots = \widehat{LA}$ (Const., Asmt. 4). 4. ∴ AG = GB = ··· = LA (§308b). 5. AGB ···L is an inscribed polygon (§288).

8-10. These are to be original constructions by the student.



11. Given ∠ABC inscribed in ⊙O, BD bisecting ∠ABC, and DE ∥ BC.

To prove DE = BA.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309. 2. We shall use § 308b, Asmt.

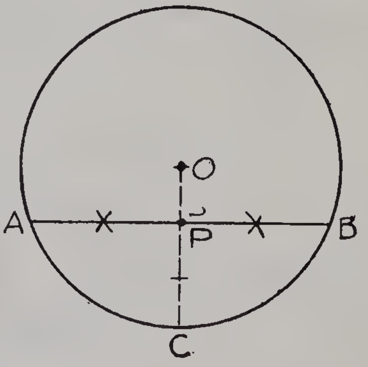
Proof: 1. ∠x = ∠y and ED ∥ BC (Given). 2. ∠x = ∠D (§158a). 3. ∴ ∠y = ∠D (Asmt. 7). 4. $\angle y \overset{\circ}{=} \frac{1}{2}\widehat{AD}$ and $\angle D \overset{\circ}{=} \frac{1}{2}\widehat{BE}$ (§ 318). 5. ∴ $\frac{1}{2}\widehat{AD} = \frac{1}{2}\widehat{BE}$ (Asmt. 7). 6. ∴ $\widehat{AD} = \widehat{BE}$ (Asmt. 3). 7. $\widehat{EA} + \widehat{AD} = \widehat{BE} + \widehat{EA}$ (Asmt. 1). 8. ∴ $\widehat{ED} = \widehat{BA}$ (Asmt. 9, 6). 9. ∴ ED = BA (§308b).

PAGES 339-340

1. AB, one diameter, can be found as shown in the diagram. Another diameter can be found by moving the square in the ⊙ so that the vertex of the rt. ∠ of the square remains on the ⊙. The 2 diameters intersect in the center of the plate (§277).

2. Given ⊙O and P, the midpoint of a chord of the ⊙.

To construct the chord.



Construction: 1. Draw OC, a radius through P (Asmt. 11). 2. Construct chord AB ⊥ OC at P (§§178b, 277).

Then AB is the required chord.

Proof: 1. AB is a chord (Const.). 2. OP ⊥ AB (Const.). 3. ∴ P is the midpoint of AB (§290).

3. Given ⊙ACB, with inscribed ∠C intercepting $\widehat{AB}$, and ∠D formed by secants DA and DB, intercepting $\widehat{EF}$ and $\widehat{AB}$.

To prove that ∠C is greater than ∠D.

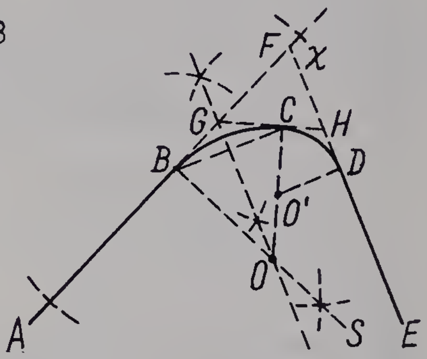
Planning the Proof: 1. We can prove ∡ unequal by §93, Asmt. 2. We shall use Asmt. 9.

Proof: 1. $\angle C \overset{\circ}{=} \frac{1}{2}\widehat{AB}$ (§318). 2. $\angle D \overset{\circ}{=} \frac{1}{2}(\widehat{AB} - \widehat{EF})$ (§324). 3. $\angle C - \angle D \overset{\circ}{=} \frac{1}{2}\widehat{AB} - \frac{1}{2}\widehat{AB} + \frac{1}{2}\widehat{EF} = \frac{1}{2}\widehat{EF}$ (Asmt. 2). 4. $\angle C \overset{\circ}{=} \angle D + \frac{1}{2}\widehat{EF}$ (Asmt. 1). 5. ∴ ∠C is greater than ∠D (Asmt. 9).

4. 1. $\angle C \overset{\circ}{=} \frac{1}{2}\widehat{AB}$ (§318). 2. The ship is inside the ⊙, on the ⊙, or outside the ⊙ (§281b). 3. If D is inside the ⊙, $\angle D = \frac{1}{2}(\widehat{AB} + \text{another arc})$ and $\angle D > \angle C$ (§323). 4. If D is on the ⊙, $\angle D = \angle C$ (§320). 5. Since $\angle D$ is neither $>$ nor $=$ $\angle C$, D is neither inside nor on the ⊙; and hence D is outside the ⊙ and the ship is out of danger (Asmt. 10).

5. Construct a $\perp$ to AB at E and bisect $\angle A$. Using the intersection of these two lines as a center and a radius equal to the distance from this point to E, construct the arc of the circle.

6. The points B and D are fixed and the point C is determined by the terrain.



Construction: 1. Construct BS $\perp$ AB at B (§178b). 2. Draw BC (Asmt. 11). 3. Construct the $\perp$ bisector of BC intersecting BS at O (§177b). 4. With BO as a radius construct BC (Asmt. 15). 5. In like manner, construct $\widehat{CD}$ (Statements 1-4).

To prove that $\angle x = \angle O + \angle CO'D$.

Proof: 1. Construct GH $\perp$ OC at C (§178b). 2. $\angle x = \angle FGH + \angle FHG$ (§123). 3. $\angle BOC + \angle OCG + \angle CGB + \angle GBO = 360°$ (§210). 4. $\angle OCG = 90°$ and $\angle GBO = 90°$ (Const.). 5. $\therefore \angle OCG + \angle GBO = 180°$ (Asmt. 1). 6. $\therefore \angle BOC + \angle CGB = 180°$ (Asmt. 2). 7. $\angle FGH + \angle HGB = 180°$ (Asmt. 24). 8. $\therefore \angle BOC = \angle FGH$ (§50c). 9. In like manner, it can be shown that $\angle CO'D = \angle FHG$ (Statements 3-8). 10. $\angle x = \angle BOC + \angle CO'D$ (Statement 2, Asmt. 6).

7. 1. $\triangle$ BCD is equilateral (Const.). 2. $\therefore$ BCD is equiangular (§84). 3. $\therefore \angle EDB = 120°$ (§123). 4. BD = DE (Const.). 5. $\therefore \angle DBE = \angle BED$ (§98a). 6. $\angle DBE + \angle DEB + \angle EDB = 180°$ (§120). 7. $\therefore 2\angle DBE + 120° = 180°$ (Asmt. 6). 8. $\therefore 2\angle DBE = 60°$ (Asmt. 2). 9. $\therefore \angle DBE = 30°$ (Asmt. 4). 10. $\angle DBC = 60°$ (§120, Asmt. 6, 4). 11. $\therefore \angle CBE = 90°$ (Asmt. 1). 12. $\therefore$ EB $\perp$ AB (§14).

PAGES 340-341

1. Valid. 2. Not valid. 3. Not valid. 4. Valid. 5. Not valid. 6. Not valid. 7. Not valid. 8. Not valid. 9. Not valid. 10. Not valid. 11. Not valid. 12. Not valid. 13. Not valid. 14. Not valid. 15. Not valid. 16. Valid because the contrapositive of the assumption is true. 17. Not valid.

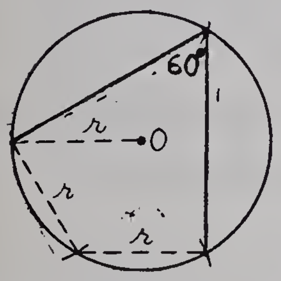
PAGES 341-342

1. An angle degree is one ninetieth of a right angle and an arc degree is the arc which a central angle of one degree intercepts on a circle (§314). 2. $\overset{\frown}{AB}$. 3. 90°. 4. Point of tangency, or point of contact (§277). 5. c, a, d, b. 6. No. 7. Yes (§331). 8. §§ 23, 279; § 277; § 277; § 277; § 278. 9. Two. 10. Minor. 11. An infinite number. 12. 70°; 35°; 65°; 80°. 13. 21 in. 14. 60°; 120°; 60°; 60°; 30°. 15. 8 in.; 16 in.; 8 in.; $8\sqrt{3}$ in.; $8\sqrt{3}$ in. 16. The opposite $\angle$s are supp. (§321). 17. No. 18. 5 in. 19. 39 in. 20. 80°.

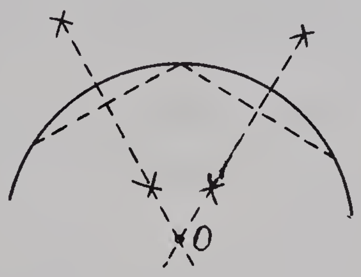
PAGES 343-344 (Test 25)

1. a right angle. 2. its intercepted arc. 3. is doubled. 4. one half its intercepted arc. 5. acute. 6. supplementary. 7. equal. 8. one half the difference of its intercepted arcs. 9. one half the sum. 10. chords.

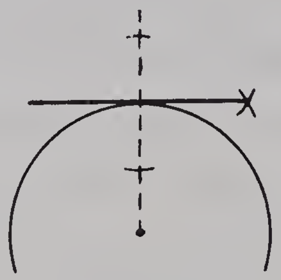
PAGE 344 (Test 26)



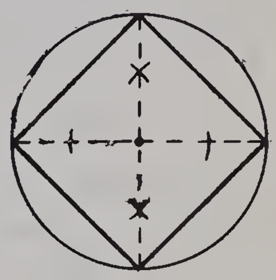
1. Depends on §§332, 323, 315.



2. Depends on § 292.



3. Depends on § 329.



4. Depends on § 331.

PAGES 344-345 (Test 27)

1. 86°. 2. 48°. 3. 19°. 4. 94°. 5. 90°. 6. 24°. 7. 72°. 8. 7 in. 9. 45°; 135°. 10. 120°. 11. 88°. 12. 5 in. 13. 100°.

Chapter 11. Loci

PAGES 348-349

1. The locus of points which are 3 in. from a given point is a circle with the given point as the center and 3 in. as the radius.

2. The locus of the center of a ball as it rolls on a straight line over a level floor is a straight line parallel to the given line and at a distance of the radius of the ball from the floor.

3. The locus of the center of a circle that rolls around the outside of another circle is a circle with a center the same as the fixed circle and a radius equal to the sum of the radii of the two given circles.

4. The locus of points on this page 3 in. from the top is a line parallel to the top edge of the page and 3 in. from it.

5. The locus of all cities that are 5 mi. from a given railroad is a pair of lines parallel to the railroad and each 5 mi. from the railroad.

6. The locus of points 5 in. from a circle whose radius is 5 in. is the center of the given circle and a circle having the same center as the given circle and a radius of 10 in.

7. The locus of points equidistant from the top and bottom of this page is the line parallel to the top and bottom edges and midway between them.

8. The locus of points less than 5 in. from a given point is the portion of the plane enclosed by a circle having the given point as a center and 5 in. as a radius.

9. The locus of a point equidistant from two given points is the perpendicular bisector of the line segment joining the two points.

10. The locus of points within an angle equidistant from the sides of the angle is the bisector of the angle.

11. The locus of the vertex of a right triangle with a given hypotenuse as a base is the circle having the hypotenuse as a diameter.

12. The locus of the centers of all circles tangent to a given line at a given point is the perpendicular to the line at that point

13. The locus of points equidistant from the points of intersection of two given intersecting circles is the line of centers of the two given circles.

14. The locus of the midpoints of parallel chords of a given circle is the diameter perpendicular to any one of the parallel chords.

15. The locus of the midpoints of all equal chords of a given circle is a circle having the same center as the given circle and a radius equal to the perpendicular from the center of the given circle to one of the equal chords.

16. The locus of the midpoints of the radii of a given circle is a circle having the same center as the given circle and a radius equal to half the radius of the given circle.

17. A 2-inch square inside the 4-inch square and equidistant from it on all 4 sides, and the extensions of the sides of the 2-inch square to meet the sides of the 4-inch square.

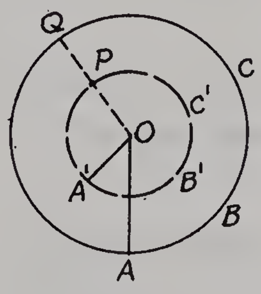
18. The locus of points inside a circle whose radius is 3 inches is the plane enclosed by the circle.

19. The locus inside a circle of a point whose distance from the circle is equal to a radius is the center of the circle.

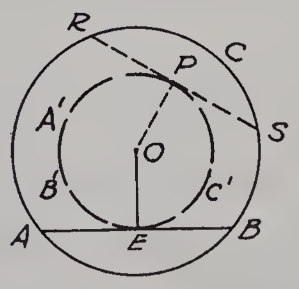
20. The locus of the center of a circle that rolls around the inside of a larger circle and is always tangent to the larger circle is a circle whose center is the center of the larger circle and whose radius is the difference of the radii of the two given circles.

PAGE 352

1. The locus of the midpoints of the radii of a given circle is a circle having the same center as the given circle and a radius equal to half the radius of the given circle.



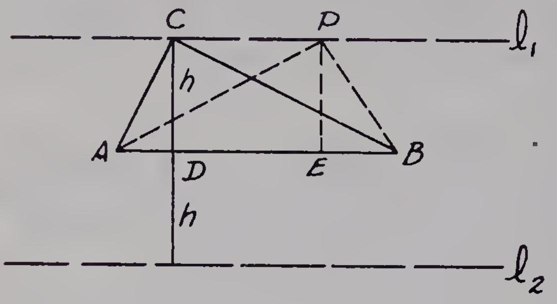
(Ex. 1)



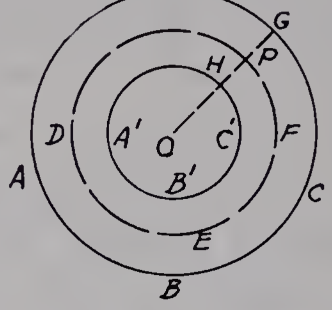
(Ex. 2)

2. The locus of the midpoints of all chords of a given length in a given circle is a circle having the same center as the given circle and a radius equal to the perpendicular from the center to one of the chords.

3. The locus of the vertices of all triangles having a given base and a given altitude is a pair of lines parallel to and at an altitude's distance from the base of the given triangle.



(Ex. 3)



(Ex. 4)

4. The locus of points equidistant from two circles having the same center is a circle having the same center as the two given circles and a radius equal to half the sum of the radii of the two given circles.

PAGES 354-355

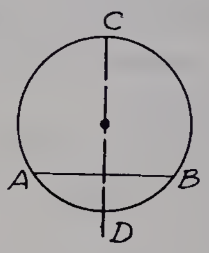
1. The locus of the centers of all circles passing through two given points is the perpendicular bisector of the line segment joining the two points.

Given line segment AB, and CD the ⊥ bisector of AB.

To prove that CD is the locus of the centers of Ⓢ passing through A and B.

Planning the Proof: 1. We can prove loci by §§341, 343-347. 2. We shall use §345.

Proof: 1. The center of any circle passing through A and B is equidistant from A and B (§276) 2. Then CD is the locus of centers of circles passing through A and B (§345).



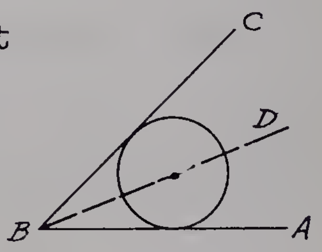
2. The locus of the center of a circle tangent to a given angle is the bisector of the angle.

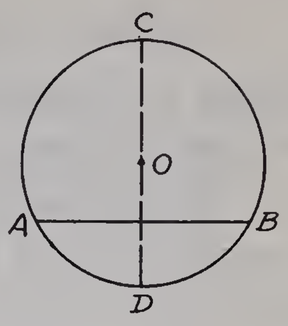
Given BD the bisector of ∠ABC.

To prove that BD is the locus of the center of a ⊙ tangent to BA and BC.

Planning the Proof: 1. We can prove loci by §§341, 343-347. 2. We shall use §343.

Proof: 1. The center of a ⊙ tangent to BA and BC is equidistant from BA and BC (§§295, 276). 2. ∴ BD is the locus of the center of a ⊙ tangent to BA and BC (§343).





3. The locus of points within a given circle which are equidistant from the ends of a given chord is the diameter perpendicular to the given chord.

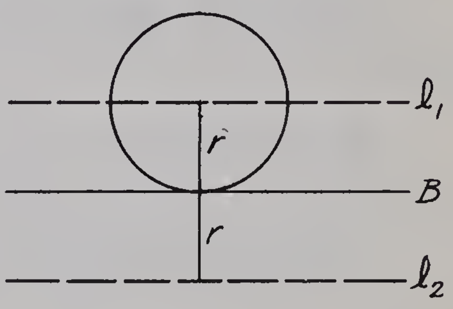
Given ⊙O with chord AB, and diameter CD ⊥ AB.

To prove that CD is the locus of points within ⊙O equidistant from A and B.

Planning the Proof: 1. We can prove loci by §§ 341, 343-347. 2. We shall use §345.

Proof: 1. CD is a diameter of ⊙O (Given). 2. Any point on CD is within ⊙O (Def. of a diameter, §277). 3. CD ⊥ AB (Given). 4. CD bisects AB (§290). 5. A line through C and D is the locus of points equidistant from A and B (§345). 6. ∴ CD is the locus of points within ⊙O equidistant from A and B (Statements 2 and 5).

4. The locus of the center of a circle which has a given radius and is tangent to a given line is a pair of lines parallel to the given line and at the distance of the given radius from it.



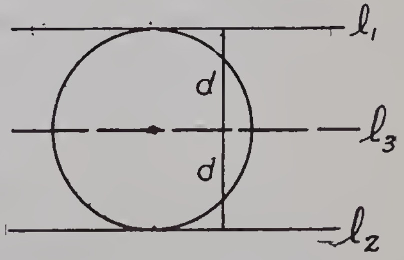
Given line AB, and lines ℓ_1 and $\ell_2 \parallel$ AB and at the distance r from AB.

To prove that the pair of lines ℓ_1 and ℓ_2 is the locus of the center of a ⊙ tangent to AB and having the radius r.

Planning the Proof: 1. We can prove loci by §§341, 343-347. 2. We shall use § 341, Asmt. 48.

Proof: 1. The centers of all circles tangent to AB and having radii = r, have the distance r from AB (§§295, 15, 276). 2. Then ℓ_1 and ℓ_2 are the required locus (Asmt. 48, § 341).

5. The locus of the centers of all circles tangent to two given parallel lines is the line parallel to each of the given lines and midway between them.



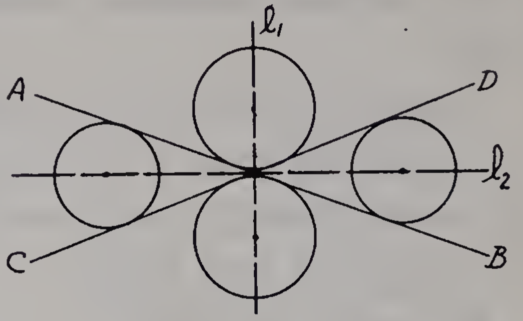
Given the ∥ lines ℓ_1, ℓ_2, and ℓ_3, with ℓ_3 the distance d from ℓ_1 and ℓ_2.

To prove that ℓ_3 is the locus of the centers of all Ⓢ tangent to ℓ_1 and ℓ_2.

Planning the Proof: 1. We can prove loci by §§ 341, 343-347. 2. We shall use Asmt. 49, §341.

Proof: 1. The centers of all circles tangent to ℓ_1 and ℓ_2 have a distance d from each line (§§295, 15, 276). 2. Then ℓ_1 is the required locus (Asmt. 49).

(b) The locus of the centers of all circles tangent to two given intersecting lines is the pair of lines bisecting the angles formed by the given lines.



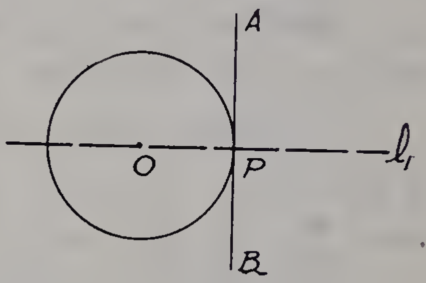
Given ℓ_1 and ℓ_2 bisecting the ∠s formed by the intersecting lines AB and CD.

To prove that the pair of lines ℓ_1 and ℓ_2 is the locus of the centers of all Ⓢ tangent to AB and CD.

Planning the Proof: 1. We can prove loci by §§341, 343-347. 2. We shall use §343.

Proof: 1. The center of any circle tangent to the intersecting lines AB and CD is equidistant from the lines (Asmt. 26, §§295, 15). 2. ∴ ℓ_1 and ℓ_2 are the required locus (§343).

6. The locus of the centers of all circles tangent to a given circle at a given point on the circle is the line perpendicular to the tangent to the given circle at the given point.



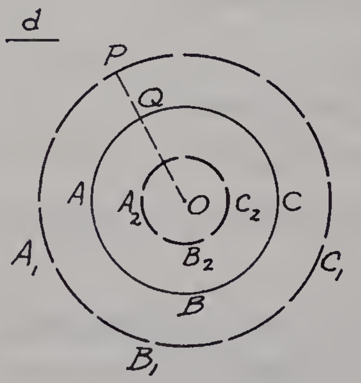
Given AB tangent to ⊙O at P, and $\ell_1 \perp AB$ at P.

To prove that ℓ_1 is the locus of the centers of all Ⓢ tangent to ⊙O at P.

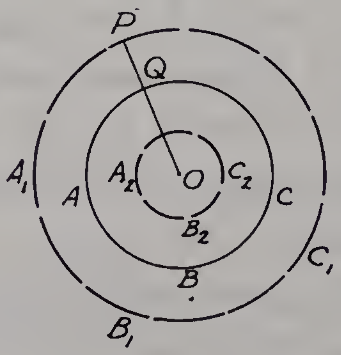
Planning the Proof: 1. We can prove loci by §§341, 343-347. 2. We shall use §347.

Proof: 1. ℓ_1 is the locus of the centers of all Ⓢ tangent to AB at P (§347). 2. ℓ_1 is the locus of the centers of all Ⓢ tangent to ⊙O at P (§302).

7. The locus of points at a given distance from a given circle is two circles concentric with the given circle and having radii of r + d and r - d, where r is the radius of the given circle and d is the given distance.



Part I



Part II

PART I.

Any point on either of two circles which are concentric with the given circle and have radii of $r + d$ and $r - d$ respectively is at the given distance d from the given circle of radius r.

Given the distance d and concentric circles ABC, $A_1B_1C_1$, and $A_2B_2C_2$ with radii of r, $r + d$, and $r - d$, respectively.

To prove that any point on either $\odot A_1B_1C_1$ or $\odot A_2B_2C_2$ is at the distance d from $\odot ABC$.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309, 343, 344, 345. 2. We shall use Asmt.

Proof: 1. Let P be any point on either $\odot A_1B_1C_1$ or $\odot A_2B_2C_2$, say $\odot A_1B_1C_1$, and draw OP intersecting $\odot ABC$ in Q (Asmt. 11). 2. $OP = r + d$ (Given). 3. $OQ = r$ (Given). 4. $OP - OQ = d$ (Asmt. 2). 5. $PQ = d$ (Asmt. 6 and §340); that is, any point on either Ⓢ$A_1B_1C_1$ or $A_2B_2C_2$ is at the distance d from $\odot ABC$.

PART II.

Any point at a given distance d from a given circle lies on one of two circles which are concentric with the given circle and have radii of $r + d$ and $r - d$, respectively.

Given the distance d and the concentric circles ABC, $A_1B_1C_1$, and $A_2B_2C_2$ with radii of r, $r + d$, and $r - d$, respectively.

To prove that any point at the distance d from $\odot ABC$ is on either $\odot A_1B_1C_1$ or $\odot A_2B_2C_2$.

Planning the Proof: 1. We can prove that a point is on a circle by §281b; §341, Asmt. 47; §346. 2. We shall use §341, Asmt. 47.

Proof: 1. Let OQ be the radius of $\odot ABC$, and point P be on line OQ such that $PQ = d$ (Given and §340). 2. $OQ = r$ (Given). 3. $OQ + PQ = r + d$ (Asmt. 1). 4. $OP = r + d$ (Asmt. 6 and 9). 5. P is on $\odot A_1B_1C_1$ (§341, Asmt. 47); that is, any point at the distance d from $\odot ABC$ is on either $\odot A_1B_1C_1$ or $\odot A_2B_2C_2$.

∴ Ⓢ $A_1B_1C_1$ and $A_2B_2C_2$ form the locus of all points at the distance d from $\odot ABC$ (§§339, 342).

Discussion: The proof above covers the case where $d < r$.

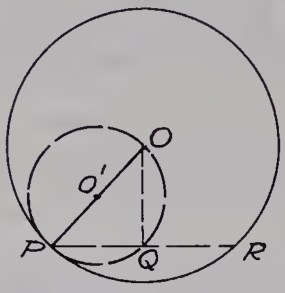
If $d = r$, $\odot A_2B_2C_2$ has the radius zero, and hence is the point O.

If $d > r$ but $< 2r$, $\odot A_2B_2C_2$ lies inside $\odot ABC$ as above, but the distance is measured on PO extended through O across $\odot A_2B_2C_2$.

If $d = 2r$, $\odot A_2B_2C_2$ coincides with $\odot ABC$.

If $d > 2r$, $\odot A_2B_2C_2$ lies outside $\odot ABC$.

8. The locus of the midpoints of all chords which can be drawn from a given point on a given circle is a circle having the radius of the given circle to the given point as a diameter.



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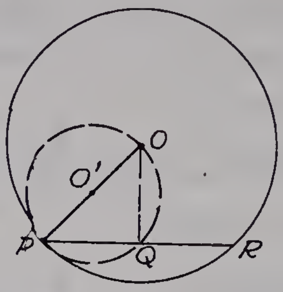


II

PART I.

Any point on the circle having the radius of the given circle to the given point as a diameter is the midpoint of a chord of the given circle from the given point.

Given $\odot O$ with radius OP and $\odot O'$ on OP as a diameter.

To prove that any point on $\odot O'$ is the midpoint of a chord of $\odot O$ from P.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309, 343, 344, 345. 2. We shall use §309a.

Proof: 1. Let Q be any point on $\odot O'$ except P and draw PQ intersecting $\odot O$ in R (Asmt. 11). 2. Draw OQ (Asmt. 11). 3. $OQ \perp PR$ (§335). 4. $PQ = QR$ (§309a); that is, any point on $\odot O'$ is the midpoint of a chord of $\odot O$ from P.

PART II.

The midpoint of any chord of a given circle from a given point lies on a circle having the radius of the given circle to the given point as a diameter.

Given $\odot O$ with radius OP and $\odot O'$ on OP as a diameter.

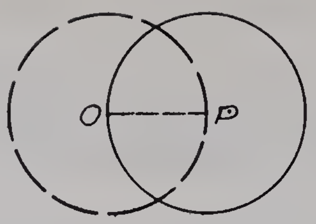
To prove that the midpoint of any chord of $\odot O$ from P lies on $\odot O'$.

Planning the Proof: 1. We can prove that a point is on a circle by §§281b, 341, Asmt. 47; §346. 2. We shall use §346.

Proof: 1. Let PR be a chord of $\odot O$ from P, and Q the midpoint of PR (Asmt. 17). 2. Draw OQ (Asmt. 11). 3. $OQ \perp PR$ (§311a). 4. $\angle PQO$ is a rt. $\angle$ (§14). 5. $\triangle PQO$ is a rt. $\triangle$ (§57). 6. Q is on $\odot O'$ (§346), that is, the midpoint of any chord of $\odot O$ from P lies on $\odot O'$.

$\therefore$ $\odot O'$ is the locus of the midpoints of all chords drawn in $\odot O$ from P (§§339, 342).

9. The locus of the centers of all circles with a given radius and passing through a given point is a circle having the given point as a center and the given radius as a radius.



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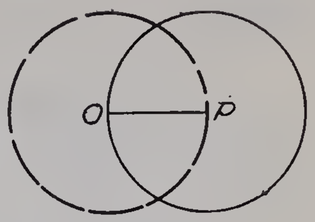


II

PART I.

Any point on the circle having the given point as center and the given radius as a radius is the center of a circle with the given radius and passing through the given point.

Given ⊙O with radius r.

To prove that any point on ⊙O is the center of a ⊙ with radius r and passing through O.

Planning the Proof: 1. We can prove that a point is on a circle by §§281b; 341, Asmt. 47; §346. 2. We shall use Asmt. 47.

Proof: 1. Let P be any point on ⊙O and draw OP (Asmt. 11). 2. OP = r (Given). 3. Draw the ⊙ whose center is P and radius r (Asmt. 15). 4. ⊙P passes through O (Asmt. 47).

PART II.

Any circle with the given radius and passing through the given point has its center on the given circle.

Given ⊙O with radius r.

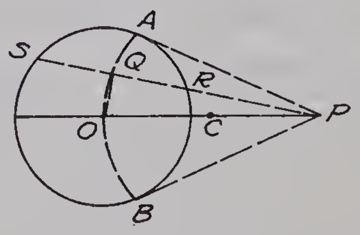
To prove that any circle with radius r and passing through O has its center on ⊙O.

Planning the Proof: (Same as in Part I.)

Proof: 1. Let P be the center and r the radius of ⊙P which passes through O. 2. P is on ⊙O (Asmt. 47).

∴ ⊙O is the locus of the centers of ⓢ with radius r and passing through O (§§339, 342).

10. The locus of the midpoints of all chords formed by secants drawn to a given circle from a given external point is the arc of another circle, which lies within the given circle and which has the line segment joining the external point to the center of the given circle as a diameter.



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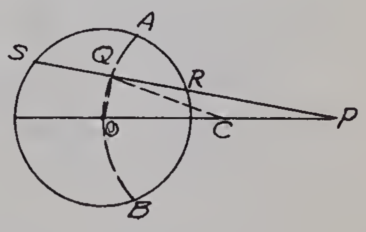


II

PART I.

Any point on the arc is the midpoint of a chord of the given circle formed by a secant drawn to the given circle from the given point.

Given $\odot O$, external point P, and arc AB of $\odot C$ whose diameter is PO.

To prove that any point on $\widehat{AB}$ is the midpoint of a chord of $\odot O$ formed by a secant drawn to $\odot O$ from P.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 308, 309, 343, 344, 345. 2. We shall use § 309a.

Proof: 1. Let Q be any point on $\widehat{AB}$ except A or B and draw PQ intersecting $\odot O$ in R and S (Asmt. 11). 2. Draw OQ (Asmt. 11). 3. $OQ \perp RS$ (§335). 4. $SQ = QR$ (§309a); that is, any point on $\widehat{AB}$ is the midpoint of a chord of $\odot O$ formed by a secant drawn to $\odot O$ from P.

PART II.

The midpoint of any chord formed by a secant drawn to a given circle from a given point lies on the arc of another circle, which lies within the given circle and which has the line segment joining the given point to the center of the given circle as a diameter.

Given $\odot O$, external point P, and arc AB of $\odot C$ whose diameter is PO.

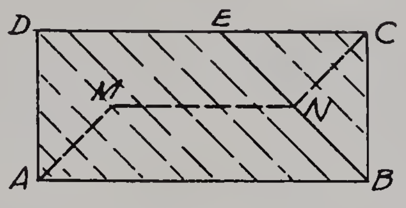
To prove that the midpoint of any chord formed by a secant drawn from P to $\odot O$ lies on $\widehat{AB}$.

Planning the Proof: 1. We can prove that a point is on a circle by §§281b; 341, Asmt. 47; §346. 2. We shall use §346.

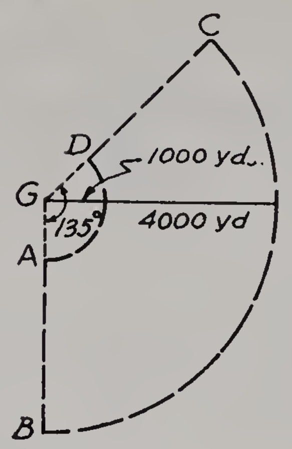
Proof: 1. Let PRS be a secant to $\odot O$ from P, and Q the midpoint of chord RS (Asmt. 17). 2. Draw OQ (Asmt. 11). 3. $OQ \perp RS$ (§ 311a). 4. $\angle OQP$ is a rt. $\angle$ (§14). 5. $\triangle OQP$ is a rt. $\triangle$ (§57). 6. Q is on $\widehat{AB}$ (§346); that is, the midpoint of any chord formed by a secant drawn from P to $\odot O$ lies on $\widehat{AB}$.

$\therefore$ $\widehat{AB}$ is the locus of the midpoints of all chords of $\odot O$ formed by secants drawn to $\odot O$ from P (§§339, 342).

11. The broken line AMNC is the locus. AM and CN are $\perp$ to the lines $\parallel$ EB, and MN is $\parallel$ AB and DC.



(Ex. 11)



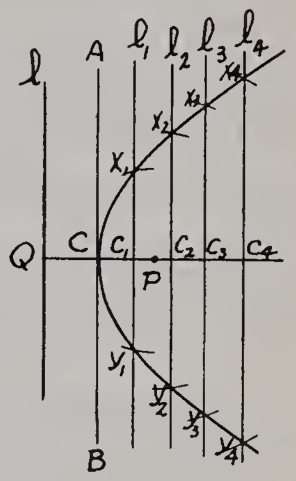
(Ex. 12)

12. The portion of the plane enclosed by ABCD is the locus of points where the projectiles may fall.

13. a. Given the line ℓ and the point P.

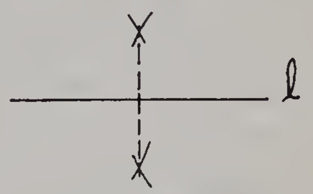
To find the locus of points equidistant from ℓ and P.

Solution: 1. Draw PQ $\perp$ ℓ (§178a). 2. Construct AB bisecting PQ at C (§177b). 3. Construct a series of lines ℓ_1, ℓ_2, ℓ_3, ℓ_4, etc., $\parallel$ ℓ and intersecting PQ in C_1, C_2, C_3, C_4, etc. (§179). 4. Using P as a center and C_1Q, C_2Q, C_3Q, C_4Q, etc., as radii, draw arcs intersecting ℓ_1, ℓ_2, ℓ_3, ℓ_4 in points x_1, y_1, x_2, y_2, etc. (Asmt. 15). 5. Draw a smooth curve through x_4, x_3, x_2, x_1, C, y_1, y_2, y_3, and y_4.



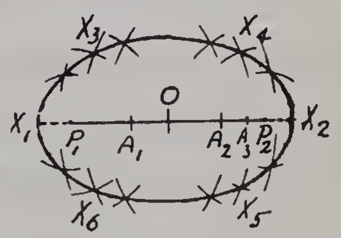
The required curve is a parabola.

b. Given two points P_1 and P_2 and a line $\ell > P_1P_2$.



To find the locus of points the sum of whose distances from P_1 and P_2 is equal to ℓ.

Solution: 1. Draw P_1P_2 and extend it; bisect ℓ and P_1P_2. 2. Take OX_1 and $OX_2 = \frac{1}{2}\ell$. 3. Then X_1 and X_2 are two points of the locus. 4. Take any point A_1 on X_1X_2 and using P_1 and P_2 as centers and A_1X_1 and A_1X_2 as radii draw arcs intersecting at X_3, X_4, X_5, and X_6. 5. By taking several other points on P_1P_2 and repeating Step 4, several points of the locus can be obtained.



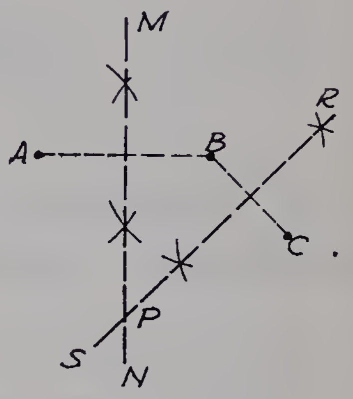
The locus is an ellipse.

PAGE 356

1. Two angles of a triangle are equal if and only if the sides opposite are equal.

2. The necessary and sufficient condition for two angles of a triangle to be equal is that the sides opposite are equal.

PAGES 357-358

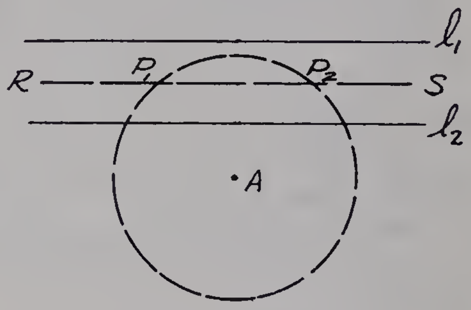


1. Given the 3 points A, B, and C.

To find all points equidistant from A, B, and C.

Solution: 1. MN, the ⊥ bisector of AB, the line segment joining A and B, is the locus of points equidistant from A and B (§345). 2. RS, the ⊥ bisector of BC, the line segment joining B and C, is the locus of points equidistant from B and C (§345). 3. The required point is P, the intersection of MN and RS (§350).

Discussion: 1. If A, B, and C are in a st. line, MN ∥ RS and there are no points. 2. In all other positions there is one point.



2. Given point A and the ∥ lines ℓ_1 and ℓ_2 which are 1 in. apart.

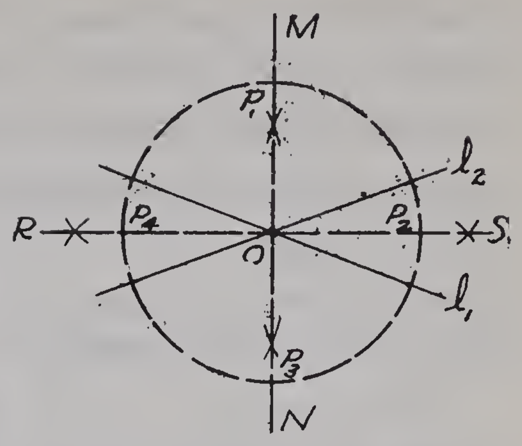
To find all points $1\frac{1}{2}$ in. from A and equidistant from ℓ_1 and ℓ_2.

Solution: 1. The ⊙ with A as a center and $1\frac{1}{2}$ in. as a radius is the locus of points $1\frac{1}{2}$ in. from A (Asmt. 47). 2. RS, the line ∥ ℓ_1 and ℓ_2 and midway between them, is the locus of points equidistant from ℓ_1 and ℓ_2 (Asmt. 49). 3. The required points are P_1 and P_2, the intersection of the ⊙ and RS (§350).

Discussion: 1. If A is on the same side of ℓ_1 and ℓ_2 and at a distance of 1 in. from the nearer line, there is 1 point. 2. If A is on the same side of ℓ_1 and ℓ_2 and at a distance greater than 1 in. from the nearer line, there is no point. 3. In all other positions there are 2 points.

3. Given the 2 lines ℓ_1 and ℓ_2 intersecting at O.

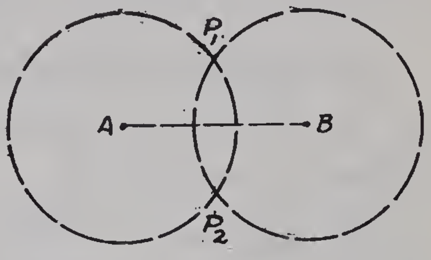
To find all points equidistant from ℓ_1 and ℓ_2 and at a distance of 1 in. from O.



Solution: 1. MN and RS, the bisectors of the $\angle$s formed by ℓ_1 and ℓ_2, are the loci of points equidistant from ℓ_1 and ℓ_2 (§344). 2. The ⊙ with center O and a radius of 1 in. is the locus of points 1 in. from O (Asmt. 47). 3. P_1, P_2, P_3, and P_4, the intersections of MN and RS with the ⊙, are the required points (§350).

4. Given the points A and B, 3 in. apart.

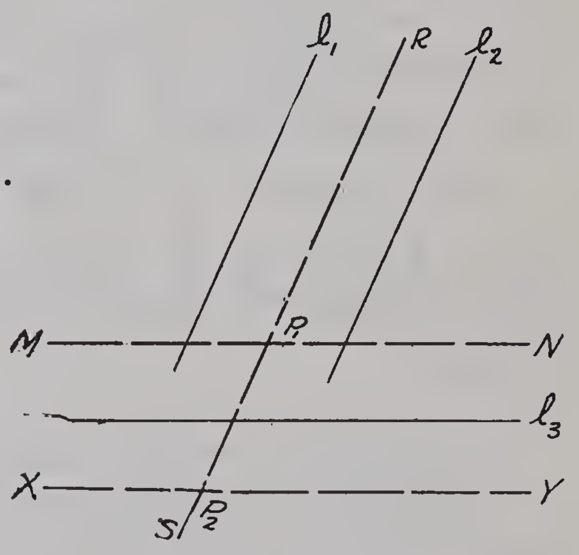
To find all points at a distance of 2 in. from A and B.



Solution: 1. The ⊙ with A as a center and a radius of 2 in. is the locus of points 2 in. from A (Asmt. 47). 2. The ⊙ with B as a center and a radius of 2 in. is the locus of points 2 in. from B (Asmt. 47). 3. P_1 and P_2, the intersection of the 2 ⓢ, are the required points (§350).

5. Given the ∥ lines ℓ_1 and ℓ_2, ℓ_3 another line, and the distance d.

To find all points equidistant from ℓ_1 and ℓ_2 and at the distance d from ℓ_3.



Solution: 1. RS, the line ∥ ℓ_1 and ℓ_2 and midway between them, is the locus of points equidistant from ℓ_1 and ℓ_2 (Asmt. 49). 2. MN and XY, the two lines ∥ ℓ_3 and at a distance d from it, are the locus of points at a distance d from ℓ_3 (Asmt. 48). 3. P_1 and P_2, the intersections of MN and XY with RS, are the required points (§350).

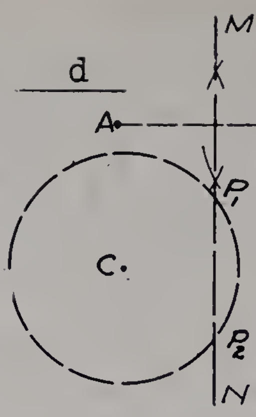
Discussion: 1. If ℓ_3 ∥ ℓ_1 and ℓ_2, there are no points, except that if RS coincides with MN or XY, every point in RS is a required point. 2. In all other positions of ℓ_3, there are two points.

6. Given points A, B, and C, and the distance d.

To find all points equidistant from A and B and at the distance d from C.

Solution: 1. MN, the ⊥ bisector of the line segment joining

A and B, is the locus of points equidistant from A and B (§345). 2. The ⊙ with C as a center and d as a radius is the locus of points at a distance d from C (Asmt. 47). 3. P_1 and P_2, the intersection of the ⊙ and MN, are the required points (§350).

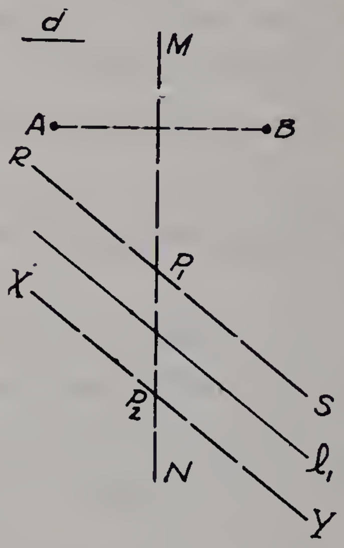


Discussion: 1. If MN $<$ the distance d from C, there are 2 points. 2. If MN $>$ the distance d from C, there are no points. 3. If MN = the distance d from C, there is 1 point.

7. Given the points A and B, the line ℓ_1, and the distance d.

To find all points equidistant from A and B, and at the distance d from ℓ_1.

Solution: 1. MN, the ⊥ bisector of the line segment joining A and B, is the locus of points equidistant from A and B (§345). 2. RS and XY, 2 lines ∥ ℓ_1 and at a distance d from ℓ_1, are the locus of points at the distance d from ℓ_1 (Asmt. 48). 3. P_1 and P_2, the intersections of RS and XY with MN, are the required points (§350).

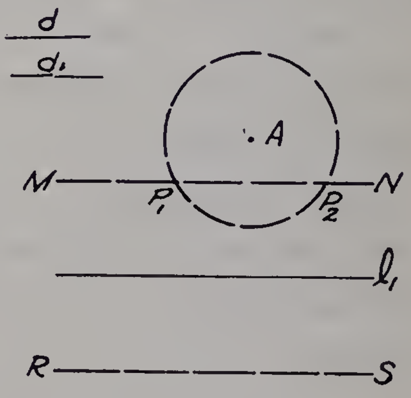


Discussion: 1. If RS and XY ∥ MN, there are no points. 2. If MN coincides with RS or XY, every point in MN is a required point. 3. In all other positions there are 2 points.

8. Given line ℓ_1, point A, and distances d_1 and d.

To find all points at the distance d_1 from ℓ_1 and at the distance d from A.

Solution: 1. MN and RS, ∥ ℓ_1 and at a distance d_1 from ℓ_1, are the locus of the points at the distance d_1 from ℓ_1 (Asmt. 48). 2. The ⊙ with A as a center and d as a radius is the locus of points at the distance d from A (Asmt. 47). 3. P_1 and P_2, the intersection of the ⊙ and MN, are the required points (§350).

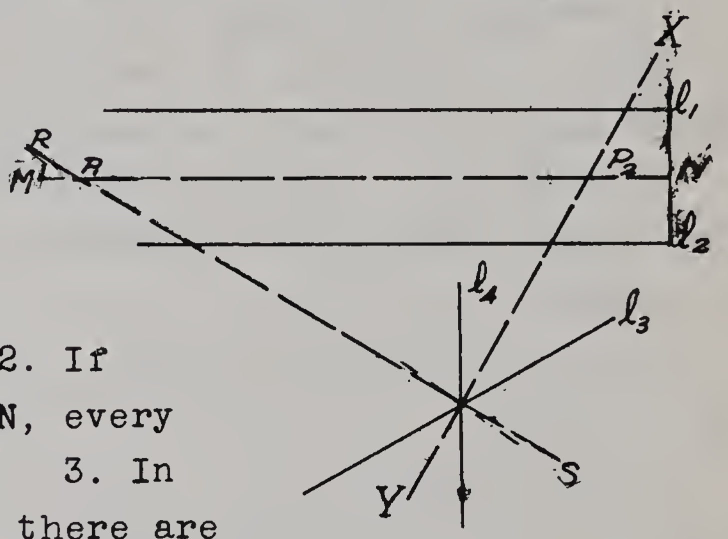


Discussion: 1. If the ⊙ intersects both MN and RS, there are 4 points. 2. If the ⊙ intersects 1 of the lines MN and RS and is tangent to the other, there are 3 points. 3. If the ⊙ is tangent to MN and RS, there are 2 points. 4. If the $\odot$ intersects one of the lines MN and RS, there are 2 points. 5. If the $\odot$ is tangent to either MN or RS and does not meet the other, there is 1 point. 6. If the $\odot$ does not meet either MN or RS, there are no points.

9. Given 2 $\parallel$ lines ℓ_1 and ℓ_2, and 2 intersecting lines ℓ_3 and ℓ_4.

To find all points equidistant from ℓ_1 and ℓ_2 and also equidistant from ℓ_3 and ℓ_4.

Solution: 1. MN, the line $\parallel$ ℓ_1 and ℓ_2 and midway between them, is the locus of points equidistant from ℓ_1 and ℓ_2 (Asmt. 49). 2. RS and XY, the bisectors of the $\angle$s formed by ℓ_3 and ℓ_4, are the loci of points equidistant from ℓ_3 and ℓ_4 (§344). 3. P_1 and P_2, the intersections of MN with RS and XY, are the required points (§350).

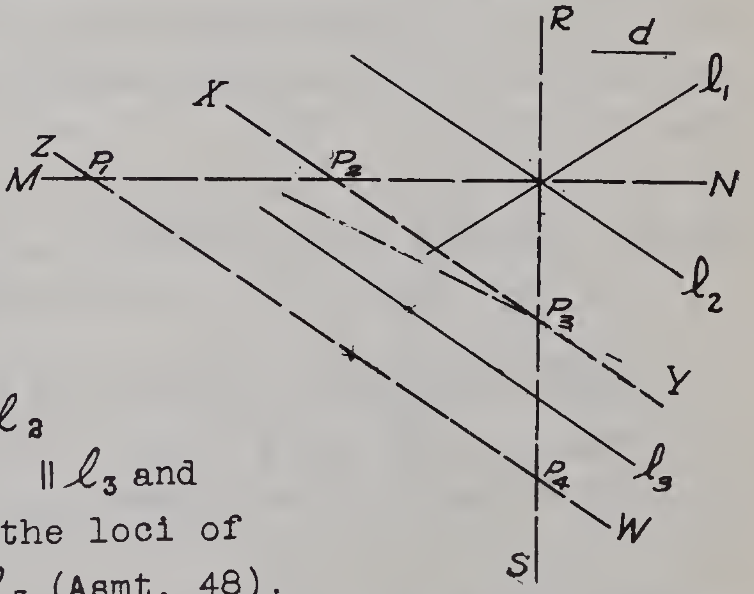


Discussion: 1. If either XY or RS $\parallel$ MN, there is 1 point. 2. If either XY or RS coincides with MN, every point in MN is a required point. 3. In all other positions of the lines there are 2 points.

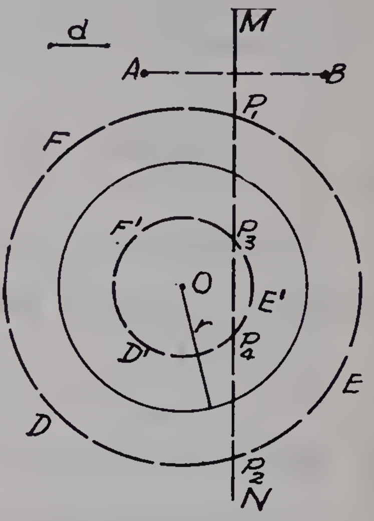
10. Given the intersecting lines ℓ_1 and ℓ_2, line ℓ_3, and the distance d.

To find all points equidistant from ℓ_1 and ℓ_2 and at the distance d from ℓ_3.

Solution: 1. MN and RS, the bisectors of the $\angle$s formed by ℓ_1 and ℓ_2, are the loci of points equidistant from ℓ_1 and ℓ_2 (§344). 2. XY and ZW, 2 lines $\parallel$ ℓ_3 and at the distance d from ℓ_3, are the loci of points at the distance d from ℓ_3 (Asmt. 48). 3. P_1, P_2, P_3, and P_4, the intersections of XY and ZW with MN and RS, are the required points (§350).



Discussion: 1. If either MN or RS coincides with XY or ZW, the required points will consist of a line of points and one point outside the line. 2. If XY and ZW are $\parallel$ either to RS or to MN, there will be 2 required points. 3. If either XY or ZW passes through the point of intersection of ℓ_1 and ℓ_3, there will be 3 points. 4. In all other positions there are 4 points.

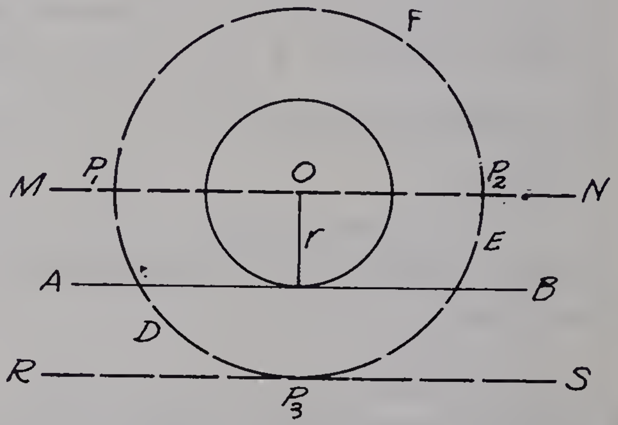


11. Given points A and B, $\odot$O with radius r, and distance d.

To find all points equidistant from A and B and at the distance d from $\odot$O.

Solution: 1. MN, the ⊥ bisector of the line segment joining A and B, is the locus of points equidistant from A and B (§345). 2. ⓢ DEF and D'E'F', having O as a center and radii of r + d and r − d, are the loci of points at a distance d from $\odot$O (Ex. 7, p. 354). 3. P_1, P_2, P_3, and P_4, the intersections of MN with ⓢDEF and D'E'F', are the required points (§350).

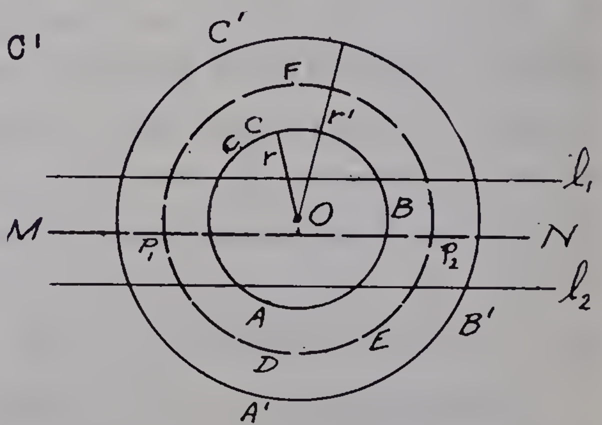
Discussion: There are from none to 4 possible points, depending on the positions of points A, B, and $\odot$O, and on the lengths of r and d.



12. Given $\odot$O with radius r, and AB a tangent to $\odot$O.

To find all points at the distance r from $\odot$O and also at the distance r from AB.

Solution: 1. Point O and $\odot$DEF, whose center is O and radius 2r, are the loci of points at the distance r from $\odot$O (Ex. 7, p. 354). 2. MN and RS, ∥ AB and at the distance r from AB, are the loci of points at a distance r from AB (Asmt. 48). 3. O, P_1, P_2, and P_3 are the required points (§350).

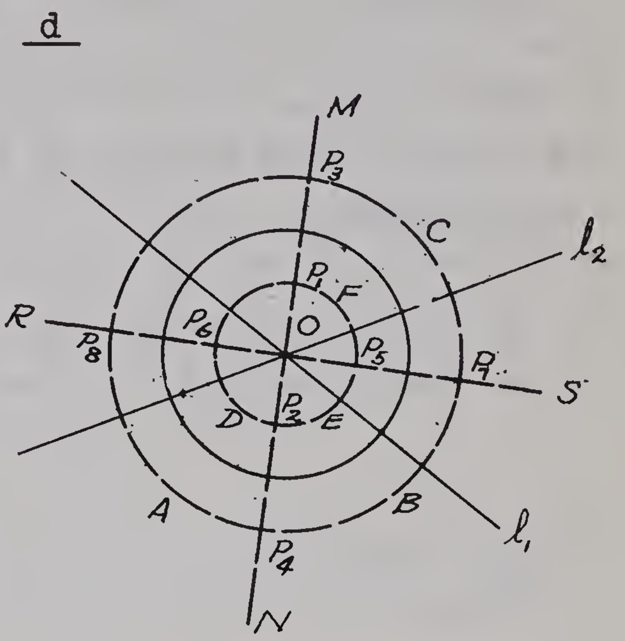


13. Given concentric ⓢABC and A'B'C' with center O and radii r and r' respectively, and ∥ lines ℓ_1 and ℓ_2 which intersect the ⓢ.

To find all the points equidistant from ⓢABC and A'B'C' and also equidistant from ℓ_1 and ℓ_2.

Solution: 1. $\odot$DEF, having center O and a radius = $\frac{r + r'}{2}$, is the locus of points equidistant from

Ⓢ ABC and A'B'C' (Ex. 4, p. 352). 2. MN, a line ∥ ℓ_1 and ℓ_2 and midway between them, is the locus of points equidistant from ℓ_1 and ℓ_2 (Asmt. 49). 3. P_1 and P_2, the intersections of MN and ⊙ DEF, are the required points (§350).



14. Given ⊙ O with radius r, ℓ_1 and ℓ_2 two lines intersecting at O, and line segment d.

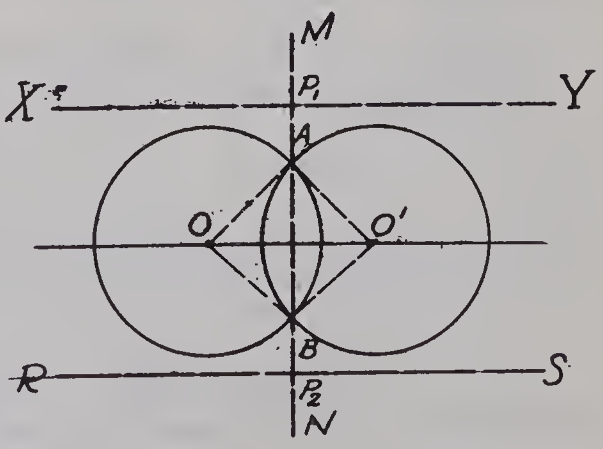
To find all the points at the distance d from ⊙ O and also equidistant from ℓ_1 and ℓ_2.

Solution: 1. Ⓢ ABC and DEF, having O as a center and radii of r + d and r − d respectively, are the loci of points at the distance d from ⊙ O (Ex. 7, p. 354). 2. MN and RS, the lines bisecting the ∠s formed by ℓ_1 and ℓ_2, are the loci of points equidistant from ℓ_1 and ℓ_2 (§344). 3. P_1, P_2, P_3, P_4, the intersections of Ⓢ ABC and DEF with MN, and P_5, P_6, P_7, P_8, the intersections of Ⓢ ABC and DEF with RS, are the required points (§350).

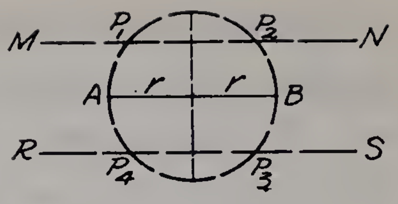
Discussion: 1. If the loci of Step (1) are a point and a ⊙, there are 5 required points. 2. If the locus of Step (1) is a ⊙, there are 4 required points.

15. Given the = Ⓢ O and O' intersecting at A and B, OO' the line of centers, and AB the common chord of Ⓢ O and O'.

To find all points equidistant from Ⓢ O and O' and at the distance AB from OO'.



Solution: 1. AB is the ⊥ bisector of line segment OO' (§§308a, 140). 2. All points equidistant from O and O' are equidistant from the Ⓢ O and O' (§§308a, 340, Asmt. 9, 2). 3. ∴ MN, the ⊥ bisector of the line segment joining O and O', is the locus of points equidistant from Ⓢ O and O' (Statement 1 and §345). 4. XY and RS, ∥ OO' and at the distance AB from OO', are the loci of points at the distance AB from OO' (Asmt. 48). 5. ∴ P_1 and P_2, the intersections of RS and XY with MN, are the required points (§350).



16. Given AB, a line segment 3 in. long.

To find all points which are 1 in. from AB and are vertices of rt. ∠s whose sides pass through A and B.

Solution: 1. MN and RS, each ∥ AB and at a distance of 1 in. from AB, are the loci of points 1 in. from AB (Asmt. 48). 2. The ⊙ whose center is the midpoint of AB and whose radius is $\frac{1}{2}$AB is the locus of the vertex of a rt. ∠ whose sides will pass through A and B (§346). 3. ∴ P_1, P_2, P_3, and P_4, the intersections of MN and RS with the ⊙ having diameter AB, are the required points.

If the locus consisted of only 2 points, the vertices of the rt. ∠s would be $1\frac{1}{2}$ in. from AB.

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Ex. OB = OC = 5.

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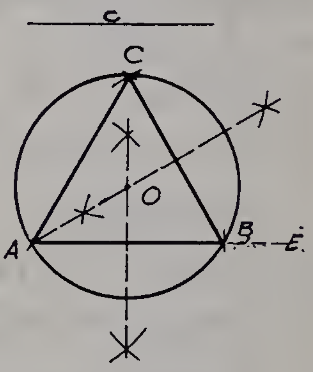


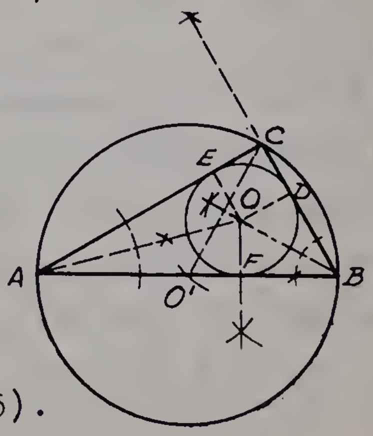
1. Given c the side of an equilateral △.

To construct the △ and circumscribe a ⊙ about it.

Construction: 1. On any line AE construct △ABC with AB = BC = AC = c (§181a). 2. Construct the ⊥ bisectors of sides AB and BC, calling the point of intersection of these bisectors O (§177b). 3. With O as a center and OA as a radius construct ⊙O (Asmt. 15).

Then △ABC is the required △, and ⊙O is the required circumscribed ⊙.

Proof: 1. AB = BC = AC = c (Const.). 2. ∴ △ ABC is equilateral (§56). 3. O is the intersection of the ⊥ bisectors of AB and BC (Const.). 4. ∴ O is equidistant from A, B, and C (§356). 5. ∴ A, B, and C lie on ⊙O (Const., §281b). 6. ∴ ⊙O is circumscribed about the equilateral △ABC (§288).



2. (1) Construction: 1. On any line of convenient length such as AB as a hypotenuse, construct a 30°-60° rt. △ (Ex. 12, page 195). 2. Construct the bisectors of ∠BAC and ∠ABC, calling the point of intersection O (§177a). 3. From O, construct OF ⊥ AB (§178a). 4. With O as center and OF as a radius draw ⊙O (Asmt. 15).

Then $\triangle$ ABC is a 30°-60° rt. $\triangle$ with $\odot$O an inscribed $\odot$.

Proof: 1. $\angle$BAC = 30° and $\angle$ABC = 60° (Const.). 2. $\therefore$ $\angle$ACB = 90° (§120, Asmt. 2). 3. $\therefore$ $\triangle$ABC is a rt. $\triangle$ (§57). 4. OF $\perp$ AB (Const.). 5. Draw OE $\perp$ AC and OD $\perp$ BC (§178a). 6. O is the intersection of the bisectors of $\angle$s BAC and ABC (Const.). 7. $\therefore$ O is equidistant from AB, BC, and AC (§358). 8. $\therefore$ OF = OD = OE (§15). 9. F is on $\odot$O (Const.). 10. $\therefore$ D and E are on $\odot$O (§281b). 11. $\therefore$ From 4, 5, 9, and 10, $\odot$O is tangent to AB at F, to BC at D, and to AC at E (§298). 12. $\therefore$ $\odot$O is inscribed in $\triangle$ABC (§325).

(2) Construction: 1. On AB, construct AO' = BC (§9). 2. With O' as a center and BC as a radius draw $\odot$O' (Asmt. 15).

Then $\odot$O' is circumscribed about $\triangle$ABC.

Proof: 1. BC = AO' = O'B (Const., §136). 2. Draw O'C (Asmt. 11). 3. BC is the radius of $\odot$O' (Const.). 4. AO' = O'C = O'B (§205). 5. $\therefore$ A, B, and C are on $\odot$O' (§281b). 6. $\therefore$ $\odot$O' is circumscribed about $\triangle$ABC (§288).

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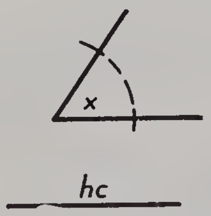
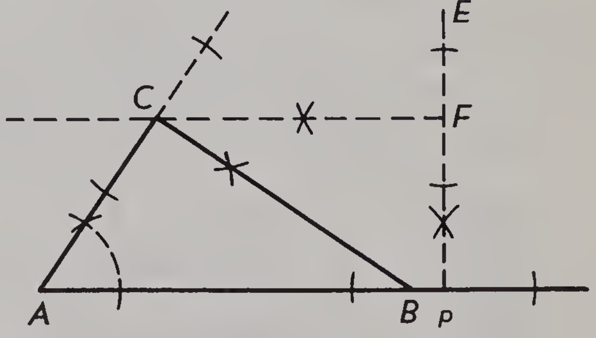
1. §334. 2. $\overset{\frown}{AHC}$ is obtuse.

PAGES 371-374

1. Given h_c the altitude on the hypotenuse and $\angle$x an acute $\angle$ of a rt. $\triangle$.

To construct the rt. $\triangle$.



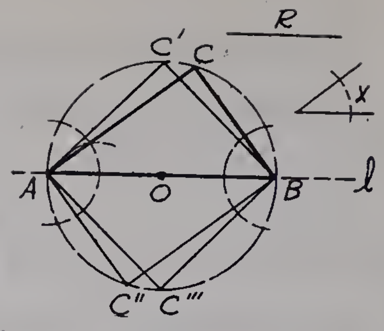


Construction: 1. At P any point in AX construct PE $\perp$ AX (§173). 2. On PE construct PF = h_c (§9). 3. Through F, construct FG $\parallel$ AX (§179). 4. With A as a vertex and AX as a side, construct $\angle$XAD = $\angle$x, letting AD intersect FG in C (§180). 5. At C construct BC $\perp$ AD (§178b).

Then $\triangle$ABC is the required rt. $\triangle$.

Proof: 1. $\angle$BAC = $\angle$x and $\angle$ACB is a rt. $\angle$ (Const.). 2. $\therefore$ $\triangle$ABC is a rt. $\triangle$ (§57). 3. The altitude on the hypotenuse of $\triangle$ABC = h_c (Const., Asmt. 48).

2. Given $\angle$x the acute $\angle$ of a rt. $\triangle$ and R the radius of the circumcircle.



To construct the rt. $\triangle$.

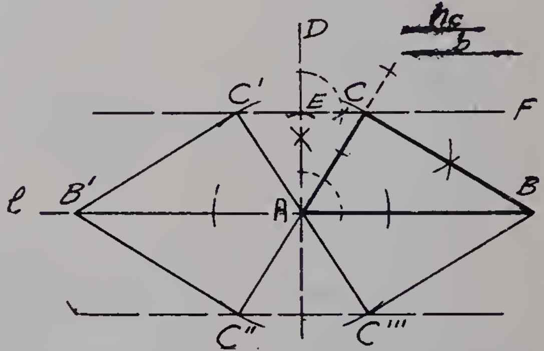
Construction: 1. With O any point in line ℓ as a center and R as a radius draw $\odot$O cutting line ℓ in A and B (Asmt. 15). 2. With A as a vertex and AB as a side, construct $\angle$BAC = $\angle$x, letting C be the point in which AC meets $\odot$O (§180). 3. Draw BC (Asmt. 11).

Then $\triangle$ABC is the required rt. $\triangle$.

Proof: 1. AO = OB = R (Const.). 2. $\therefore$ A and B lie on $\odot$O (§281b). 3. C is on $\odot$O (Const.). 4. $\therefore$ R is the radius of the circumscribed $\odot$ (§288). 5. $\angle$ACB is a rt. $\angle$ (§§277, 283, 279, 319). 6. $\therefore$ $\triangle$ABC is a rt. $\triangle$ (§57). 7. $\angle$BAC = $\angle$x (Const.).

Discussion: 1. As shown in the figure, there are 3 other rt. ⟁ ($\triangle$ABC', $\triangle$ABC", $\triangle$ABC"') fulfilling the given conditions. 2. If $\angle$x = 45°, there would be 2 rt. ⟁, both isos., 1 above AB and 1 below.

3. Given b a leg of a rt. $\triangle$ and h_c the altitude on the hypotenuse.



To construct the rt. $\triangle$.

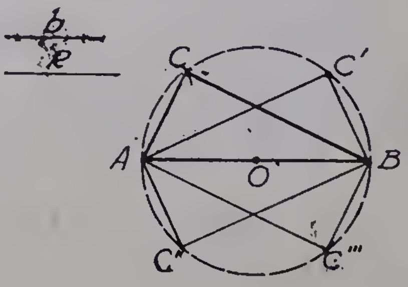
Construction: 1. At A any point in any line ℓ, construct AD $\perp$ ℓ (§178b). 2. On AD construct AE = h_c (§9). 3. Through E, construct EF $\parallel$ ℓ (§179). 4. With A as a center and a radius = b, draw an arc cutting EF at C (Asmt. 15). 5. Draw AC (Asmt. 11). 6. At C construct CB $\perp$ AC (§178b).

Then $\triangle$ABC is the required rt. $\triangle$.

Proof: 1. $\angle$ACB is a rt. $\angle$ (Const.). 2. $\therefore$ $\triangle$ABC is a rt. $\triangle$ (§57). 3. AC = b (Const.). 4. The altitude on the hypotenuse of $\triangle$ABC = h_c (Const., Asmt. 48).

Discussion: 1. As shown in the figure, there are 3 other rt. ⟁ ($\triangle$AC'B', $\triangle$AB'C", $\triangle$ABC"') having $h_c < b$, which fulfill the given conditions. 2. If b = the other leg, there would be 2 ⟁, both isos., one above AB and one below. 3. If $b < h_c$, the construction would be impossible.

4. Given b a leg of a rt. $\triangle$ and R the radius of the circumcircle.



To construct the rt. $\triangle$.

Construction: 1. Construct a $\odot$ with any point O as a center and R as a radius (Asmt. 15). 2. Draw any diameter AB of ⊙O (Asmt. 11, §277). 3. With A as a center and a radius = b draw an arc intersecting ⊙O in C (Asmt. 15). 4. Draw AC and BC (Asmt. 11).

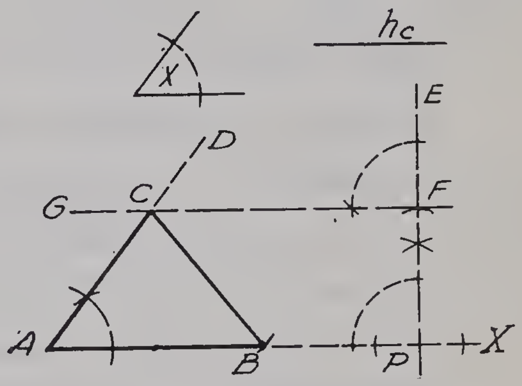
Then △ACB is the required rt. △.

Proof: 1. AB is a diameter of ⊙O (Const.). 2. ∴ ∠ACB is a rt. ∠ (§§283, 279, 319). 3. ∴ △ACB is a rt. △ (§57). 4. AC = b (Const.). 5. A, B, and C are on ⊙O and R is the radius of ⊙O (Const.). 6. ∴ The radius of the circumcircle is R (§§365, 288).

Discussion: 1. As shown in the figure, there is another rt. △ (△ABC") having $b < R$, and there are 2 other rt. △s (△ABC', △ABC‴) having $R < b < 2R$ which fulfill the given conditions. 2. If $b = R\sqrt{2}$, there would be 2 rt. △s, both isos., 1 above AB and 1 below. 3. If $b \geqq 2R$, the construction would be impossible.

5. Given ∠x a base ∠ of an isos. △ and h_c the altitude on the base.

To construct the isos. △.



Construction: 1. With A, on any line AX, as a vertex and AX as a side, construct ∠XAD = ∠x (§180). 2. At any point in AX, as P, construct PE ⊥ AX (§178b). 3. On PE construct PF = h_c (§9). 4. Through F construct FG ∥ AX and intersecting AD in C (§179). 5. With C as a center and CA as a radius draw an arc intersecting AX in B (Asmt. 15). 6. Draw CB (Asmt. 11).

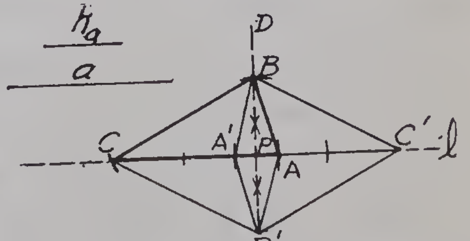
Then △ABC is the required isos. △.

Proof: 1. CA = CB (Const.). 2. ∴ △ABC is isos. (§56). 3. ∠A = ∠x (Const.). 4. The altitude on the base of △ABC = h_c (Const., Asmt. 48).

Discussion. If $\angle x \geqq 90$, the construction is impossible.

6. Given a, 1 of the = sides of an isos. △, and the altitude h_a.

To construct the isos. △.



Construction: 1. At any point P in any line ℓ construct PD ⊥ ℓ (§178b). 2. On PD construct PB = h_a (§9). 3. With B as a center and a as a radius draw an arc intersecting ℓ in C (Asmt. 15). 4. On ℓ construct CA = a (§9). 5. Draw CB and BA (Asmt. 11).

Then △ABC is the required isos. △.

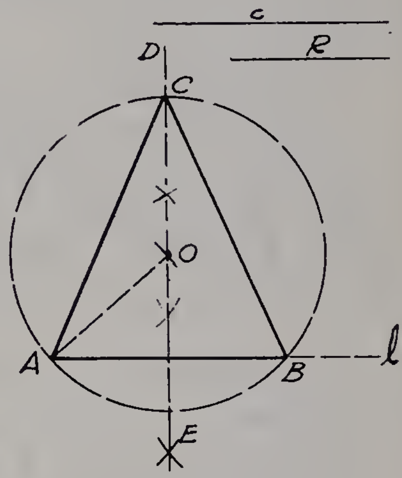
Proof: 1. CB = a and CA = a (Const.). 2. ∴ CB = CA (Asmt. 7). 3. ∴ △ABC is isos. (§56). 4. BP = h_a (Const.).

Discussion: 1. As shown in the figure, there are 3 other isos. △s (△A'BC', △AB'C, △A'B'C') having $h_a < a$, which fulfill the given conditions. 2. If $h_a = a$, there would be 4 isos. rt. △s. 3. If $h_a > a$, the construction would be impossible.

7. Given c the base of an isos. △ and R the radius of the circumcircle.

To construct the isos. △.

Construction: 1. On any line ℓ construct AB = c (§9). 2. Construct DE the ⊥ bisector of AB (§177b). 3. With A as a center and R as a radius draw an arc intersecting DE at O (Asmt. 15). 4. Draw OA (Asmt. 11). 5. With O as a center and OA as a radius draw a ⊙ intersecting DE in C (Asmt. 15). 6. Draw CA and CB (Asmt. 11).



Then △ABC is the required isos. △.

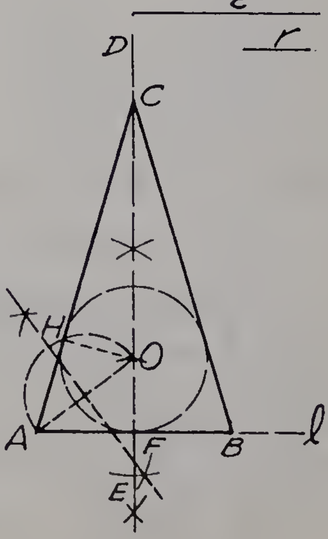
Proof: 1. AB, the base of △ABC, = c, OA = R = the radius of ⊙O (Const.). 2. DE is the ⊥ bisector of AB (Const.). 3. ∴ A and B are on ⊙O (§§345, 281b). 4. C is on ⊙O (Const.). 5. ∴ From 1, R is the radius of the circumcircle (§§365, 288). 6. From 2, CA = CB (§345). 7. ∴ △ABC is isos. (§56).

Discussion: 1. As shown in the figure, △ABC having $c > R > \frac{1}{2}c$ fulfills the given conditions. 2. If $R = \frac{1}{2}c$, △ABC would be an isos. rt. △ with the base c. 3. If $R < \frac{1}{2}c$, the construction would be impossible.

8. Given c the base of an isos. △ and r the radius of the incircle.

To construct the isos. △.

Construction: 1. On any line ℓ construct AB = c (§9). 2. Construct DE the ⊥ bisector of AB intersecting AB in F (§177b). 3. On FD construct FO = r (§9). 4. With O as a center and r as a radius draw a ⊙ (Asmt. 15). 5. From A draw AC tangent to ⊙O at H and intersecting DE in C (§336c). 6. Draw CB (Asmt. 11).



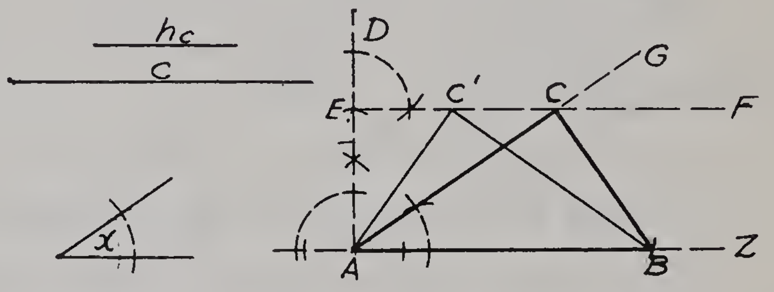
Then △ABC is the required isos. △.

Proof: 1. AB, the base of $\triangle ABC$, = c, DE is the $\perp$ bisector of AB, and C is in DE (Const.). 2. $\therefore$ CB = CA (§345). 3. $\therefore$ $\triangle ABC$ is isos. (§56). 4. O and F are in DE, OF = r, and r is the radius of $\odot O$ (Const.). 5. $\therefore$ AB is tangent to $\odot O$ at F (§298). 6. CF bisects $\angle ACB$ (§89, Ex. 4, page 119). 7. $\therefore$ From 1 and 4, O is equidistant from AC and BC (§343). 8. AC is tangent to $\odot O$ at H (Const.). 9. $\therefore$ CB is tangent to $\odot O$ (Asmt. 21, §298). 10. $\therefore$ From 4, r is the inradius of $\triangle ABC$ (§§370, 325).

Discussion: 1. As shown in the figure $\triangle ABC$ with $r < \frac{1}{2}c$ fulfills the given conditions. 2. If $r \geqq \frac{1}{2}c$, the construction would be impossible.

9. Given the base c, the altitude h_c, and $\angle x$ an $\angle$ at the base of a $\triangle$.

To construct the $\triangle$.



Construction: At A in line AZ construct AD $\perp$ AZ (§178b). 2. On AD construct AE = h_c (§9). 3. Through E construct EF $\parallel$ AZ (§179). 4. With A as a vertex and AZ as a side, construct $\angle ZAG = \angle x$, AG intersecting EF in C (§180). 5. On AZ construct AB = c (§9). 6. Draw CB (Asmt. 11).

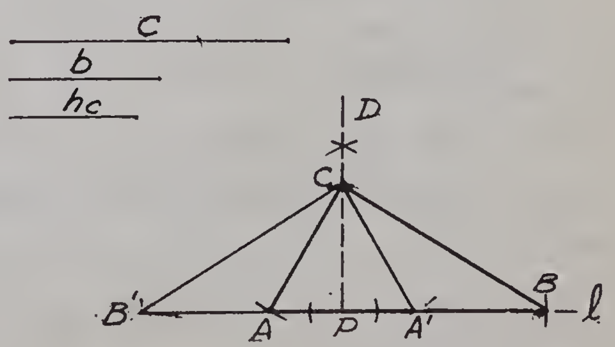
Then $\triangle ABC$ is the required $\triangle$.

Proof: 1. AB, the base of $\triangle ABC$, equals c and $\angle CAB = \angle x$ (Const.). 2. The altitude on the base of $\triangle ACB = h_c$ (Const., §341, Asmt. 48).

Discussion: 1. As shown in the figure, there is another $\triangle$ ($\triangle ABC'$), having its base $\angle$ at B = $\angle x$ and its altitude $h_c < c$, which fulfills the given conditions. 2. If $\angle x = 45°$, there would be only 1 $\triangle$. 3. If $\angle x = 90°$, there would be 2 rt. $\triangle$.

10. Given the base c, the side b, and the altitude h_c of a $\triangle$.

To construct the $\triangle$.



Construction: 1. At any point P in any line ℓ construct PD $\perp$ ℓ (§178b). 2. On PD construct PC = h_c (§9). 3. With C as a center and a radius = b draw an arc intersecting ℓ at A (Asmt. 15). 4. On ℓ construct AB = c (§9). 5. Draw CA and CB (Asmt. 11).

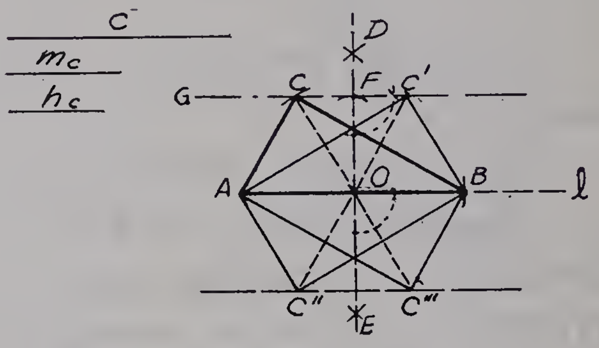
Then $\triangle ABC$ is the required $\triangle$.

Proof: 1. AB, the base of △ABC, = c and AC = b (Const.). 2. The altitude of △ABC = h_c (Const.).

Discussion: 1. As shown in the figure, there is another △ (△A'B'C) having $h_c < b < c$, which fulfills the given conditions. 2. If $h_c = b \gtrless c$, there would be 2 rt. △s. 3. If $h_c = b = c$, there would be 2 isos. rt. △s. 4. If $h_c < b = c$, there would be 2 isos. △s. 5. If $h_c > b$ and b is the shorter side, the construction would be impossible.

11. Given the side c, the altitude h_c, and the median m_c of a △.

To construct the △.



Construction: 1. On any line ℓ construct AB = c (§9). 2. Construct DE the ⊥ bisector of AB, intersecting AB in O (§177b). 3. On OD construct OF = h_c (§9). 4. Through F construct FG ∥ ℓ (§179). 5. With O as a center and m_c as a radius, draw an arc intersecting FG in C (Asmt. 15). 6. Draw AC, BC, and CO (Asmt. 11).

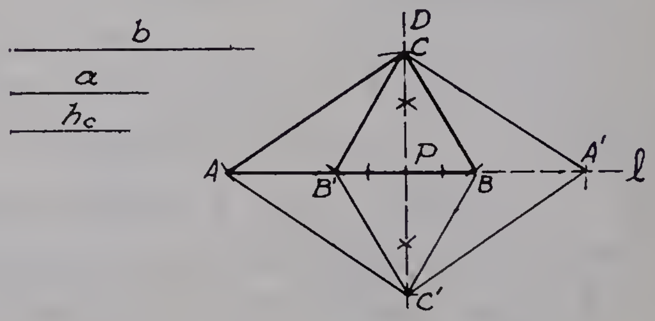
Then △ABC is the required △.

Proof: 1. AO = OB (Const.). 2. ∴ CO is the median on side c (§89). 3. AB = c and CO = m_c (Const.). 4. The altitude of △ABC on side c = h_c (Const., Asmt. 48).

Discussion: 1. As shown in the figure, there are 3 other △s (△ABC', △ABC", △ABC''') with $h_c < m_c < c$ which fulfill the given conditions. 2. If $h_c = m_c \gtrless \frac{1}{2}c$, there would be 2 △s, both isos., 1 above AB and 1 below. 3. If $h_c = m_c = \frac{1}{2}c$, there would be 2 isos. rt. △s, 1 above AB and 1 below. 4. If $h_c > m_c$, the construction would be impossible.

12. Given the sides a and b and the altitude h_c of a △.

To construct the △.



Construction: 1. At any point P in any line ℓ construct PD ⊥ ℓ (§178b). 2. On PD construct PC = h_c (§9). 3. With C as a center and b as a radius draw an arc intersecting ℓ in A (Asmt. 15). 4. With C as a center and a as a radius draw arcs intersecting ℓ in B and B' (Asmt. 15). 5. Draw CA, CB, and CB' (Asmt. 11).

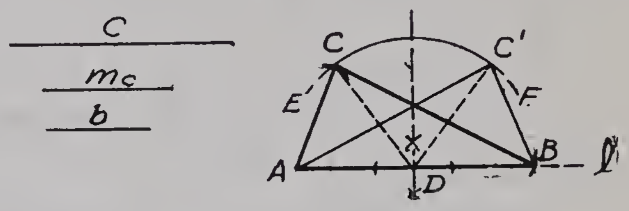
Then either $\triangle$ABC or $\triangle$AB'C is the required $\triangle$.

Proof: AC = b, B'C = BC = a, and the altitude of $\triangle$ABC or $\triangle$AB'C on side c = h_c (Const.).

Discussion: 1. As shown in the figure, there are 6 other $\triangle$s ($\triangle$A'BC, $\triangle$A'B'C, $\triangle$AB'C', $\triangle$ABC', $\triangle$A'BC', $\triangle$A'B'C') with $h_c < a < b$ which fulfill the given conditions. 2. If $h_c < a = b$, there would be 2 $\triangle$s, both isos., one above AB and one below. 3. If h_c = either a or b, there would be 4 rt. $\triangle$s. 4. If $h_c = a = b$ or $h_c >$ either a or b, the construction would be impossible.

13. Given the base c, the median m_c, and b, a side of a $\triangle$.

To construct the $\triangle$.



Construction: 1. On any line ℓ construct AB = c (§9). 2. Bisect AB calling the midpoint D (§177b). 3. With D as a center and m_c as a radius draw an arc EF (Asmt. 15). 4. With A as a center and b as a radius draw an arc intersecting $\overset{\frown}{EF}$ in C (Asmt. 15). 5. Draw AC, DC, and BC (Asmt. 11).

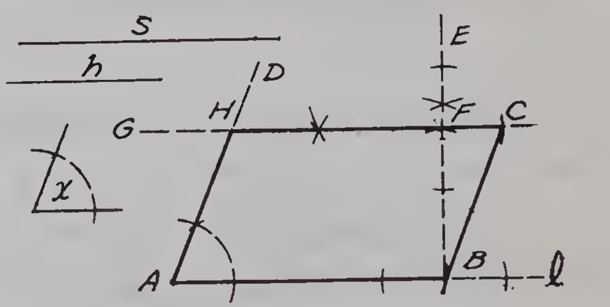
Then $\triangle$ABC is the required $\triangle$.

Proof: 1. AB = c, AC = b, AD = DB (Const.). 2. ∴ CD is the median of $\triangle$ABC on side c (§89). 3. CD = m_c (Const.).

Discussion: 1. As shown in the figure, there is another $\triangle$ ($\triangle$ABC') having $b < m_c < c$, which fulfills the given conditions. 2. If $m_c = \frac{1}{2}c$, there would be 2 rt. $\triangle$s. 3. If $b = m_c = \frac{1}{2}c$, there would be two 30°-60° rt. $\triangle$s. 4. If $(b + m_c) \leqq \frac{1}{2}c$, the construction would be impossible.

14. Given a side s, h the altitude on s, and ∠x an ∠ of a ▱.

To construct the ▱.



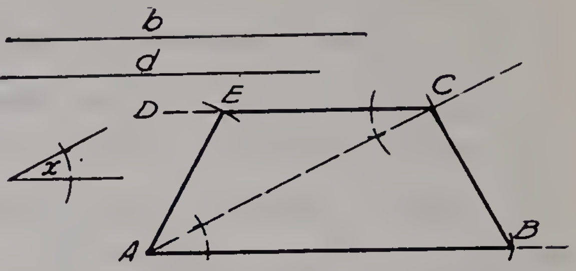
Construction: 1. On any line ℓ construct AB = s (§9). 2. At A construct ∠BAD = ∠x (§180). 3. At B construct BE ⊥ ℓ (§178b). 4. On BE construct BF = h (§9). 5. Through F construct FG ∥ AB, intersecting AD in H (§179). 6. On GF produced construct HC = s (§9). 7. Draw BC (Asmt. 11).

Then ABCH is the required ▱.

Proof: 1. HC ∥ AB, HC = s, and AB = s (Const.). 2. ∴ ABCH is a ▱ (Asmt. 7, §223c). 3. ∠A = ∠x, and the altitude on AB = BF = h (Const.). Discussion: If ∠x = 90°, the ▱ would be a rect.

15. Given a base b, a diagonal d, and $\angle x$ the $\angle$ included by b and d of an isos. trapezoid.

To construct the isos. trapezoid.



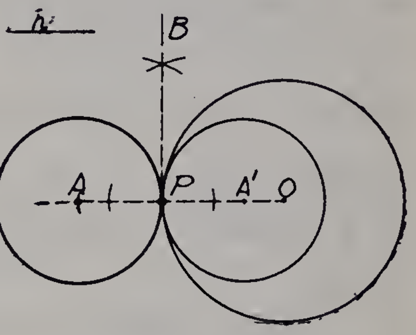
Construction: 1. Construct a $\triangle ABC$ having AB = b, AC = d, and $\angle BAC = \angle x$ (§181b). 2. Through C construct CD $\parallel$ AB (§179). 3. With A as a center and BC as a radius draw an arc intersecting CD at E (Asmt. 15). 4. Draw AE (Asmt. 11).

Then ABCE is the required isos. trapezoid.

Proof: 1. AB = b, AC = d, and $\angle BAC = \angle x$ (Const.). 2. EC $\parallel$ AB and AE = CB (Const.). 3. $\therefore$ ABCE is an isos. trapezoid (§206).

16. Given the $\odot O$, P a point on $\odot O$, and the line segment r.

To construct a $\odot$ having a radius = r and tangent to $\odot O$ at P.



Construction: 1. Draw a line through O and P (Asmt. 11). 2. On OP produced construct PA = r (§9). 3. With A as a center and PA as a radius draw a $\odot$ (Asmt. 15).

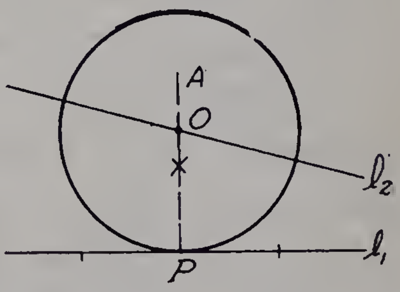
Then $\odot A$ is the required $\odot$.

Proof: 1. OP is a radius of $\odot O$ (§277). 2. PA is a radius of $\odot A$ and OA is a st. line (Const.). 3. At P construct PB $\perp$ OA (§178b). 4. PB is tangent to $\odot O$ and $\odot A$ (§298). 5. $\therefore$ $\odot A$ is tangent to $\odot O$ at P (§295). 6. PA = r (Const.).

Discussion: 1. As shown in the figure, there is another $\odot$ ($\odot A'$) with radius = r, which fulfills the given conditions. 2. If r = the radius of the given $\odot$, there would be only 1 $\odot$, externally tangent to the given $\odot$ at the given point.

17. Given the lines ℓ_1 and ℓ_2 with P a point on ℓ_1.

To construct a $\odot$ tangent to ℓ_1 at P and having its center on ℓ_2.



Construction: 1. At P construct PA $\perp \ell_1$ and intersecting ℓ_2 at O (§178b). 2. With O as a center and OP as a radius draw a $\odot$ (Asmt. 15).

Then $\odot O$ is the required $\odot$.

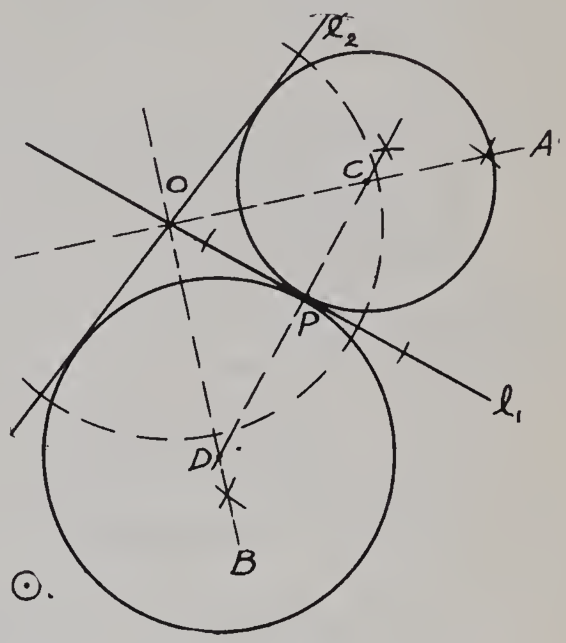
Proof: 1. OP is the radius of $\odot O$ and OP $\perp \ell_1$ at P (Const.). 2. $\therefore$ $\odot O$ is tangent to ℓ_1 at P (§298). 3. $\odot O$ has its center on ℓ_2 (Const.).

Discussion: If $\ell_2 \perp \ell_1$ at any point except P, the construction would be impossible.

18. Given the lines ℓ_1 and ℓ_2 intersecting at O and P a point on ℓ_1.

To construct a ⊙ that is tangent to ℓ_1 at P and is also tangent to ℓ_2.

Construction: 1. Construct OA and OB the bisectors of the $\angle$s formed by ℓ_1 and ℓ_2 (§177a). 2. At P construct a line $\perp$ ℓ_1, intersecting OA in C and OB in D (§178b). 3. With D and C as centers and PD and PC as radii respectively draw ⓢ D and C (Asmt. 15).



Then either ⊙D or ⊙C is the required ⊙.

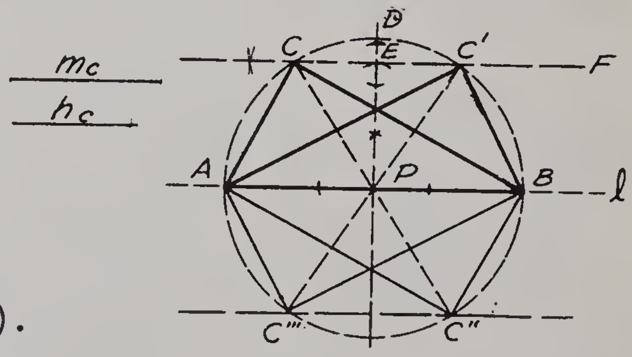
Proof: 1. OA and OB are the loci of the centers of all ⓢ tangent to ℓ_1 and ℓ_2 (Ex. 5b, page 354). 2. CD is the locus of the centers of all ⓢ tangent to ℓ_1 at P (Const., §347). 3. ∴ D and C are the centers of ⓢ tangent to ℓ_1 at P and also tangent to ℓ_2 (§350).

Discussion: If the given point is identical with the point of intersection of the given lines, the construction is impossible.

19. Given the altitude h_c and the median m_c of a rt. △.

To construct the rt. △.

Construction: 1. From any point, as P, in any line ℓ construct PA = m_c and PB = m_c (§9). 2. At P construct PD $\perp$ ℓ (§178b). 3. On PD construct PE = h_c (§9). 4. Through E construct EF $\perp$ ℓ (§179). 5. On AB as a diameter construct ⊙P intersecting EF, produced, in C and C' (Asmt. 15). 6. Draw CA, CP, CB, C'A, C'P, and C'B (Asmt. 11).



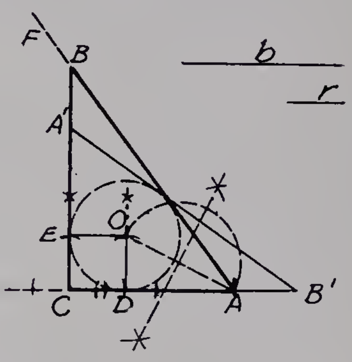
Then either △ABC or △ABC' is the required rt. △.

Proof: 1. AB is a diameter of ⊙P (Const.). 2. ∴ $\overset{\frown}{ACC'B}$ is a semicircle (§§283, 279). 3. ∴ ∠ACB and ∠AC'B are rt. $\angle$s (§319). 4. ∴ △ACB and △AC'B are rt. △s (§57). 5. PA = PB = m_c (Const., Asmt. 7). 6. ∴ PC or PC' is a median on AB, the hypotenuse (§89). 7. PC = PC' = PA = PB (§205). 8. △PC = PC' = m_c (Asmt. 7). 9. The altitude of △ABC and △ABC' = h_c (Const., Asmt. 48).

Discussion: 1. As shown in the figure, there are 2 other rt. $\triangle$ ($\triangle$ABC", $\triangle$ABC"') with $h_c < m_c$, which fulfill the given conditions. 2. If $h_c = m_c$, there would be 2 rt. $\triangle$, both isos., 1 above AB and 1 below. 3. If $h_c > m_c$, the construction would be impossible.

20. Given a leg b and the radius r of the incircle of a rt. $\triangle$.

To construct the rt. $\triangle$.



Construction: 1. Construct the square CDOE having a side = r (Ex. 1, page 224). 2. Extend CD and construct CA = b (Asmt. 13, §9). 3. With O as a center and OD as a radius draw ⊙O (Asmt. 15). 4. From A construct AF tangent to ⊙O (§336c). 5. Extend CE to meet AF in B (Asmt. 13).

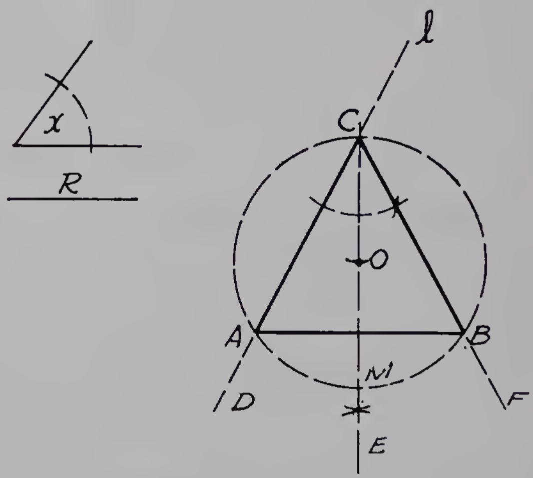
Then $\triangle$ABC is the required rt. $\triangle$.

Proof: 1. CDOE is a square (Const.). 2. ∴ $\angle$DCE, $\angle$CEO, and $\angle$CDO are rt. $\angle$ (§185). 3. ∴ $\triangle$ACB is a rt. $\triangle$ (§57). 4. EO = OD = r (§185). 5. ∴ AC and BC are tangent to ⊙O (§298). 6. AB is tangent to ⊙O (Const.). 7. ∴ r is the radius of the incircle of $\triangle$ABC (Const., §§370, 325). 8. CA = b (Const.).

Discussion: 1. As shown in the figure, there is another rt. $\triangle$ ($\triangle$A'B'C), with A'C = b and r < b, which fulfills the given conditions. 2. If $b \leqq 2r$, the construction would be impossible.

21. Given $\angle x$ the vertex $\angle$ and R the radius of the circumcircle of an isos. $\triangle$.

To construct the isos. $\triangle$.



Construction: 1. With C any point in any line ℓ as vertex and ℓ as a side, construct $\angle$DCF = $\angle x$ (§180). 2. Construct CE the bisector of $\angle$DCF (§177a). 3. On CE construct CO = R (§9). 4. With O as a center and R as a radius draw a ⊙ intersecting CD in A and CF in B (Asmt. 15). 5. Draw AB (Asmt. 11).

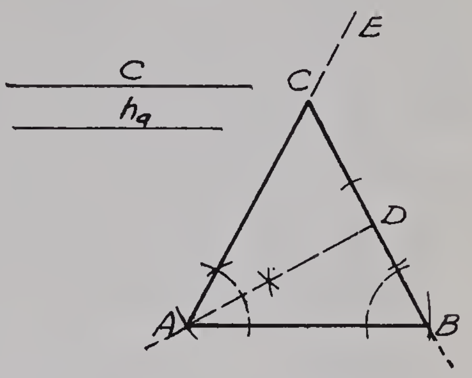
Then $\triangle$ABC is the required isos. $\triangle$.

Proof: 1. R is the radius of ⊙O and A and B are on ⊙O (Const.). 2. OC = R (Const.). 3. ∴ C is on ⊙O (§281b). 4. ∴ R is the radius of the circumcircle (§§365, 288). 5. OA =

OB and C is in CO (Const.). 6. ∴ C and O are in the ⊥ bisector of AB, and CA = CB (§345). 7. ∴ △ABC is isos. (§56).

22. Given the base c and the altitude h_a of an isos. △.

To construct the isos. △.



Construction: 1. Construct the rt. △ ADB with the hypotenuse AB = c and 1 leg AD = h_a (Ex. 13, page 195). 2. With A as a vertex and AB as 1 side, construct ∠BAE = ∠B (§180). 3. Extend BD to meet AE at C (Asmt. 13).

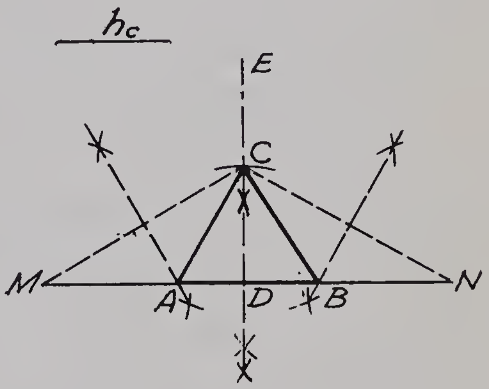
Then △ABC is the required isos. △.

Proof: 1. ∠CAB = ∠B (Const.). 2. ∴ AC = CB (§157a). 3. ∴ △ACB is isos. (§56). 4. The altitude on CB = AD = h_a and AB = c (Const.).

Discussion: If $h_a \geqq c$, the construction is impossible.

23. Given the perimeter MN and the altitude h_c of an isos. △.

To construct the isos. △.



Construction: 1. Construct DE the ⊥ bisector of MN (§177b). 2. On DE construct DC = h_c (§9). 3. Draw MC and NC (Asmt. 11). 4. Construct the ⊥ bisectors of MC and NC, intersecting MN in A and B respectively (§177b). 5. Draw CA and CB (Asmt. 11).

Then △ACB is the required isos. △.

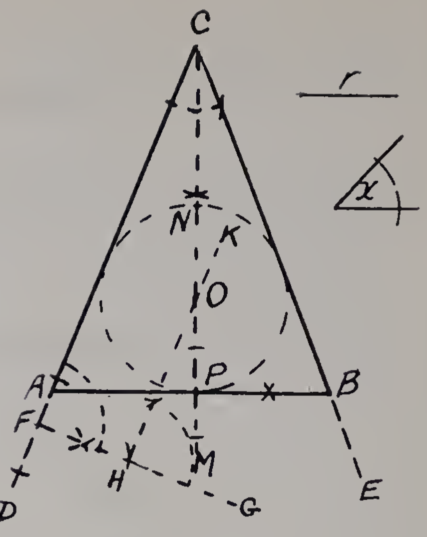
Proof: 1. CM = CN (§345). 2. ∴ ∠M = ∠N (§98a). 3. CA = AM and CB = BN (§345). 4. ∠M = ∠MCA and ∠N = ∠NCB (§98a). 5. ∴ ∠MCA = ∠NCB (Asmt. 7). 6. From 2 and 5, ∠M + ∠MCA = ∠N + ∠NCB (Asmt. 1). 7. ∠CAB = ∠M + ∠MCA and ∠CBA = ∠N + ∠NCB (§123). 8. ∴ ∠CAB = ∠CBA (Asmt. 7). 9. ∴ CA = CB (§157a). 10. ∴ △ABC is isos. (§56). 11. AB = AB (Iden.). 12. From 3 and 11, AM + AB + BN = CA + AB + BC (Asmt. 1). 13. ∴ MN = CA + AB + BC (Asmt. 9, 6). 14. The altitude on the base of △ABC = DC = h_c (Const.).

Discussion: If $h_c \geqq \frac{1}{2}$MN, the construction is impossible.

24. Given ∠x the vertex ∠ and r the inradius of an isos. △.

To construct the isos. △.

Construction: 1. With C in line CD as a vertex and CD as a side construct ∠ECD = ∠x (§180). 2. At any point, as F, in CD,



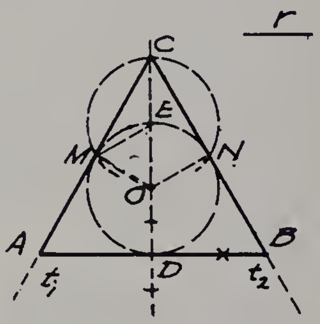
construct $FG \perp CD$ (§178b). 3. On FG construct $FH = r$ (§9). 4. Through H construct $HK \parallel DC$ (§179). 5. Construct CM the bisector of $\angle ECD$, intersecting HK in O (§177a). 6. With O as a center and r as a radius construct a ⊙, intersecting CM in N and P (Asmt. 15). 7. At P construct a tangent to ⊙O, intersecting CD in A and CE in B (§336b).

Then △ABC is the required isos. △.

Proof: 1. $\angle ACB = \angle x$, $r =$ the radius of ⊙O, and O is the intersection of KH and CM (Const.). 2. ∴ O is at a distance $= r$ from AC and BC (Const., Asmt. 48, §§343, 350). 3. ∴ AC and BC are tangent to ⊙O (Asmt. 21, §298). 4. AB is tangent to ⊙O (Const.). 5. ∴ From 1, 3, and 4, $r =$ the radius of the incircle of △ABC (§§370, 325). 6. $\angle ACP = \angle BCP$ (Const.). 7. $CP = CP$ (Iden.). 8. $\angle APC = \angle BPC$ (Const., §§311b, 14). 9. ∴ $\triangle APC \cong \triangle BPC$ (§95b). 10. ∴ $AC = BC$ (§97a). 11. ∴ △ABC is isos. (§56).

25. Given r the radius of the incircle of an equilateral △.

To construct the equilateral △.



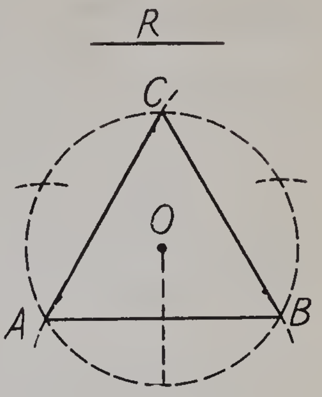
Construction: 1. Construct ⊙O with any point as a center and a radius $= r$ (Asmt. 15). 2. Draw any diameter, as DE, and extend it to C, making $EC = OE$ (§§277, 9, Asmt. 13). 3. From C construct t_1 and t_2, tangents to ⊙O at M and N respectively (§336c). 4. At D construct a tangent to ⊙O meeting t_1 and t_2 at A and B respectively (§336b).

Then △ABC is the required equilateral △.

Proof: 1. The radius of ⊙O $= r$ and AC, BC, and AB are all tangent to ⊙O (Const.). 2. ∴ r is the radius of the incircle of △ABC (§§370, 325). 3. Draw OM and ME (Asmt. 11). 4. $OM \perp AC$ (§311b). 5. ∴ △OMC is a rt. △ (§§14, 57). 6. $OE = EC$ (Const.). 7. $ME = OE$ (§205). 8. $OE = OM$ (§308a). 9. ∴ △MEO is equilateral (Asmt. 7, §57). 10. ∴ $\angle MOE = 60°$ (§§84, 120). 11. ∴ $\angle MCO = 30°$ (§161). 12. $\angle OCN = \angle MCO$ (§300). 13. ∴ $\angle MCN = 60°$ (Asmt. 9, 1). 14. $CD \perp AB$ (§311b). 15. $\triangle ADC \cong \triangle BDC$ (§95b). 16. ∴ $\angle A = \angle B$ (§98b). 17. From 13, $\angle A = \angle B = 60°$ (§120, Asmt. 2, 4). 18. ∴ △ABC is equilateral (§129).

26. Given R the radius of the circumcircle of an equilateral △.

To construct the equilateral △.



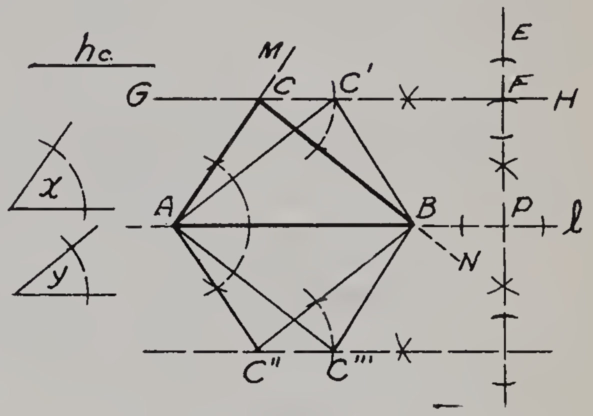
Construction: 1. With any point O as a center and R as a radius, construct ⊙O (Asmt. 15).
2. Inscribe the equilateral △ABC in ⊙O (Ex. 4, page 338).

Then △ABC is the required equilateral △.

Proof: 1. △ABC is equilateral (Const.).
2. △ABC is inscribed in a ⊙O having a radius = R (Const., §288).

27. Given ∠x and ∠y two ∠s of a △ and h_c the altitude on the side included by ∠x and ∠y.

To construct the △.



Construction: 1. At P, any point in any line ℓ, construct PE ⊥ ℓ (§178b)
2. On PE, construct PF = h_c (§9).
3. Through F, construct GH ∥ ℓ (§179).
4. With A, any point in ℓ, as a vertex and AP as a side, construct ∠PAM = ∠x (§180), letting C be the point of intersection of AM and GH.
5. With C as a vertex and HC as a side, construct ∠HCN = ∠y, letting B be the point of intersection of CN and ℓ (§180).

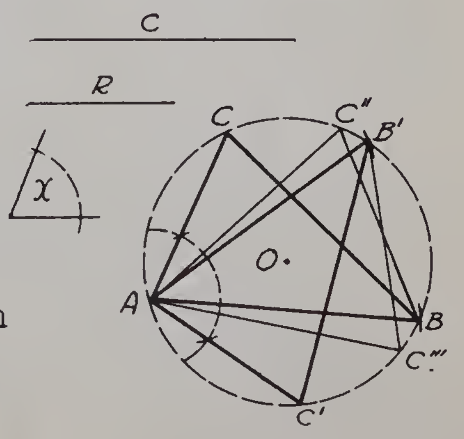
Then △ABC is the required △.

Proof: 1. ∠BAC = ∠x and ∠BCH = ∠y (Const.). 2. GH ∥ ℓ (Const.). 3. ∴ ∠ABC = ∠y (§158a, Asmt. 7). 4. The altitude on AB of △ABC = h_c (Const., §341b).

Discussion: 1. As shown in the figure, there are 3 other △s (△ABC', △ABC'', △ABC''') which fulfill the given conditions.
2. If ∠x = ∠y, there would be 2 △s, both isos., one above AB and one below. 3. If ∠x = ∠y = 45°, there would be 2 isos. rt. △s. 4. If either ∠x or ∠y = 90°, there would be 4 rt. △s.
5. If ∠x = ∠y $\geqq$ 90°, the construction would be impossible.

28. Given 1 side c, ∠x an adjacent ∠, and R the radius of the circumcircle of a △.

To construct the △.



Construction: 1. With any point O as a center and R as a radius, draw ⊙O (Asmt. 15). 2. With any point on ⊙O, as A, as a center and c as a radius, draw an arc cutting ⊙O at B and B' (Asmt. 15). 3. With A as a vertex and AB and AB' as sides respectively, construct $\angle BAC = \angle x$ and $\angle B'AC' = \angle x$ (§180). 4. Draw BC and B'C' (Asmt. 11).

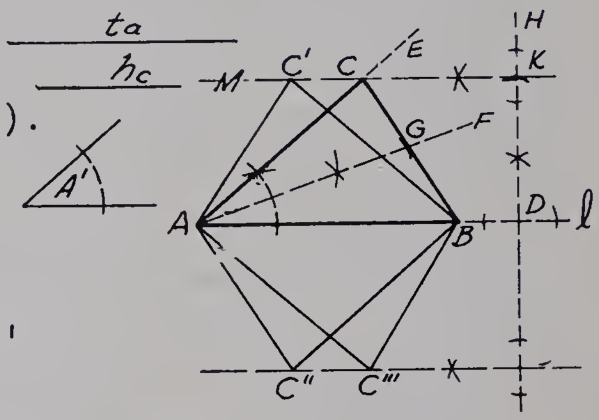
Then either $\triangle ABC$ or $\triangle AB'C'$ is the required $\triangle$.

Proof: 1. R is the radius of $\odot O$, A, B, C, B', and C' are on $\odot O$, $AB = AB' = c$, and $\angle BAC = \angle B'AC' = \angle x$ (Const.). 2. ∴ R is the radius of the circumcircle of $\triangle ABC$ or $\triangle AB'C'$ (§§365, 288).

Discussion: 1. The figure shows 2 other $\triangle$s ($\triangle ABC''$, $\triangle AB'C'''$) with $\angle x$ acute and $\frac{1}{2}c < R < c$ which fulfill the given conditions. 2. If $\angle x = 90°$ or $c = 2R$, there would be 4 rt. $\triangle$s. 3. If $\angle x = 45°$ and $c = 2R$, there would be 2 isos. rt. $\triangle$s. 4. If $\angle x = 90°$ and $c = 2R$ or if $c > 2R$, the construction would be impossible.

29. Given $\angle A'$ an $\angle$ of a $\triangle$, t_a the bisector of $\angle A'$, and h_c the altitude on a side adjacent to $\angle A'$.

To construct the $\triangle$.



Construction: 1. At D, any point in any line ℓ, construct $DH \perp \ell$ (§178b). 2. On DH, construct $DK = h_c$ (§9). 3. Through K, construct $KM \parallel \ell$ (§179). 4. With A, any point in ℓ, as a vertex and AD as a side construct $\angle DAE = \angle A'$ (§180) letting C be the point of intersection of AE and MK. 5. Construct AF, the bisector of $\angle DAC$ (§177a). 6. On AF, construct $AG = t_a$ (§9). 7. Draw CG and extend it to meet ℓ in B (Asmt. 11, 13).

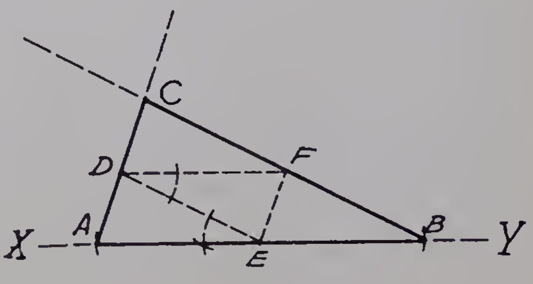
Then $\triangle ABC$ is the required $\triangle$.

Proof: 1. The altitude on side AB of $\triangle ABC = h_c$ (Const., §341b). 2. $\angle BAC$ is adjacent to side AB and $= \angle A'$ (Const.). 3. AG is the bisector of $\angle BAC$ and $AG = t_a$ (Const.).

Discussion: 1. As shown in the figure, there are 3 other $\triangle$s ($\triangle ABC'$, $\triangle ABC''$, $\triangle ABC'''$) having an $\angle = \angle A'$ adjacent to side AB and $h_c < t_a$ which fulfill the given conditions. 2. If $h_c = t_a$ and $\angle A' = 60°$, there would be 2 equilateral $\triangle$s, one above AB and one below.

30. Given D, E, and F the midpoints of the 3 sides of a $\triangle$.

To construct the $\triangle$.



Construction: 1. Draw DF, FE, and ED (Asmt. 11). 2. Through E, construct $XY \parallel DF$ (§179). 3. On XY, construct

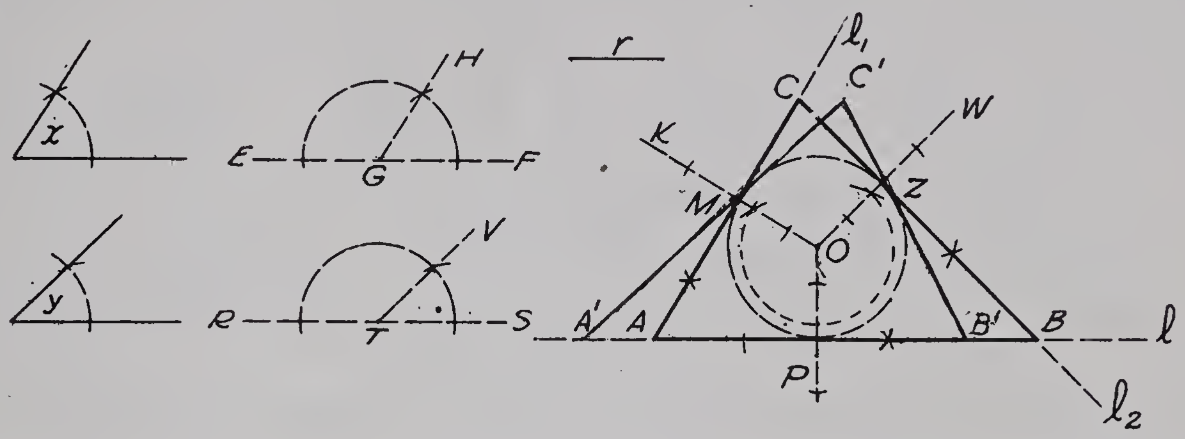
AE = EB = DF (§9). 4. Draw AD and BF and extend them to meet in C (Asmt. 11, 13).

Then $\triangle$ABC is the required $\triangle$.

Proof: 1. AB $\parallel$ DF, AE = DF, and EB = DF (Const.). 2. ∴ AEFD and EBFD are ▱s (§223c). 3. ∴ EF $\parallel$ AD, and DE $\parallel$ FB (§221a). 4. E is the midpoint of AB (§22). 5. ∴ F is the midpoint of BC and D is the midpoint of AC (§203).

Discussion: If D, E, and F lie in a st. line, the construction is impossible.

31.



Given $\angle x$, $\angle y$, and r the radius of the incircle of a $\triangle$.

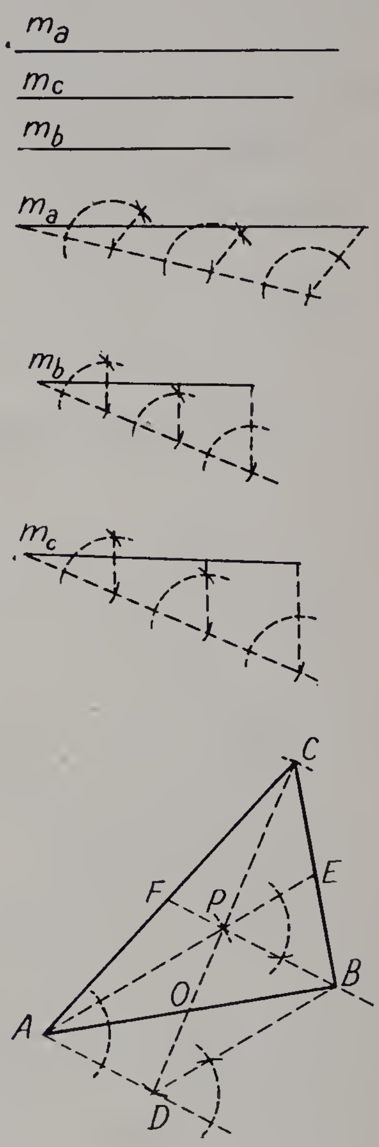
To construct the $\triangle$.

Construction: 1. With any point O as a center and a radius = r, draw $\odot$O (Asmt. 15). 2. Draw any radius, as OP (§277, Asmt. 11). 3. On any line, as EF, construct $\angle$FGH = $\angle x$ and on any line, as RS, construct $\angle$STV = $\angle y$ (§180). 4. With O as a vertex, and OP as 1 side, construct $\angle$POK = $\angle$EGH and construct $\angle$POW = $\angle$RTV, letting OK intersect $\odot$O in M and OW intersect $\odot$O in Z (§180). 5. At P, M, and Z, respectively, construct ℓ, ℓ_1, and ℓ_2 tangents to $\odot$O intersecting in A, C, and B (§336b).

Then $\triangle$ABC is the required $\triangle$.

Proof: 1. r is the radius of $\odot$O and AB, BC, and AC are tangent to $\odot$O (Const.). 2. ∴ r is the radius of the incircle of $\triangle$ABC (§§370, 325). 3. $\angle$s OMA and OPA are rt. $\angle$s (§§311b, 14). 4. $\angle$A + $\angle$OPA + $\angle$MOP + $\angle$OMA = 360° (§210). 5. ∴ $\angle$A + $\angle$MOP = 180° (Asmt. 2). 6. ∴ In quad. APOM, $\angle$A is supp. to $\angle$POM, and, similarly, in quad. BZOP, $\angle$B is supp. to $\angle$POZ (§21). 7. $\angle$POM = $\angle$EGH, $\angle$EGH is supp. to $\angle$FGH, and $\angle$FGH = $\angle x$ (Const.). 8. ∴ $\angle$A = $\angle x$ (§50c). 9. Similarly, $\angle$B = $\angle y$ (Statements 3-8).

Discussion: 1. As shown in the figure, there is another $\triangle$ ($\triangle$A'B'C') having $\angle$A' = $\angle y$, $\angle$B' = $\angle x$, and an inradius = r, which fulfills the given conditions. 2. If $\angle x$ = $\angle y$ and both are acute, there would be one $\triangle$, isos. 3. If $\angle x$ = $\angle y$ = 45°, the △ would be an isos. rt. △. 4. If either ∠x or ∠y = 90°, there would be 2 rt. △s. 5. If ∠x = 90° and ∠y = 45° or vice versa, there would be 2 isos. rt. △s. 6. If both ∠x and ∠y ≧ 90°, the construction would be impossible.



32. Construct a triangle given the three medians.

Given the medians m_a, m_b, and m_c.

To construct the △.

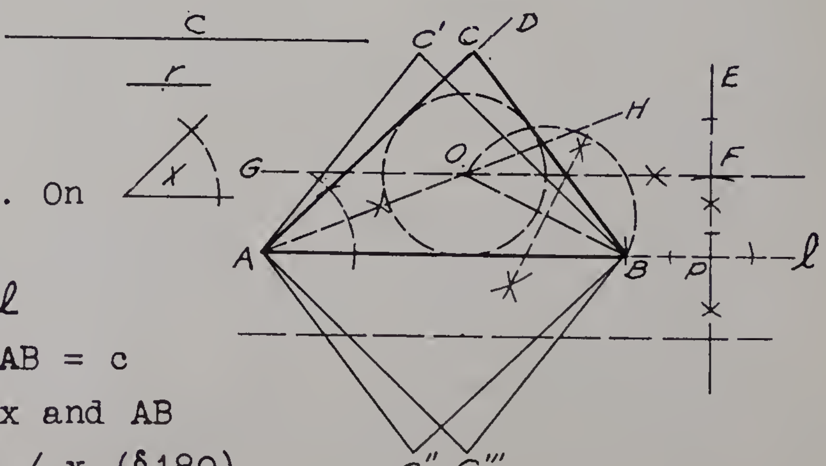
Construction: 1. Divide m_a, m_b, and m_c into 3 equal parts (§224). 2. Construct △APD having AP = $\frac{2}{3}m_a$, PD = $\frac{2}{3}m_c$, and AD = $\frac{2}{3}m_b$ (§181a). 3. Construct PB ∥ AD, and DB ∥ AP (§179). 4. Extend DP to C making PC = DP (§9). 5. Draw AB, AC and BC (Asmt. 11).

Then △ABC is the required △.

Proof: 1. Extend BP to intersect AC in F and AP to intersect BC in E (Asmt. 13). 2. DB ∥ AP and PB ∥ AD (Const.). 3. ADBP is a ▱ (§223a). 4. CO bisects AB (§217c). 5. ∴ CO is a median. 6. PD = $\frac{2}{3}m_c$ (Const.). 7. PC = PD (Const.). 8. ∴ PC = $\frac{2}{3}m_c$ (Asmt. 6). 9. PO = OD (§217c). 10. PO = $\frac{1}{2}$PC (Asmt. 6). 11. CO = m_c (Asmt. 1). 12. P is the centroid of △ABC (§363). 13. Then BF and AE are its medians (§362). 14. PB = AD = $\frac{2}{3}m_b$ (Const. and §217a). 15. PB = $\frac{2}{3}$BF (§362). 16. $\frac{2}{3}$BF = $\frac{2}{3}m_b$ (Asmt. 6). 17. BF = m_b (Asmt. 4). 18. In like manner we can prove that AE is a median and = m_a (Statements 12-18).

33. Given one side c, ∠x adjacent to c, and r the radius of the incircle of a △.

To construct the △.



Construction: 1. At any point, as P, in line ℓ, construct PE ⊥ ℓ (§178b). 2. On PE, construct PF = r (§9). 3. Through F construct FG ∥ ℓ (§179). 4. On ℓ construct AB = c (§9). 5. With A as a vertex and AB as a side, construct ∠BAD = ∠x (§180). 6. Construct AH the bisector of ∠BAD intersecting FG in O (§177a). 7. With O as a center and r as a radius draw ⊙O (Asmt. 15). 8. From B construct a tangent to ⊙O meeting AD in C (§336c).

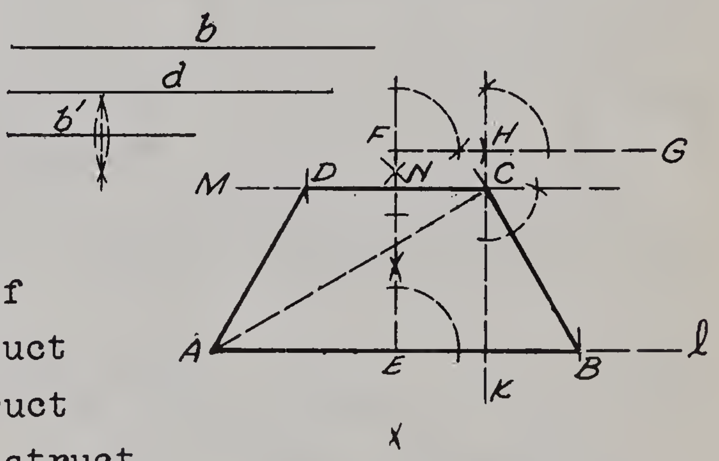
Then △ABC is the required △.

Proof: 1. ∠BAC = ∠x, AB = c, and r is the radius of ⊙O (Const.). 2. FG is the locus of points at the distance r from AB (Const., Asmt. 48). 3. AH is the locus of points equidistant from AB and AC (Const., §343). 4. ∴ O is at the distance r from AB and BC (§350). 5. ∴ AB and AC are tangents to ⊙O (§§15, 277, 298). 6. BC is tangent to ⊙O (Const.). 7. ∴⊙O is the incircle of △ABC (§§370, 325).

Discussion: 1. As shown in the figure, there are 3 other △s (△ABC', △ABC", △ABC‴) with $r < \frac{1}{2}c$ and the ∠ adjacent to the given side an acute ∠, which fulfill the given conditions. 2. If ∠x = 90°, there would be 4 rt. △s. 3. If $r \geqq \frac{1}{2}c$, the construction would be impossible.

34. Given the bases b and b', and the diagonal d of an isos. trapezoid.

To construct the isos. trapezoid.



Construction: 1. On any line ℓ construct AB = b (§9). 2. Construct EF the ⊥ bisector of AB (§177b). 3. Through F construct FG ∥ AB (§179). 4. On FG construct FH = $\frac{1}{2}$b' (§9). 5. Through H construct HK ∥ FE (§179). 6. With A as a center and d as a radius draw an arc intersecting HK in C (Asmt. 15). 7. Through C construct CM ∥ AB, intersecting EF in N (§179). 8. On CM construct CD = b' (§9). 9. Draw AD, CB, and AC (Asmt. 11).

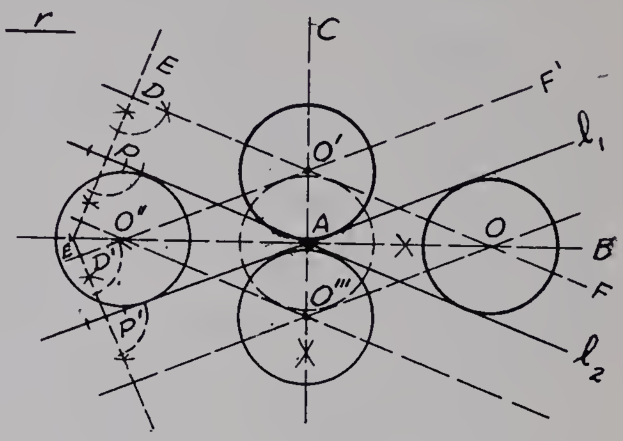
Then quad. ABCD is the required isos. trapezoid.

Proof: 1. AC = d, AB = b, and CD = b' (Const.). 2. DC ∥ AB (Const.). 3. Place quad. AEND on quad. ENCB so that NE coincides with its equal NE (Asmt. 16 and §6). 4. ND falls along NC and EA falls along EB, since ∠DNE = ∠CNE and ∠AEN = ∠BEN (Asmt. 19). 5. D falls on C and A falls on B since DN = CN and AE = EB (Const.). 6. ∴ DA = CB (§6). 7. ∴ ABCD is an isos. trapezoid (§206).

35. Given ℓ_1 and ℓ_2, 2 lines intersecting at A, and the line segment r.

To construct a ⊙ tangent to ℓ_1 and ℓ_2 and having r as a radius.

Construction: 1. Construct AB and AC the bisectors of the $\angle$s formed by the lines ℓ_1 and ℓ_2 (§177a). 2. At P and P' respectively, construct PE ⊥ ℓ_2 and P'E' ⊥ ℓ_1 (§178b). 3. On PE construct PD = r and on P'E' construct P'D' = r (§9). 4. Through D and D', respectively, construct DF ∥ ℓ_2, intersecting AB in O and D'F' ∥ ℓ_1, intersecting AC in O' (§179). 5. With O and O' as centers and r as a radius draw Ⓢ O and O' (Asmt. 15).



Then either ⊙O or ⊙O' is the required ⊙.

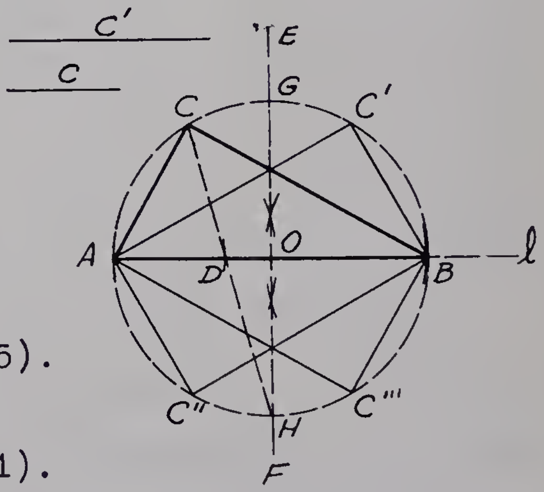
Proof: 1. AB and AC are the loci of the centers of Ⓢ tangent to ℓ_1 and ℓ_2 (Ex. 5b, page 354). 2. All points on DF and D'F' are the centers of Ⓢ with the radius r and tangent to ℓ_2 and ℓ_1 respectively (Ex. 4, page 354). 3. ∴ O and O' are the centers of Ⓢ tangent to ℓ_1 and ℓ_2 and having a radius = r (§350). 4. ∴ Ⓢ O and O' are the required Ⓢ (Const.).

Discussion: As shown in the figure, there are 2 other Ⓢ (⊙O'' and ⊙O''') which fulfill the given conditions.

36. Given c and c', segments of the hypotenuse of a rt. △ made by the bisector of the rt. ∠.

To construct the rt. △.

Construction: 1. On any line ℓ construct AD = c and DB = c' (§9). 2. Construct EF the ⊥ bisector of AB, bisecting AB in O (§177b). 3. With O as a center and a radius = OA, construct a ⊙ intersecting EF in G and H (Asmt. 15). 4. Draw HD extending it to meet ⊙O in C (Asmt. 13). 5. Draw AC and CB (Asmt. 11).



Then △ABC is the required rt. △.

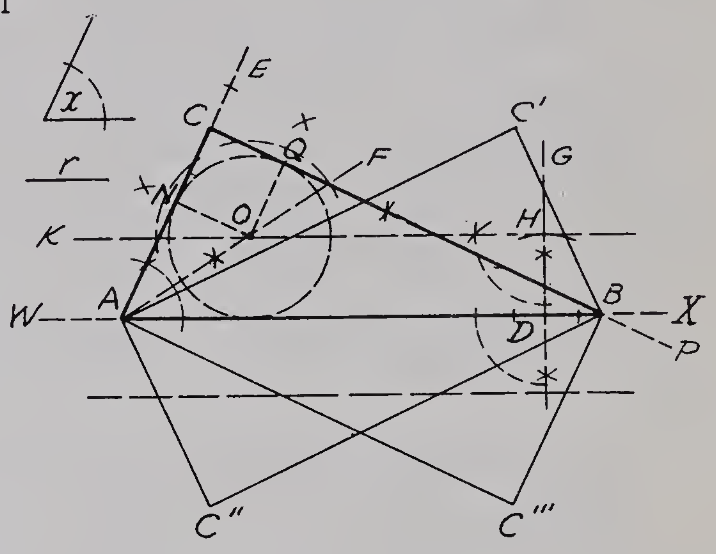
Proof: 1. AD = c and DB = c' (Const.). 2. AB is the diameter of ⊙O and C is on ⊙O (Const.). 3. ∴ △ACB is a rt. △ (Const., §346). 4. $\angle ACH \stackrel{\circ}{=} \frac{1}{2}\widehat{AH}$ and $\angle BCH \stackrel{\circ}{=} \frac{1}{2}\widehat{BH}$ (§333b). 5. $\widehat{AH} = \widehat{BH}$ (Const., §336a). 6. ∴ $\frac{1}{2}\widehat{AH} = \frac{1}{2}\widehat{BH}$ (Asmt. 4). 7. ∴ ∠ACH = ∠BCH (Asmt. 7). 8. ∴ CD is the bisector of ∠ACB (§22).

Discussion: 1. As shown in the figure, there are 3 other ⟁ (△ABC', △ABC", △ABC"') with c < c', which fulfill the given conditions. 2. If c = c', there would be 2 isos. rt. ⟁, one above AB and one below.

37. Given ∠x an acute ∠ and r the inradius of a rt. △.

To construct the rt. △.

Construction: 1. With A any point on any line WX as a vertex and AX as a side, construct ∠XAE = ∠x (§180). 2. Construct AF the bisector of ∠XAE (§177a). 3. At any point in AX, as D, construct DG ⊥ AX (§178b). 4. On DG construct DH = r (§9).
5. Through H construct HK ∥ AX, intersecting AF in O (§179).
6. Construct ON ⊥ AC (§178a).
7. With O as a center and r as a radius draw a ⊙ (Asmt. 15).
8. On AE construct NC = ON (§9).
9. Construct CP ⊥ AE at C, intersecting AX in B (§178b).



Then △ABC is the required △.

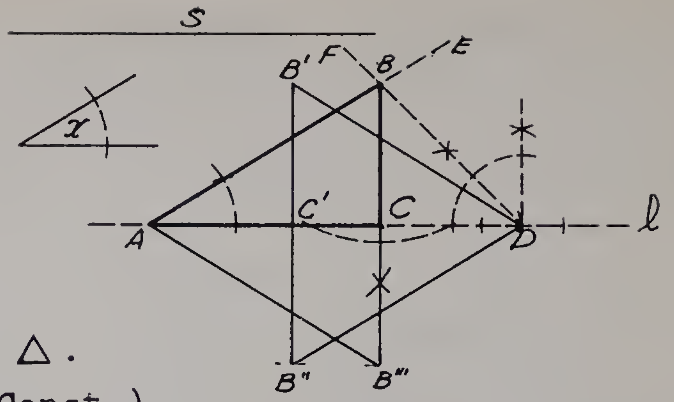
Proof: 1. ∠BAC = ∠x and ∠ACB = 1 rt. ∠ (Const.). 2. ∴ △ABC is a rt. △ (§57). 3. AF is the locus of points equidistant from AC and AB (Const., §343). 4. All points in HK are at the distance r from AB (Const., Asmt. 48). 5. O is on AF and KH, ON ⊥ AC, and the radius of ⊙O = r (Const.). 6. ∴ From 3, 4, and 5, AB and AC are tangents to ⊙O (§§15, 347, 350). 7. BC ⊥ AC (Const.). 8. ∴ BC ∥ ON (§160d). 9. From O, construct OQ ⊥ BC (§178a). 10. ∴ OQ ∥ NC (§160d). 11. ∴ OQ = NC = r (§217b, Asmt. 7). 12. ∴ Q is on ⊙O (§281b). 13. ∴ BC is tangent to ⊙O (§298). 14. ∴ r is the inradius of △ABC (§§370, 325).

Discussion: 1. As shown in the figure, there are 3 other ⟁ (△ABC', △ABC", △ABC"'), which fulfill the given conditions. 2. If ∠x = 45°, there would be 2 isos. rt. ⟁, 1 above AB and the other below.

38. Given ∠x an acute ∠ of a rt. △ and the line segment s = the sum of the legs.

To construct the rt. △.

Construction: 1. On any line segment ℓ, construct AD = s (§9). 2. With A as a vertex and AD as a side, construct ∠DAE =



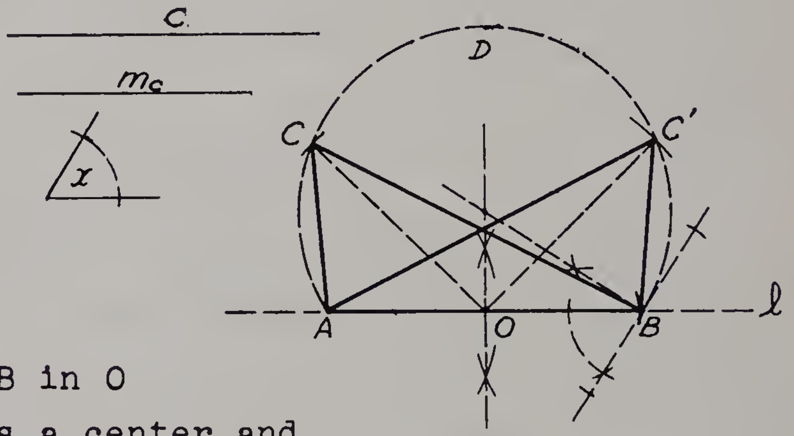
∠x (§180). 3. With D as a vertex and AD as a side, construct ∠ADF = 45°, letting B be the point of intersection of AE and DF (Ex. 1, page 192). 4. From B construct BC ⊥ DA (§178a).

Then △ABC is the required rt. △.

Proof: 1. ∠ACB is a rt. ∠ (Const.). 2. ∴ △ABC is a rt. △ (§57). 3. ∠A = ∠x (Const.). 4. ∠CDB = 45° and ∠DCB = 90° (Const.). 5. ∴ ∠DBC = 45° (§120, Asmt. 2). 6. ∴ ∠CDB = ∠DBC (Asmt. 7). 7. ∴ BC = CD (§157a). 8. AC + CD = s (Const.). 9. ∴ AC + BC = s (Asmt. 6).

Discussion: 1. As shown in the figure, there are 3 other △s (△DC'B', △ACB''', △DC'B") which fulfill the given conditions. 2. If ∠x = 45°, there would be 4 isos. rt. △s.

39. Given the base c, the median m_c, and ∠x the vertex ∠ of a △.



To construct the △.

Construction: 1. On any line ℓ construct AB = c (§9). 2. On AB as a chord construct $\overset{\frown}{ADB}$ in which ∠x can be inscribed (§372). 3. Let the ⊥ bisector of AB intersect AB in O (Asmt. 12, 17). 4. With O as a center and m_c as a radius draw arcs intersecting $\overset{\frown}{ADB}$ in C and C' (Asmt. 15). 5. Draw AC, CB, CO, AC', BC', and OC' (Asmt. 11).

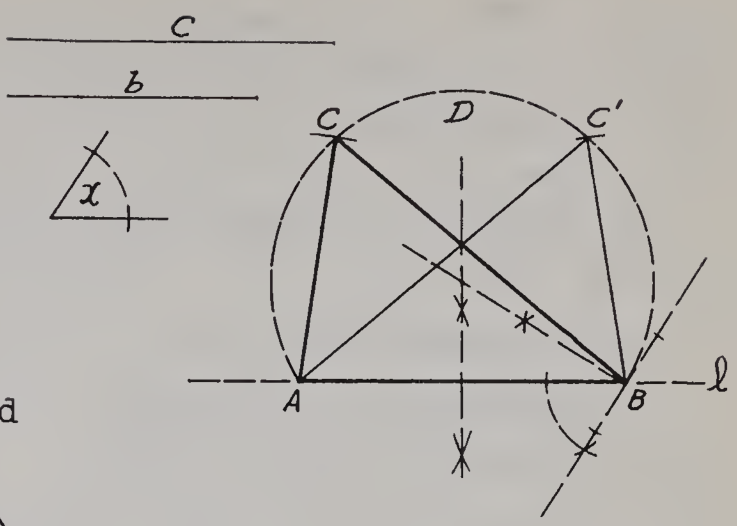
Then either △ABC or △ABC' is the required △.

Proof: 1. CO = C'O = m_c, AB = c, and ∠ACB = ∠AC'B = ∠x (Const.). 2. AO = OB (Const.). 3. ∴ CO or C'O is the median on base AB (§89).

Discussion: 1. As shown in the figure, there are 2 △s with ∠x an acute ∠ and $\frac{1}{2}c < m_c < c$ which fulfill the given conditions. 2. If ∠x = 90° or $m_c = \frac{1}{2}c$, $\overset{\frown}{ADB}$ would be a semicircle and any △ inscribed in $\overset{\frown}{ADB}$ would fulfill the given conditions. 3. If ∠x $\geqq$ 90° and $m_c > \frac{1}{2}c$, the construction would be impossible.

40. Given the base c, ∠x the ∠ at the vertex, and side b of a △.

To construct the △.



Construction: 1. On any line ℓ construct AB = c (§9). 2. On AB as a chord, construct $\overset{\frown}{ADB}$ in which ∠x can be inscribed (§372). 3. With A as a center and b as a radius draw an arc intersecting $\overset{\frown}{ADB}$ in C (Asmt. 15). 4. Draw AC and CB (Asmt. 11).

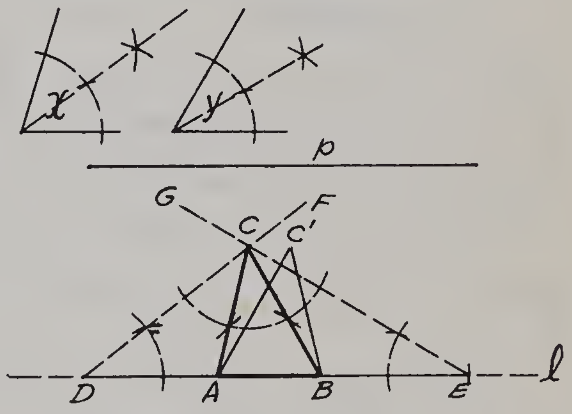
Then △ABC is the required △.

Proof: 1. AB = c, ∠ACB = ∠x, and AC = b (Const.).

Discussion: 1. As shown in the figure, there is another △ (△ABC') with BC' = b, ∠AC'B = ∠x, ∠x an acute ∠, and $\frac{1}{2}c < b < c$, which fulfills the given conditions. 2. If ∠x = 90°, there would be 2 rt. △s. 3. If ∠x = 90° and $b = \frac{1}{2}c\sqrt{2}$, there would be one isos. rt. △. 4. If ∠x = 90° and $b \geqq c$, the construction would be impossible.

41. Given the perimeter p, and ∠x and ∠y of a △.

To construct the △.



Construction: 1. On any line ℓ, construct DE = p (§9). 2. Bisect ∠x and ∠y (§177a). 3. With D as a vertex and DE as one side, construct ∠EDF = $\frac{1}{2}$∠x, and with E as a vertex and DE as a side, construct ∠DEG = $\frac{1}{2}$∠y, EG intersecting DF in C (§180). 4. With C as a vertex and DC as a side, construct ∠DCA = $\frac{1}{2}$∠x, and with C as a vertex and CE as a side, construct ∠ECB = $\frac{1}{2}$∠y (§180).

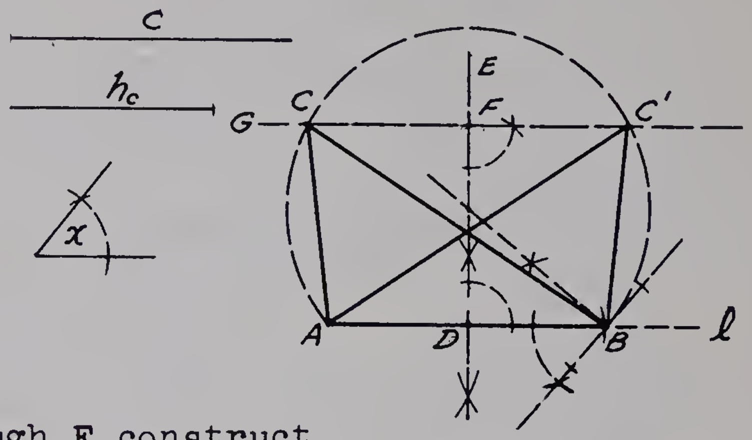
Then △ABC is the required △.

Proof: 1. ∠ADC = ∠DCA = $\frac{1}{2}$∠x (Const.). 2. ∠BAC = ∠ADC + ∠DCA (§123). 3. ∴ ∠BAC = ∠x (Asmt. 6). 4. Similarly, ∠ABC = ∠y (Statements 1-3). 5. From 1, AD = AC (§157a). 6. Similarly, BE = BC (Statements 1 and 5). 7. DA + AB + BE = p (Const., Asmt. 9). 8. ∴ AC + AB + BC = p (Asmt. 6).

Discussion: 1. As shown in the figure, there is another △, (△ABC') with ∠BAC' = ∠y, and ∠ABC' = ∠x, which fulfills the given conditions. 2. If ∠x = ∠y, there would be 1 isos. △. 3. If ∠x = ∠y = 45°, there would be 1 isos. rt. △. 4. If ∠x = ∠y = 60°, there would be 1 equilateral △. 5. If either ∠x or ∠y = 90°, there would be 2 rt. △s. 6. If ∠x = 90° and ∠y = 45° or vice versa, there would be 2 isos. rt. △s. 7. If ∠x and ∠y are both $\geqq$ 90°, the construction is impossible.

42. Given the base c, the altitude h_c, and ∠x the vertex ∠ of a △.

To construct the △.



Construction: 1. On any line ℓ construct AB = c (§9). 2. On AB as a chord construct $\overset{\frown}{AB}$ in which ∠x can be inscribed (§372). 3. On DE the ⊥ bisector of AB, construct DF = h_c (§9). 4. Through F construct FG ∥ BA, intersecting $\overset{\frown}{AB}$ in C and C' (§179). 5. Draw CA, CB, C'A, and C'B (Asmt. 11).

Then either △ABC or △ABC' is the required △.

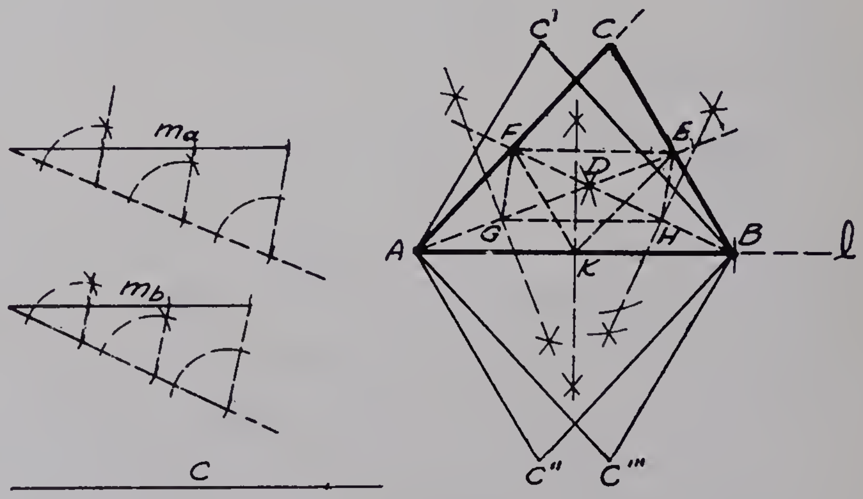
Proof: 1. ∠ACB = ∠x and AB = c (Const.). 2. The altitude of either △ABC or △ABC' is h_c (Const., Asmt. 48).

Discussion: 1. As shown in the figure, there are 2 △s with the vertex ∠ = ∠x, an acute angle, and $\frac{1}{2}c < h_c < c$, which fulfill the given conditions. 2. If ∠x = 90° and $h_c < \frac{1}{2}c$, there would be 2 rt. △s. 3. If ∠x = 90° and $h_c = \frac{1}{2}c$, there would be 1 isos. rt. △. 4. If ∠x = 90° and $h_c > \frac{1}{2}c$ or if ∠x > 90° and $h_c \geqq \frac{1}{2}c$, the construction would be impossible.

43. Given the side c and the medians m_a and m_b of a △.

To construct the △.

Construction:



1. Divide both m_a and m_b into 3 = parts (§224). 2. On any line ℓ construct AB = c (§9). 3. With A and B as centers and radii = $\frac{2}{3}m_a$ and $\frac{2}{3}m_b$ respectively draw arcs intersecting at D (Asmt. 15). 4. Draw AD and BD (Asmt. 11). 5. Extend AD to E making AE = m_a, and extend BD to F, making BF = m_b (Asmt. 13, §9). 6. Draw AF and BE and extend them to intersect at C (Asmt. 11, 13).

Then $\triangle$ ABC is the required $\triangle$.

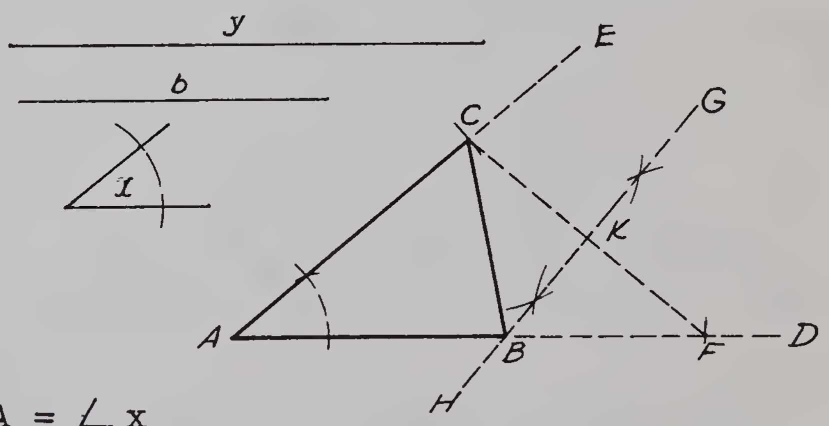
Proof: 1. AB = c, BF = m_b, and AE = m_a (Const.). 2. Bisect AD and BD at G and H respectively (§177b). 3. Draw FG, GH, HE, and EF (Asmt. 11). 4. GD = DE and FD = DH (Const. and Asmt. 7). 5. ∴ GHEF is a ▱ (§223d). 6. ∴ FE ∥ GH (§221a). 7. GH = $\frac{1}{2}$AB and GH ∥ AB (§204). 8. FE = GH (§217a). 9. ∴ FE = $\frac{1}{2}$AB (Asmt. 6). 10. FE ∥ AB (§160e). 11. Let K be the midpoint of AB (Asmt. 17). 12. Draw KE and KF (Asmt. 11). 13. AK = $\frac{1}{2}$AB (Const.). 14. ∴ From 9, FE = AK (Asmt. 7). 15. ∴ From 10 and 14, AKEF is a ▱ (§223c). 16. ∴ KE ∥ AF (§221a). 17. ∴ KE bisects BC (§203). 18. Similarly, KF bisects AC (Statements 9-17). 19. ∴ AE and BF are medians of $\triangle$ ABC (§89).

Discussion: 1. As shown in the figure, there are 3 other $\triangle$s ($\triangle$ ABC', $\triangle$ ABC", $\triangle$ ABC‴) with $m_b < m_a < c$ which fulfill the given conditions. 2. If $m_b = m_a < c$, there would be 2 $\triangle$s, both isos., 1 above AB and 1 below. 3. If m_a and m_b are both > c, the construction would be impossible.

44. Given a side b, $\angle x$ adj. b, and the line segment y = the sum of the other 2 sides of a $\triangle$.

To construct the $\triangle$.

Construction: 1. With A, any point in any line AD, as a vertex and AD as a side, construct $\angle$DAE = $\angle x$ (§180). 2. On AE construct AC = b, and on AD construct AF = y (§9). 3. Draw CF (Asmt. 11). 4. Construct GH, the ⊥ bisector of CF, intersecting CF in K and AF in B (§177b). 5. Draw BC (Asmt. 11).



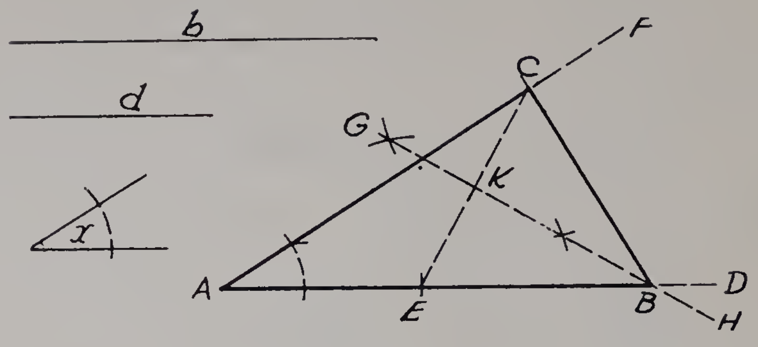
Then $\triangle$ ABC is the required $\triangle$.

Proof: 1. AC = b and $\angle$A = $\angle x$ (Const.). 2. BC = BF (Const., §345). 3. AB + BF = AF (Asmt. 9). 4. ∴ AB + BC = AF (Asmt. 6). 5. But AF = y (Const.). 6. ∴ AB + BC = y (Asmt. 7).

Discussion: 1. The figure shows a $\triangle$, with $\angle x$ acute and $\frac{1}{2}y < b < y$, which fulfills the given conditions. 2. If $\angle x = 60°$ and $b = \frac{1}{2}y$, the $\triangle$ would be an equilateral $\triangle$. 3. If $\angle x = 90°$, the $\triangle$ would be a rt. $\triangle$. 4. If $b \geqq y$, the construction would be impossible.

45. Given 1 side b, $\angle x$ the adj. $\angle$, and d the difference of the other 2 sides of a $\triangle$.

To construct the $\triangle$.



Construction: 1. On any line AD construct AE = d (§9). 2. With A as a vertex and AE as a side, construct $\angle EAF = \angle x$ (§180). 3. On AF construct AC = b (§9). 4. Draw CE (Asmt. 11). 5. Construct GH the $\perp$ bisector of CE, intersecting CE in K and AD in B (§177b). 6. Draw BC (Asmt. 11).

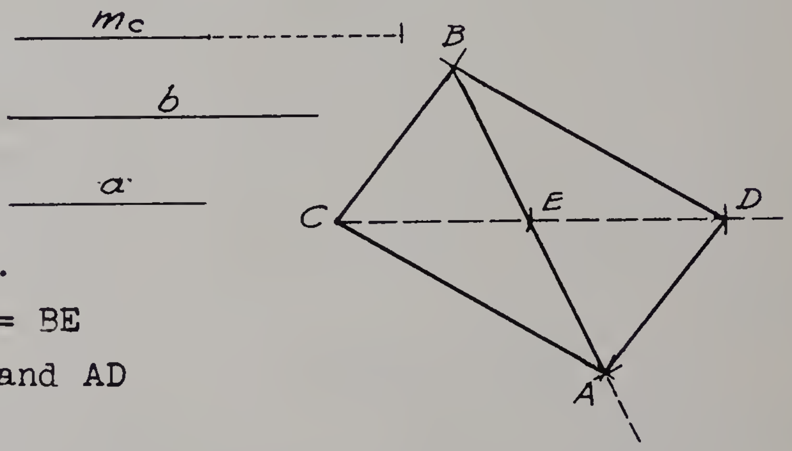
Then $\triangle$ ABC is the required $\triangle$.

Proof: 1. $\angle A = \angle x$ and AC = b (Const.). 2. BC = BE (Const., §345). 3. AB - BE = AE (Asmt. 9, 2). 4. $\therefore$ AB - BC = AE (Asmt. 6). 5. AE = d (Const.). 6. $\therefore$ AB - BC = d (Asmt. 7).

Discussion: 1. The figure shows a $\triangle$ with $\angle x$ an acute $\angle$ and $d < b$, which fulfills the given conditions. 2. If $\angle x = 90°$, the $\triangle$ would be a rt. $\triangle$. 3. If $d \geqq b$, the construction would be impossible.

46. Given 2 sides a and b and the median m_c of a $\triangle$.

To construct the $\triangle$.



Construction: 1. Construct $\triangle$ CDB with CD = $2m_c$, CB = a, and BD = b (§181a). 2. On CD construct CE = m_c (§9). 3. Draw BE (Asmt. 11). 4. Extend BE to A so that EA = BE (Asmt. 13, §9). 5. Draw AC and AD (Asmt. 11).

Then either $\triangle$ ABC or $\triangle$ ABD is the required $\triangle$.

Proof: 1. CB = a and CE = m_c (Const.). 2. BE = EA (Const.). 3. CE = ED (Const., Asmt. 7). 4. $\therefore$ CBDA is a ▱ (§223d). 5. $\therefore$ CA = BD and CB = AD (§217a). 6. BD = b and CB = a (Const.). 7. $\therefore$ CA = b and AD = a (Asmt. 7).

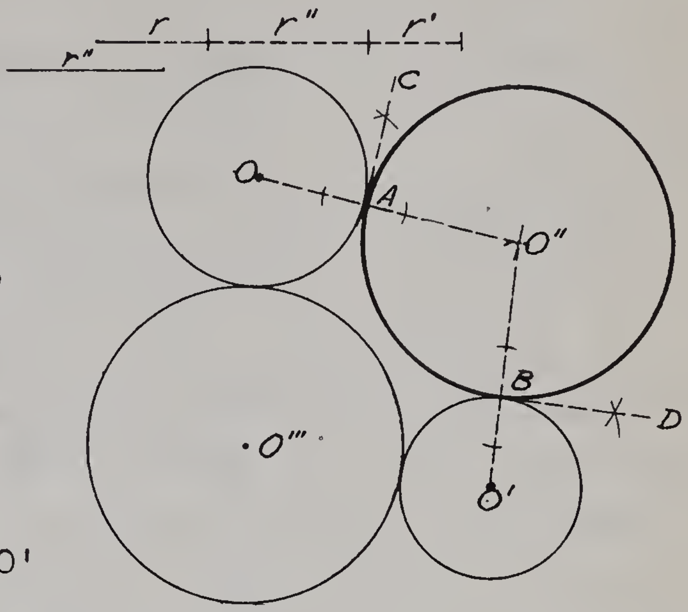
Discussion: 1. The figure shows 2 $\triangle$s with $a < m_c < b$ which fulfill the given conditions. 2. If $a = b > m_c$, the $\triangle$s would be isos. with the common side AB. 3. If $a = b \leqq m_c$ or if $(a + b) \leqq 2m_c$, the construction is impossible.

47. Given the unequal Ⓢ O and O' having the radii r and r' respectively, and the line segment r".

To construct a ⊙ tangent to O and O' and having a radius = r".

Construction: 1. With O and O' as centers and radii of (r + r") and (r' + r") respectively, draw arcs intersecting at O" (Asmt. 15). 2. With O" as a center and r" as a radius draw a ⊙ (Asmt. 15).

Then ⊙ O" is the required ⊙.



Proof: 1. The radius of ⊙ O" is r" (Const.). 2. Draw OO" and O'O", intersecting ⊙ O in A and ⊙ O' in B respectively (Asmt. 11). 3. OA + O"A = r + r" and O'B + BO" = r' + r" (Const.). 4. OA = r and O'B = r' (Given). 5. ∴ O"A = r" and O"B = r" (Asmt. 2). 6. ∴ A and B are on ⊙ O" (§281b). 7. Construct CA ⊥ OO" at A and BD ⊥ O'O" at B (§178b). 8. CA is tangent to Ⓢ O and O" and BD is tangent to Ⓢ O' and O" (§298). 9. ∴ ⊙ O" is tangent to Ⓢ O and O' (§302).

Discussion: 1. As shown in the figure, there are 2 Ⓢ (⊙ O", ⊙ O'") with a radius = r" and $r'' > r > r'$, which fulfill the given conditions. 2. If 2r" = the distance between the given Ⓢ, there would be only one ⊙. 3. If 2r" < the distance between the given Ⓢ, the construction is impossible.

4. If the given Ⓢ are tangent externally and r" = r + r', there would be 5 Ⓢ, but if $r'' \gtrless r + r'$, there would be only 4.

5. If the given Ⓢ are internally tangent, there would be 2Ⓢ except when r" = r − r', in which case there would be 3, or when r" < r − r', in which case there would be 4.

6. If the given Ⓢ are concentric and $r'' = \frac{r + r'}{2}$ or $\frac{r - r'}{2}$, there would be an infinite number of Ⓢ. For all other values of r", the construction would be impossible.

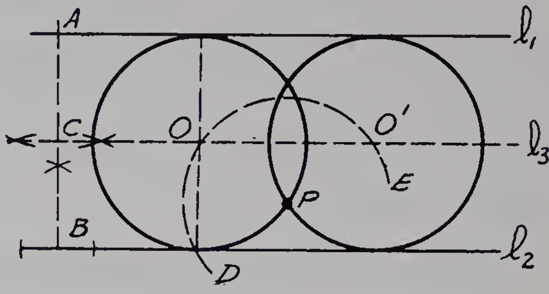
7. If the given Ⓢ are one within the other but not concentric, there are the following possible constructions where d is the distance between the centers of the given Ⓢ:

a. If $r'' = \frac{r - d - r'}{2}$ or $r'' = \frac{r - d + r'}{2}$ or $r'' = \frac{r + d + r'}{2}$, or $r'' = \frac{r + d - r'}{2}$, there would be one ⊙.

b. If $\frac{r - d - r'}{2} < r'' < \frac{r + d - r'}{2}$, there would be 2 ⓢ.

c. If $r'' < \frac{r - d - r'}{2}$ or if $r'' > r$, the construction would be impossible.

48. Given the ∥ lines ℓ_1 and ℓ_2, and a point P between ℓ_1 and ℓ_2.



To construct a ⊙ tangent to ℓ_1 and ℓ_2 which will pass through P.

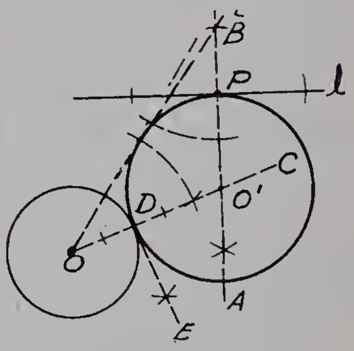
Construction: 1. At B, any point in ℓ_2, construct BA ⊥ ℓ_2 and intersecting ℓ_1 in A (§178b). 2. Construct ℓ_3, the ⊥ bisector of AB, bisecting AB in C (§177b). 3. With P as a center and CB as a radius draw $\overset{\frown}{ED}$ intersecting ℓ_3 at O and O' (Asmt. 15). 4. With O and O' as centers and CB as a radius draw ⓢO and O' (Asmt. 15).

Then either ⊙O or ⊙O' is the required ⊙.

Proof: 1. ℓ_1 ∥ ℓ_2 (Given). 2. ℓ_3 is the locus of the centers of all ⓢ tangent to ℓ_1 and ℓ_2 (Const., Ex. 5, page 354). 3. All points on $\overset{\frown}{DE}$ are at a distance = BC from P (Const., Asmt. 47). 4. OP = O'P = BC (Const.). 5. ∴ ⊙O and ⊙O' are tangent to ℓ_1 and ℓ_2 and pass through P (Const., §§350, 281b).

49. Given P, a point in a given line ℓ, and ⊙O with radius r.

To construct a ⊙ tangent to ⊙O and tangent to ℓ at P.

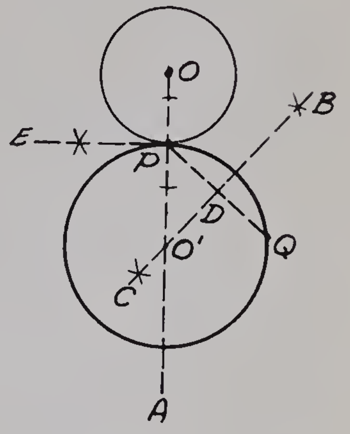


Construction: 1. Construct PA ⊥ ℓ at P (§178b). 2. On AP extended construct PB = r (Asmt. 13, §9). 3. Draw OB (Asmt. 11). 4. With O as a vertex and OB as a side, construct ∠BOC = ∠OBA, OC intersecting ⊙O in D and AB in O' (§180). 5. With O' as a center and O'P as a radius draw a ⊙ (Asmt. 15).

Then ⊙O' is the required ⊙.

Proof: 1. AP ⊥ ℓ and O'P is a radius of ⊙O' (Const.). 2. ∴ ⊙O' is tangent to ℓ at P (§298). 3. Construct DE ⊥ OO' at D (§178b). 4. ∠OBO' = ∠BOO' (Const.). 5. ∴ O'B = O'O (§157a). 6. PB = OD (Const.). 7. ∴ O'P = O'D (Asmt. 2). 8. ∴ D is on ⊙O' (§281b). 9. ∴ DE is tangent to ⓢO and O' (§298). 10. ∴ ⊙O' is tangent to ⊙O (§302).

50. Given the ⊙O with the point P on ⊙O and an external point Q.



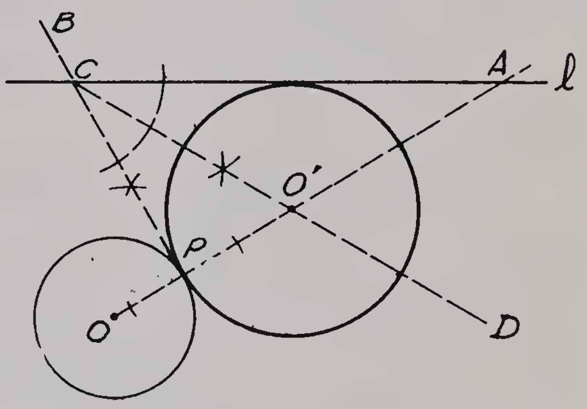
To construct a ⊙ passing through Q and tangent to ⊙O at P.

Construction: 1. Draw the line OA through O and P and draw PQ (Asmt. 11). 2. Construct BC the ⊥ bisector of PQ, intersecting OA in O' and PQ in D (§177b). 3. With O' as a center and O'P as a radius draw a ⊙ (Asmt. 15).

Then ⊙O' is the required ⊙.

Proof: 1. CB is the locus of the centers of ⓢ passing through P and Q (Ex. 1, page 354). 2. ∴ ⊙O' passes through P and Q (Const.). 3. AP is the locus of the centers of all ⓢ tangent to ⊙O at P (Ex. 6, page 354). 4. O' is the point of intersection of CB and AP and O'P is a radius of ⊙O' (Const.). 5. ∴ ⊙O' is tangent to ⊙O at P and passes through Q (§350).

51. Given ⊙O with the point P on ⊙O, and the line ℓ.



To construct a ⊙ tangent to ⊙O at P and also tangent to ℓ.

Construction: 1. Draw a line through O and P intersecting ℓ in A (Asmt. 11, 13). 2. Construct PB ⊥ OA at P, intersecting ℓ in C (§178b). 3. Construct CD, the bisector of ∠PCA, intersecting PA in O' (§177a). 4. With O' as a center and O'P as a radius draw a ⊙ (Asmt. 15).

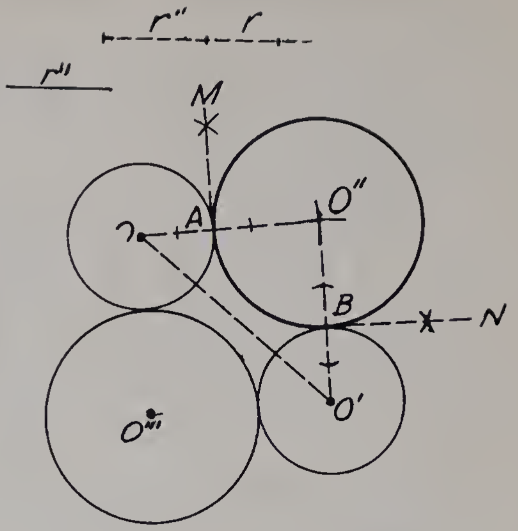
Then ⊙O' is the required ⊙.

Proof: 1. CD is the bisector of ∠PCA and is the locus of the centers of all ⓢ tangent to CA and CP (Const., Ex. 5b, page 354). 2. AP is the locus of the centers of all ⓢ tangent to ⊙O at P (Const., Ex. 6, page 354). 3. O' is the intersection of CD and PA, and ⊙O' has O'P as a radius (Const.). 4. ∴ ⊙O' is tangent to ℓ and also tangent to ⊙O at P (§350).

52. Given the = ⓢO and O' with radii = r and the line segment r".

To construct a ⊙ with radius r" and tangent to ⓢO and O'.

Construction: 1. Construct △OO'O" with OO' as a side, OO" = r + r", and O'O" = r + r" (§181a). 2. With O" as a center and r" as a radius draw a ⊙, intersecting OO" in A and O'O" in B (Asmt. 15).



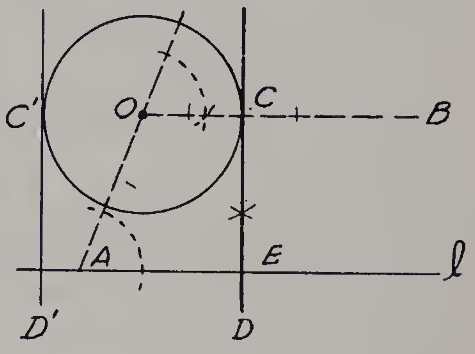
Then ⊙ O" is the required ⊙.

Proof: 1. Construct AM ⊥ OO" at A and BN ⊥ O'O" at B (§178b). 2. OO" = r + r" and O'O" = r + r" (Const.). 3. O"A = r" and O"B = r" (Const.). 4. OA = r and O'B = r (Asmt. 9, 2). 5. ∴ A is on ⊙ O and B is on ⊙ O' (§281b). 6. ∴ Ⓢ O and O" are tangent to AM at A and Ⓢ O' and O" are tangent to BN at B (§298). 7. ∴ ⊙ O" is tangent to ⊙ O and ⊙ O' (§302).

Discussion: 1. As shown in the figure, there is another ⊙ (⊙ O"') with a radius r" > r and r" > $\frac{1}{2}$ the distance between the given Ⓢ, which fulfills the given conditions. 2. If r" = $\frac{1}{2}$ the distance between the given Ⓢ, there would be only 1 ⊙. 3. If r" < $\frac{1}{2}$ the distance between the given Ⓢ, the construction would be impossible.

53. Given ⊙ O and line ℓ.

To construct a line tangent to ⊙ O and ⊥ ℓ.



Construction: 1. From any point A in ℓ, draw AO (Asmt. 11). 2. Through O construct OB ∥ ℓ, intersecting ⊙ O in C (§179). 3. At C construct CD ⊥ OB, intersecting ℓ in E (§178b).

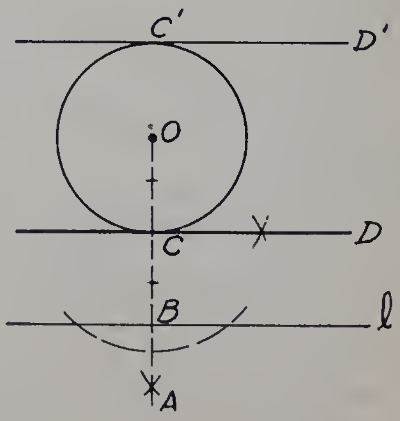
Then CD is the required line.

Proof: 1. CD ⊥ OC and C is on ⊙ O (Const.). 2. ∴ CD is tangent to ⊙ O (§298). 3. OC ∥ ℓ (Const.). 4. ∴ CD ⊥ ℓ (§159a).

Discussion: As shown in the figure, there is another line (C'D') which fulfills the given conditions.

54. Given ⊙ O and line ℓ.

To construct a line tangent to ⊙ O and ∥ ℓ.



Construction: 1. From O, construct OA ⊥ ℓ, intersecting ⊙ O in C and ℓ in B (§178a). 2. At C, construct CD ⊥ OB (§178b).

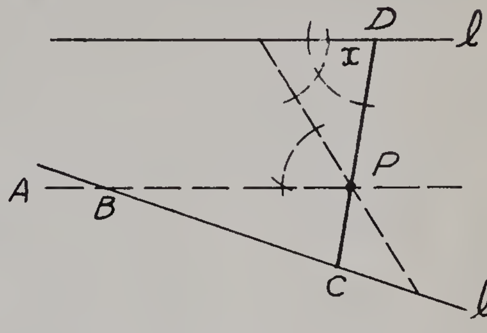
Then CD is the required line.

Proof: 1. C is on ⊙ O and CD ⊥ OB at C (Const.). 2. ∴ CD is tangent to ⊙ O (§298). 3. OB ⊥ ℓ at B (Const.). 4. ∴ CD ∥ ℓ (§160d).

Discussion: As shown in the figure, there is another line (C'D') which fulfills the given conditions.

55. Given the lines ℓ and ℓ_1, and the point P.

To construct a line through P making = ∠s with ℓ and ℓ_1.



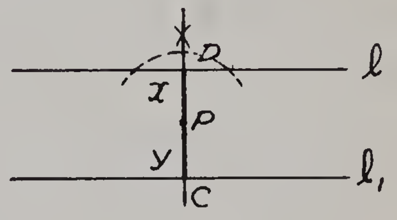
Case I. When the given lines are not ∥.

Construction: 1. Through P construct PA ∥ ℓ, intersecting ℓ_1 in B (§179). 2. On ℓ_1 construct BC = BP (9). 3. Draw CP, extending it to intersect ℓ in D (Asmt. 11, 13).

Then CD is the required line.

Proof: 1. BC = BP (Const.). 2. ∴ ∠BPC = ∠BCP (§98a). 3. PB ∥ ℓ (Const.). 4. ∴ ∠BPC = ∠x (§158a). 5. ∴ ∠BCP = ∠x (Asmt. 7).

Case II. When the given lines are ∥.



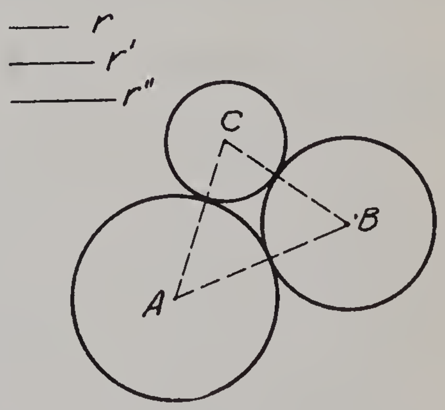
Construction: From P, construct PD ⊥ ℓ and extend it to intersect ℓ_1 in C (§178a, Asmt. 13).

Then CD is the required line.

Proof: 1. DC ⊥ ℓ_1 (§159a). 2. ∴ ∠x = ∠y (Asmt. 19).

56. Given the line segments r, r', and r".

To construct 3 ⓢ having radii r, r', and r" respectively, so that each ⊙ is tangent externally to the other 2.



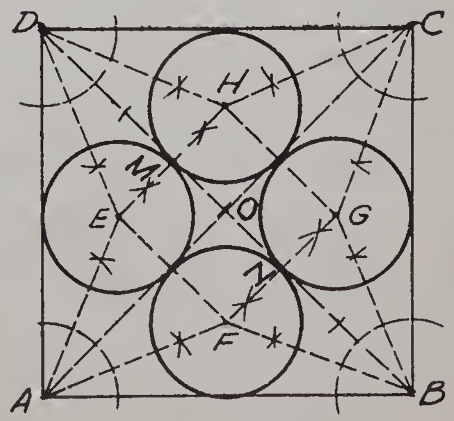
Construction: 1. Construct △ABC with AB = r" + r', AC = r" + r, and BC = r' + r (§181a). 2. With A, B, and C as centers and radii of r", r' and r respectively, draw ⓢA, B, and C (Asmt. 15).

Then ⓢA, B, and C are the required ⓢ.

Proof: (See Ex. 52, page 373).

57. Given the square ABCD.

To construct 4 = ⓢ inside the square so that each ⊙ is tangent to a side of the square and to 2 of the ⓢ.



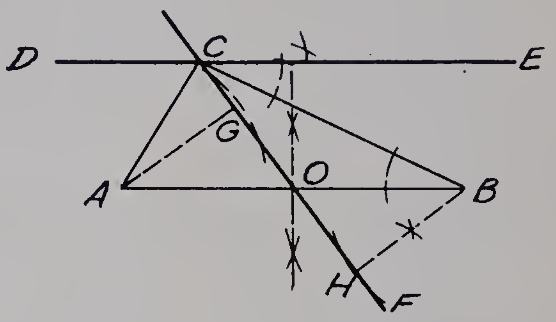
Construction: 1. Draw the diagonals AC and BD intersecting in O (Asmt. 11). 2. Inscribe ⓢE, F, G, and H in △s AOD, AOB, BOC, and COD (§369).

Then ⓢE, F, G, and H are the required ⓢ.

Proof: 1. Each ⊙ is tangent to a side of the square and to both diagonals (Const., §325). 2. △s AOB, BOC, COD, and AOD are = (§§95c, 185, 195, Ex. 8, page 206). 3. ∴ The △s can be made to coincide and the inscribed ⓢ are = (§62). 4. HM ⊥ DB and GN ⊥ DB (Const.). 5. Draw HG (Asmt. 11). 6. HM ∥ GN (§160d). 7. HM = GN (Asmt. 26). 8. ∴ MNGH is a ▱ (§223c). 9. ∴ HG ∥ DB (§217a). 10. BD ⊥ AC (§220a). 11. ∴ HG ⊥ AC (§159a). 12. ∴ From 1, ⓢH and G are tangent (Asmt. 11, 20, §302). 13. In like manner, each ⊙ is tangent to 2 others (Statements 4-12).

58. Given the △ABC.

To draw 2 lines through C, each equidistant from A and B.



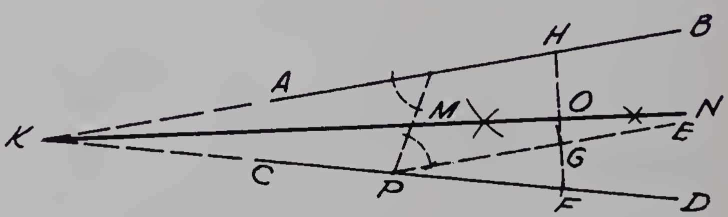
Construction: 1. Through C construct DE ∥ AB (§179). 2. Find O, the midpoint of AB (§177b). 3. Draw line CF through C and O (Asmt. 11, 13).

Then DE and CF are each equidistant from A and B.

Proof: 1. DE and CF both pass through C (Const.). 2. A and B are equidistant from DE (Const., §193). 3. Construct AG ⊥ CF and BH ⊥ CF (§178a). 4. AO = OB (Const.). 5. ∠AOG = ∠BOH (§50d). 6. △s AGO and BHO are rt. △s (Const.). 7. ∴ △s AGO and BHO are rt. △s (§57). 8. ∴ △AGO ≅ △BHO (§156a). 9. ∴ AG = BH; that is, CF is equidistant from A and B (§§97a, 15).

59. Given AB and CD, 2 nonparallel lines.

To construct the bisector of the ∠ formed by AB and CD without extending them to their point of intersection.



Construction: 1. Through P any point in CD, construct PE ∥ AB (§179). 2. On PD and PE construct = line segments PF and PG (§9). 3. Draw FG and extend it to meet AB in H (Asmt. 11, 13). 4. Construct MN the ⊥ bisector of HF, intersecting HF in O (§177b).

Then MN is the required ∠ bisector.

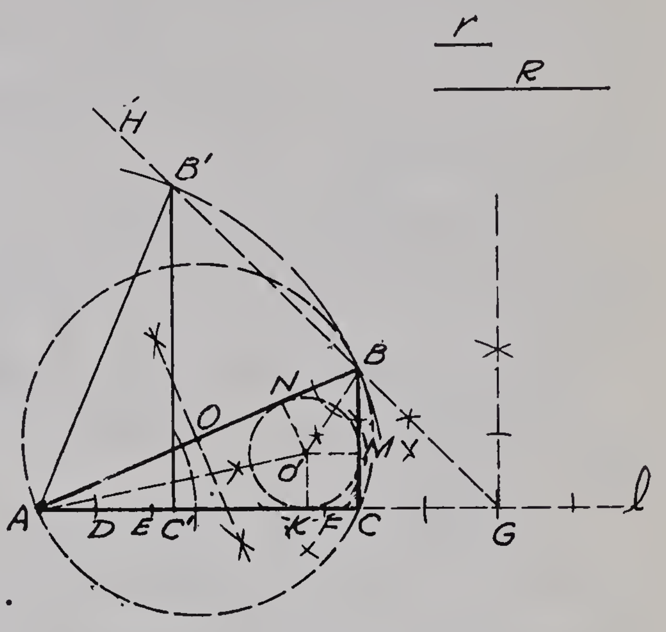
Proof: 1. Extend AB and CD to intersect at K (Asmt. 13, 35). 2. PF = PG (Const.). 3. ∴ ∠PFG = ∠PGF (§98a). 4. ∠AHO = ∠PGF (§158a). 5. ∴ ∠PFG = ∠AHO (Asmt. 7). 6. ∴ KH = KF

(§157). 7. KN will pass through K (Const., §345). 8. △KOH ≅ △KOF (§95c). 9. ∴ ∠HKO = ∠FKO (§98b).

60. Given R and r the radii of the circumcircle and incircle respectively of a rt. △.

To construct the rt. △.

Construction: 1. On any line ℓ construct AD = r, DE = r, EF = R, and FG = R (§9). 2. With G as a vertex and AG as a side, construct ∠AGH = 45° (Ex. 1, page 192). 3. With A as a center and a radius = 2R draw a ⊙ intersecting HG at B (Asmt. 15). 4. Draw AB (Asmt. 11). 5. On AB as a diameter construct ⊙O intersecting ℓ in C (Asmt. 15). 6. Draw BC (Asmt. 11).



Then △ACB is the required rt. △.

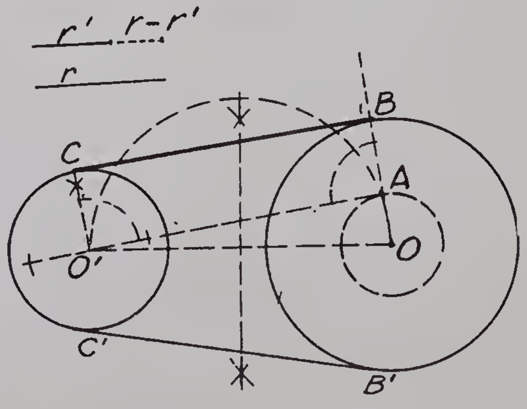
Proof: 1. ∠ACB is a rt. ∠ (§319). 2. ∴ △ACB is a rt. △ (§57). 3. R is the radius of the circumcircle and AB = 2R (Const.). 4. Inscribe ⊙O' in △ACB (§369). 5. From O', construct O'K ⊥ AC and O'M ⊥ BC (§178a). 6. MB = BN and AK = AN (§§325, 309b). 7. O'MCK is a ▱ (§§160d, 223a). 8. ∴ CK = O'M and CM = O'K (§217a). 9. AK + CK + CM + MB = AN + O'M + O'K + BN (Asmt. 1); that is, the sum of the legs of any rt. △ = the hypotenuse increased by twice the inradius. 10. AC + CG = 2R + 2r and ∠AGB = 45° (Const.). 11. BC = GC (§157). 12. AC + CB = 2R + 2r (Asmt. 6). 13. ∴ r is the inradius of rt. △ACB (Statements 9-12).

Discussion: As shown in the figure, there is another △ (△AB'C') which fulfills the given conditions.

61. Given the 2 ⓢ O and O' with radii r and r' respectively.

To construct a common external tangent to ⊙O and ⊙O'.

Construction: 1. With O as a center and a radius = r - r' draw a ⊙ (Asmt. 15). 2. From O' construct O'A tangent to this ⊙ (§336c). 3. Draw a line through O and A and extend it to meet the given ⊙O in B (Asmt. 11, 13). 4. At O' construct O'C ⊥ O'A (§178b). 5. Draw CB (Asmt. 11).



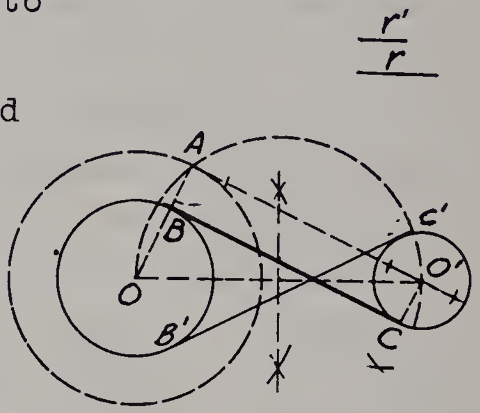
Then CB is the required common external tangent.

Proof: 1. O'C ⊥ O'A and O'A is tangent to the smaller ⊙ having O as its center (Const.). 2. BO ⊥ O'A (§311b). 3. ∴ O'C ∥ AB (§160d). 4. OB = r (Given). 5. OA = r - r' (Const.). 6. ∴ AB = r' (Asmt. 9, 6, 2). 7. O'C = r' (Given). 8. ∴ O'C = AB (Asmt. 7). 9. ∴ O'ABC is a ▱ (§ 223c). 10. ∴ CB ∥ O'A (§221a). 11. ∴ CB ⊥ O'C and CB ⊥ OB (§159a). 12. ∴ CB is tangent to ⓢ O and O' (§298).

Discussion: As shown in the figure, there is another tangent (C'B') which fulfills the given conditions.

62. Given the ⓢ O and O' with radii r and r' respectively.

To construct a common internal tangent to ⊙O and ⊙O'.



Construction: 1. With O as a center and a radius = r + r' draw a ⊙ (Asmt. 15). 2. Through O' construct O'A tangent to this ⊙ (§336c). 3. Draw OA, intersecting the given ⊙O in B (Asmt. 11). 4. At O' construct O'C ⊥ O'A (§178b). 5. Draw BC (Asmt. 11).

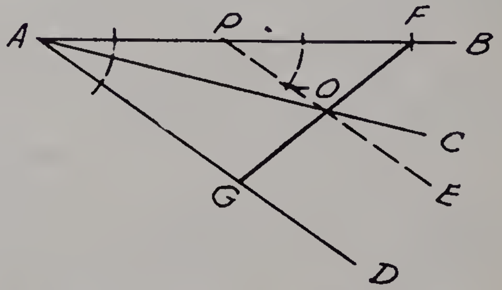
Then BC is the required common internal tangent.

Proof: 1. O'C ⊥ O'A and O'A is tangent to the larger ⊙ having O as its center (Const.). 2. ∴ OA ⊥ O'A (§311b). 3. ∴ O'C ∥ AB (§160d). 4. OA = r + r' (Const.). 5. OB = r (Given). 6. ∴ AB = r' (Asmt. 9, 6, 2). 7. O'C = r' (Given). 8. ∴ AB = O'C (Asmt. 7). 9. ∴ ABCO' is a ▱ (§159a). 10. ∴ BC ∥ AO' (§221a). 11. ∴ OB ⊥ BC and BC ⊥ O'C (§159a). 12. ∴ BC is tangent to ⓢ O and O' (§ 298).

Discussion: As shown in the figure, there is another tangent (B'C') which fulfills the given conditions.

63. Given the 3 lines, AB, AC, and AD, concurrent in the point A.

To construct a line segment terminated by AB and AD and bisected by AC.



Construction: 1. Through P, any point in AB except A, construct PE ∥ AD, intersecting AC in O (§179). 2. On AB, construct PF = AP (§9). 3. Draw FO and extend it to meet AD in G (Asmt. 11, 13).

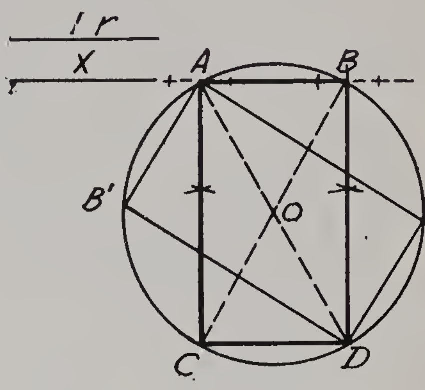
Then **FG** is the required line segment.

Proof: 1. PE ∥ AD (Const.). 2. AP = PF (Const.). 3. ∴ FO = OG (§217e).

64. Given the ⊙O with the radius r and the line segment x.

To inscribe in ⊙O a rect. having 1 side = x.

Construction: 1. With A, any point on ⊙O, as a center and a radius = x, draw an arc intersecting ⊙O in B (Asmt. 15). 2. Draw AB (Asmt. 11). 3. Draw AO and BO extending them to meet the ⊙ in D and C respectively (Asmt. 11, 13). 4. Draw AC, CD, and BD (Asmt. 11).



Then ACDB is the required rect.

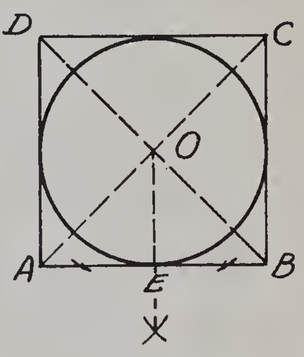
Proof: 1. AD and BC are diameters of ⊙O (Const., § 277). 2. ∴ $\overset{\frown}{ABD}$, $\overset{\frown}{BDC}$, $\overset{\frown}{DCA}$, and $\overset{\frown}{CAB}$ are semicircles (§§279, 283). 3. ∴ ∠ABD is a rt. ∠ (§319). 4. AC ⊥ AB, BD ⊥ AB, and CD ⊥ BD (§335). 5. ∴ AC ∥ BD and AB ∥ CD (§160d). 6. ∴ From 3 and 5, ABDC is a rect. (§§223a, 185). 7. AB = x, and A, B, D, and C lie on ⊙O (Const.). 8. ∴ ABDC is inscribed in ⊙O (§ 288).

Discussion: 1. As shown in the figure, there is another rect. (AB'DC') with x < r, which fulfills the given conditions. 2. If $x \geqq 2r$, the construction would be impossible.

65. Given the square ABCD.

To inscribe a ⊙ in ABCD.

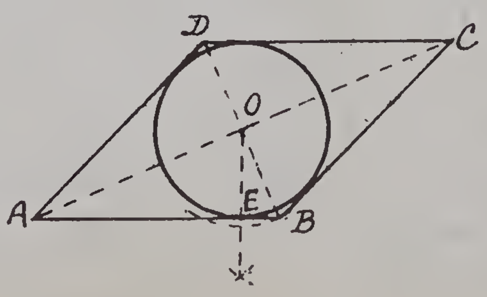
Construction: 1. Draw the diagonals AC and BD, intersecting in O (Asmt. 11). 2. From O, construct OE ⊥ AB (§178a). 3. With O as a center and OE as a radius draw ⊙O (Asmt. 15).

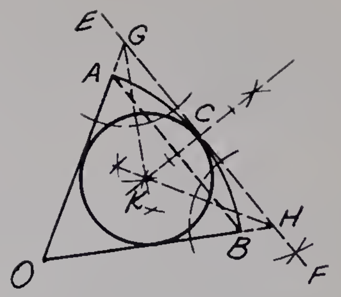


Then ⊙O is the required ⊙.

Proof: 1. AC and BD bisect the ∡ of ABCD (Ex. 7, page 206, §185). 2. O is the locus of the centers of Ⓢ tangent to AB, BC, CD, and DA (Ex. 5b, page 354, § 350). 3. OE is the distance of O from AB and OE is the radius of ⊙O (§15, Const.). 4. ∴ ⊙O is inscribed in square ABCD (§ 325).

66. (See Ex. 65, above.).





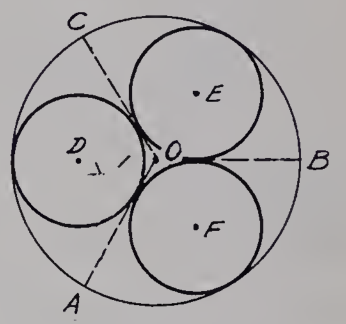
67. Given the sector OAB.

To inscribe a ⊙ in sector OAB.

Construction: 1. Bisect $\widehat{AB}$ and let C be the midpoint (§336a). 2. At C construct EF tangent to $\widehat{AB}$ (§336b). 3. Extend OA and OB to meet EF in G and H respectively (Asmt. 13). 4. In △GOH inscribe ⊙K (§369).

Then ⊙K is the required ⊙.

Proof: 1. ⊙K is tangent to OA and OB (Const.). 2. ⊙K is tangent to GH at C (Const.). 3. $\widehat{AB}$ is tangent to GH at C (Const.). 4. ∴ ⊙K is tangent to $\widehat{AB}$ (§302). 5. ∴ ⊙K is inscribed in the sector OAB (§325).



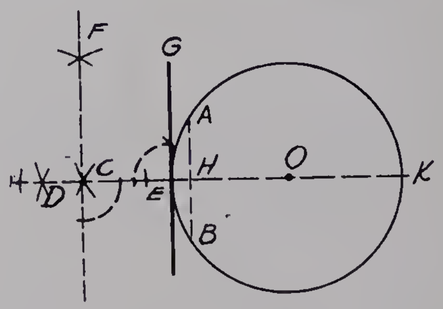
68. Given the ⊙O.

To inscribe 3 Ⓢ in ⊙O so that each ⊙ is tangent to the given ⊙ and to 2 others.

Construction: 1. At O construct ∠s AOB, BOC, and COA, each = 120° (Ex. 2, page 192 and Ex. 8, page 195). 2. Inscribe Ⓢ with centers D, E, and F in sectors AOC, COB, and BOA respectively (Ex. 67, above).

Then ⓈD, E, and F are the required Ⓢ.

Proof: 1. ⓈD, E, and F are tangent to ⊙O (Const.). 2. Place sector AOC on sector AOB so that OA coincides with its equal OA (Asmt. 16, 26). 3. OC will fall along OB, since ∠AOC = ∠AOB (Const.). 4. C will fall on B, since OB = OC (Asmt. 26). 5. $\widehat{AC}$ will coincide with $\widehat{AB}$ (§281c). 6. ∴ The inscribed Ⓢ D and F will coincide and are tangent to OA at the same point (§62). 7. ∴ ⓈD and F are tangent (§302). 8. In like manner, ⓈD and E and also ⓈE and F are tangent (Statements 2-7).



69. Given the ⊙O.

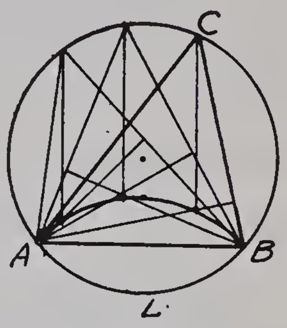
To construct a tangent to ⊙O without using any points or lines within ⊙O.

Construction: 1. With any 2 points on the ⊙, as A and B, as centers and any convenient radius, draw 2 arcs intersecting in C (Asmt. 15). 2. Repeat Step 1 with a different radius, drawing 2 arcs intersecting in D (Asmt. 15). 3. Draw DC and extend it to meet ⊙O in E (Asmt. 11, 13). 4. At C, construct CF ⊥ DE (§178b). 5. Through E construct EG ∥ CF (§179).

Then GE is the required tangent.

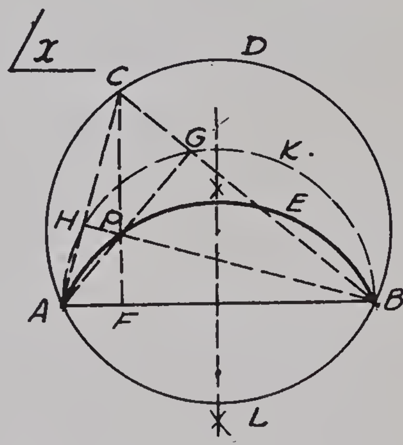
Proof: 1. Draw AB (Asmt. 11). 2. Extend DE to intersect AB in H and $\odot O$ in K (Asmt. 13). 3. DK is the $\perp$ bisector of AB (§140). 4. DK passes through O (§292). 5. CF $\parallel$ EG and CF $\perp$ DK (Const.). 6. $\therefore$ GE $\perp$ DK (§159a). 7. $\therefore$ GE is tangent to $\odot O$ (§298).

70. The following is given as a suggestion of the way in which the required locus may be found.



If we think of the acute triangles in the problem as having the base and vertex angle constant, we see that the locus of the vertex angle is found by §372. In the figure, AB and $\angle C$ are constant. By sketching several $\triangle$s as shown, it will be seen that a line through their orthocenters is = to $\overset{\frown}{ALB}$, and that the arc thus drawn is the required locus. The locus may be stated as follows:

The locus of the orthocenters of acute triangles with a given base and vertex angle is the arc of a circle on the same side of the base as the given vertex, having the given base as a chord, and equal to the minor arc which the given base cuts off on the circle circumscribed about the triangles, the arc being one in which the supplement of $\angle x$ can be inscribed.



Given base AB and vertex $\angle x$ of an acute $\triangle$; $\overset{\frown}{ADB}$ in which $\angle x$ can be inscribed; and $\overset{\frown}{AEB} = \overset{\frown}{ALB}$, in which the supplement of $\angle x$ can be inscribed.

To prove that $\overset{\frown}{AEB}$ is the locus of the orthocenters of acute $\triangle$s having AB as a base and $\angle x$ as a vertex $\angle$.

PART I.

Proof: 1. Let P be any point in $\overset{\frown}{AEB}$ and draw BP (Asmt. 11). 2. On AB as a diameter, construct semicircle AKB (§177b and Asmt. 15). 3. Extend BP to intersect semicircle AKB in H (Asmt. 13). 4. Extend AH to intersect $\overset{\frown}{ADB}$ in C (Asmt. 13). 5. Draw CB (Asmt. 11). 6. Draw CP and extend it to intersect AB in F (Asmt. 11, 13). 7. $\angle AHC$ is a st. $\angle$ (§16). 8. $\angle AHB$ is a rt. $\angle$ (§319). 9. $\therefore$ $\angle BHC$ is a rt. $\angle$ (Asmt. 2). 10. $\angle APB = \angle HPG$ (§50d). 11. $\angle HCG + \angle APB$ = 1 st. $\angle$ (Const., Given). 12. $\angle HCG + \angle HPG$ = 1 st. $\angle$ (Asmt. 6). 13. $\therefore$ $\angle HCG + \angle HPG + \angle BHC + \angle PGC$ = 2 st. $\angle$s (§210). 14. $\angle BHC + \angle PGC$ = a st. $\angle$

(Asmt. 2). 15. ∴ From 9, ∠PGC = a rt. ∠ (Asmt. 2). 16. ∴ AG ⊥BC (§14). 17. BH and AG are altitudes of △ABC (§90). 18. ∴ CF is an altitude of △ABC (§360). 19. ∴ Any point P on $\widehat{AEB}$ is the point of intersection of the altitudes of an acute △ having the base AB and the vertex ∠x.

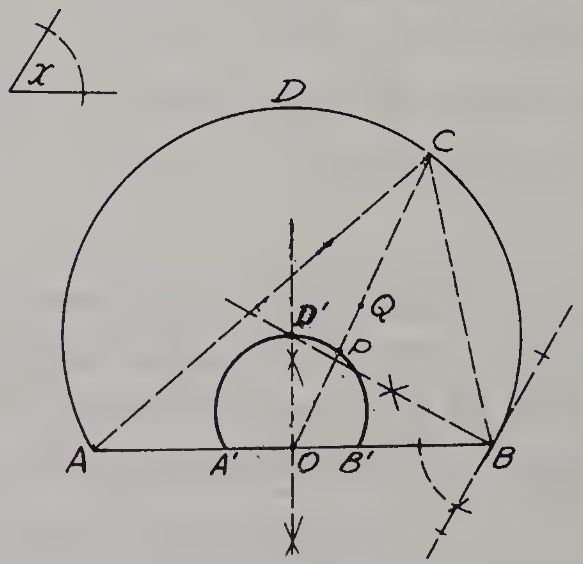
PART II.

Proof: 1. Let P be the intersection of the altitudes BH, AG, and CF of △ACB (§360). 2. ∴ ∠CHP = a rt. ∠ and ∠CGP = a rt. ∠ (§§90, 14). 3. ∠CHP + ∠CGP + ∠HPG + ∠HCG = 2 st. ∠s (§210). 4. ∴ ∠HPG + ∠HCG = a st. ∠ (Asmt. 2); that is, ∠s HPG and HCG are supp. 5. ∠HPG = ∠APB (§50d). 6. ∴ ∠s APB and HCG are supp. (Asmt. 6). 7. ∴ ∠APB can be inscribed in $\widehat{AEB}$ (Given); that is, P, the point of concurrency of the altitudes of an acute △ ACB having a base AB and a vertex ∠x, lies on $\widehat{AEB}$.

71. The locus of the centroids of triangles with a given base and a given vertex angle is the arc of a circle, all points of which are one third the distance from the midpoint of the base to the arc of a circle having the given base as a chord and in which the given vertex angle can be inscribed.

Given base AB and vertex ∠x of a △; $\widehat{ADB}$ in which ∠x can be inscribed; and $\widehat{A'D'B'}$, all points of which are one third the distance from O, the midpoint of AB, to $\widehat{ADB}$.

To prove that $\widehat{A'D'B'}$ is the locus of the centroids of △s having AB as a base and whose vertex ∠s can be inscribed in $\widehat{ADB}$.



PART I.

Proof: 1. Let P be any point in $\widehat{A'D'B'}$ and draw a line through O and P intersecting $\widehat{ADB}$ in C (Asmt. 11). 2. Draw CA and CB (Asmt. 11). 3. OP = $\frac{1}{3}$OC (Given). 4. The medians on sides AC and BC of $\triangle ABC$ will pass through P (§362). 5. Hence any point on $\overset{\frown}{A'D'B'}$ is the centroid of a $\triangle$ having AB as a base and $\angle x$ as a vertex $\angle$ (Statements 3 and 4).

PART II.

Proof: 1. Let Q be the centroid of $\triangle ABC$ having AB as a base and $\angle x$ as a vertex $\angle$, and draw a line through C and Q intersecting $\overset{\frown}{A'D'B'}$ in P (Asmt. 11). 2. CQ will pass through O (§§89, 363). 3. $OQ = \frac{1}{3}OC$ (§362). 4. $OP = \frac{1}{3}OC$ (Given). 5. OP = OQ (Asmt. 7). 6. P coincides with Q (§6); that is, the centroid of a $\triangle$, having AB as a base and $\angle x$ as a vertex $\angle$, lies on $\overset{\frown}{A'D'B'}$.

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1. The locus of points in space equidistant from two given points is the plane perpendicular to and bisecting the line joining them.

2. The locus of points in space at a given distance from a given point is a sphere having the given point as a center and the given distance as a radius.

3. The locus of points in space at a given distance from a given line is a right circular cylindrical surface having the given line as an axis and the given distance as a radius.

4. The locus of points in space at a given distance from a given plane is a pair of planes parallel to the given plane and at the given distance from it.

5. The locus of a point in space equidistant from all points on a circle is the line through the center of the circle perpendicular to the plane of the circle.

6. The locus of points in space equidistant from the vertices of a triangle is the line through the circumcenter perpendicular to the plane of the triangle.

7. The locus of points in space at a given distance from each of two given points is in general a circle formed by the intersection of two spheres having the given points as centers and the given distance as radii.

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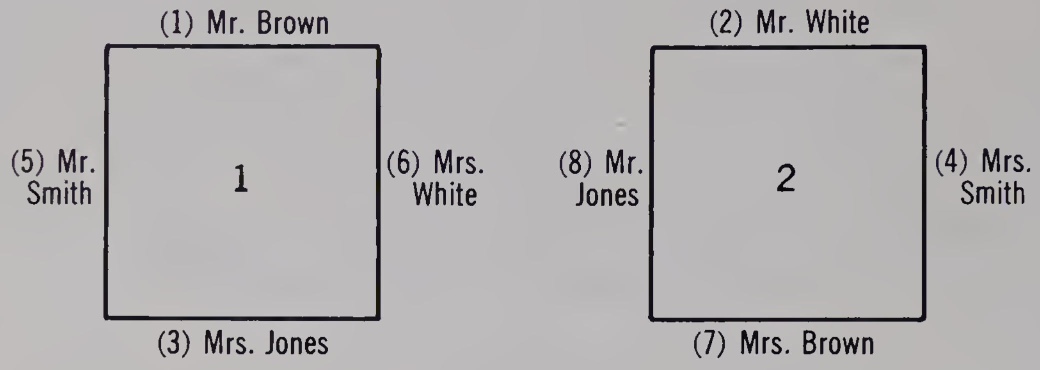
1. There were six days, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday on which the farmer could go to town. He would go in an afternoon.

He could not go on Monday or Saturday because the stockyards were not open on these days. He could not go on Wednesday or Saturday because the bank was not open in the afternoons of these days. He could not go on Tuesday, Thursday, or Saturday because the dentist's office was not open on these days. Therefore he went on Friday.

2.



Mr. Brown and Mr. White sat at different tables; Mr. Brown at Table 1, and Mr. White at table 2. Mrs. Jones sat opposite Mr. Brown at table 1. Mrs. Smith next to Mr. White. Then Mr. Smith sat at table 1. Since Mrs. White could not sit at table 2, she sat at table 1. Mr. Jones and Mrs. Brown then sat at table 2. Since Mr. White's daughter sat opposite him, the daughter was Mrs. Brown. Each of the arrangements of the people at the tables can be rotated or arranged in reverse order.

3. Mr. Robinson lived in Detroit (c). Another passenger lived midway between Chicago and Detroit (d and g), while the third passenger lived in Chicago (h). Since the quotient of 8000 divided by 3 is a never-ending decimal, the passenger living midway between Chicago and Detroit was neither Mr. Robinson (c) nor Mr. Jones (e and g). He therefore was Mr. Smith and the passenger living in Chicago was Mr. Jones. Then the engineer's name was Jones (h). Thus the fireman's name was not Jones, nor could it be Smith (f). It therefore was Robinson, and the brakeman's name was Smith.

PAGE 377 (Review Questions)

1. §338. 2. §339. 3. §341. 4. §343. 5. §345. 6. §346. 7. equilateral. 8. center. 9. A line ⊥ to the given line at the given point.

10. (1) Prove that every point of the figure is a point of the locus. (2) Prove that every point of the locus is a point of the figure or that every point outside the figure is not a point of the locus (§342).

11. A compound locus is determined by the intersection of two or more loci (§350).

12. Circumcenter (§357); incenter (§359); orthocenter (§361); centroid (§363).

13. One (§366). 14. Two (§367). 15. §370; §365.

16. Segment of a circle (§371). 17. 1 : 2 (§362).

18. On the line passing through the point and the center of the circle (§340).

19. The common internal tangent.

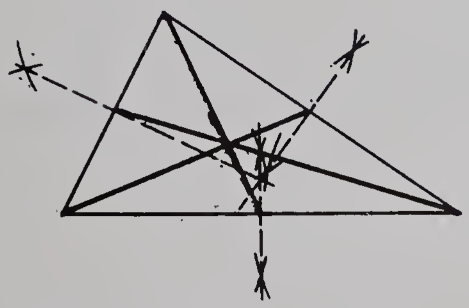
20. A circle whose diam. = $\frac{1}{2}$AC. 21. $\frac{4}{3}\sqrt{3}$ in.

PAGE 379 (Test. 28)

1. T. 2. T. 3. F. 4. F. 5. T. 6. T. 7. T.
8. T. 9. F. 10. T.

PAGE 379 (Test 29)

1.



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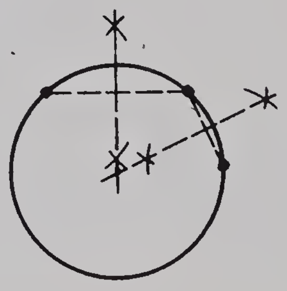

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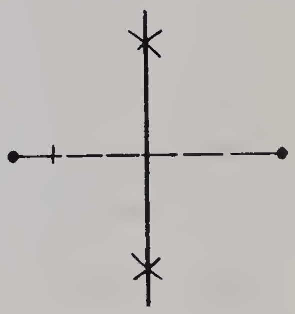

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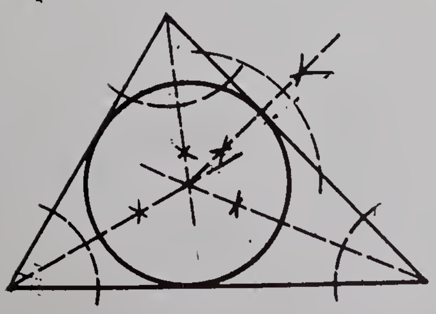

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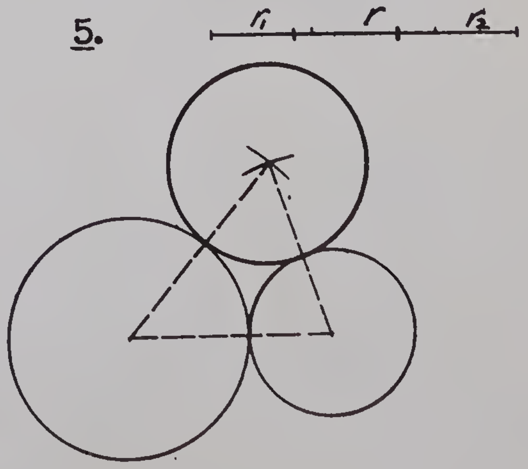

Chapter 12. Proportion and Proportional Line Segments

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1. $\frac{2}{3}$.   2. $\frac{2}{5}$.   3. $\frac{m}{2}$.   4. $\frac{1}{4x}$.   5. $\frac{5ac}{6b}$.   6. $\frac{x - 4}{4x}$.

7. Yes. 8. Yes. 9. No. 10. No. 11. Yes. 12. Yes. 13. Yes; yes.

14. Solving, $3x = 48$ and $x = 16$.

15. Solving, $24x = 72$ and $x = 3$.

16. Solving, $5x + 15 = 28$ and $x = 2\frac{3}{5}$.

17. Solving, $8x = 35 - 7x$ and $x = 2\frac{1}{3}$.

18. Solving, $2x - 1 = 9$ and $x = 5$.

19. Solving, $12x + 8 = 5x - 20$ and $x = -4$.

20. Solving, $cx = ab$ and $x = \frac{ab}{c}$.

21. Solving, $x^2 = 16$ and $x = \pm 4$.

22. Solving, $6bx = 3b$ and $x = \frac{1}{2}$.

23. 3 is the first, 4 is the second, 6 is the third, and 8 is the fourth; $a + b$ is the first, $c + d$ is the second, m is the third, and n is the fourth; $a^2 + 1$ is the first, a is the second, $a - 1$ is the third, and 5 is the fourth.

24. 6 and 15 are the means, 5 and 18 are the extremes; 5 and 7 are the means, $4x$ and 8 are the extremes; b and c are the means, a^2 and d are the extremes.

25. $4 : x = x : 9$.   26. $2 : 8 = 8 : 32$.

27. a. $\frac{1}{7} = \frac{4}{x}$; $x = 28$.
   b. $\frac{7}{21} = \frac{x}{6}$; $21x = 42$; $x = 2$.
   c. $\frac{5}{x - 3} = \frac{2}{2x + 1}$; $10x + 5 = 2x - 6$; $8x = -11$; $x = -1\frac{3}{8}$.
   d. $\frac{12.5}{x} = \frac{x}{2}$; $x^2 = 25$; $x = \pm 5$.

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1. The mean proportional between the first and fourth terms.

2. m and t are the extremes; n and s, the means.

3. A proportion may be written $a : b = c : d$. Since b and c are in the middle, they are called the means. Since a and d are on the ends, they are called the extremes.

4. a. Solving, $5x = 28$; whence $x = 5\frac{3}{5}$.
b. Solving, $16x = 21$; whence $x = 1\frac{5}{16}$.
c. Solving, $5x - 25 = x + 3$; whence $4x = 28$; and $x = 7$.
d. Solving, $x^2 = 64$; whence $x = \pm 8$.
e. Solving, $x^3 - 6x = x^2$, or $x^3 - x^2 - 6x = 0$.
Then $x(x - 3)(x + 2) = 0$; $x = 3$ and -2.
(0 is not a root.)
f. Solving, $ax + bx = ab + bx$; whence $ax = ab$, and $x = b$.

5. See §389.

6. $\frac{9}{x} = \frac{x}{16}$; whence $x^2 = 144$, and $x = \pm 12$.
$\frac{7}{x} = \frac{x}{63}$; whence $x^2 = 441$, and $x = \pm 21$.

7. a. $\frac{2}{3} = \frac{4}{x}$; whence $2x = 12$, and $x = 6$.

b. $\frac{4}{6} = \frac{10}{x}$; whence $4x = 60$, and $x = 15$.

c. $\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{\frac{1}{4}}{x}$; whence $\frac{1}{2}x = \frac{1}{12}$, and $x = \frac{1}{6}$.

d. $\frac{m}{n} = \frac{p}{x}$; whence $mx = np$, and $x = \frac{np}{m}$.

8. a. $\frac{5}{2} = \frac{35}{14}$. c. $\frac{9}{14} = \frac{22.5}{35}$. e. $\frac{b}{a + b} = \frac{d}{c + d}$.

b. $\frac{3}{8} = \frac{9}{24}$. d. $\frac{n}{m} = \frac{t}{s}$. f. $3 : x = 5 : y$.

9. (a) a. $\frac{2}{14} = \frac{5}{35}$. c. $\frac{14}{35} = \frac{9}{22.5}$. e. $\frac{a + b}{c + d} = \frac{b}{d}$.

b. $\frac{8}{24} = \frac{3}{9}$. d. $\frac{m}{s} = \frac{n}{t}$. f. $x : y = 3 : 5$.

(b) a. $\frac{7}{5} = \frac{49}{35}$. c. $\frac{23}{9} = \frac{57.5}{22.5}$. e. $\frac{a + 2b}{b} = \frac{c + 2d}{d}$.

b. $\frac{11}{3} = \frac{33}{9}$. d. $\frac{m + n}{n} = \frac{s + t}{t}$. f. $(x + 3) : 3 = (y + 5) : 5$.

(c) a. $\frac{-3}{5} = \frac{-21}{35}$. c. $\frac{5}{9} = \frac{12.5}{22.5}$. e. $\frac{a}{b} = \frac{c}{d}$.

b. $\frac{5}{3} = \frac{15}{9}$. d. $\frac{m - n}{n} = \frac{s - t}{t}$. f. $(x - 3) : 3 = (y - 5) : 5$.

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1. a. $\frac{x}{y} = \frac{3}{2}$. b. $\frac{x}{y} = \frac{7}{5}$. c. $\frac{x}{y} = \frac{1}{4}$. d. $\frac{x}{y} = \frac{1}{5}$. e. $24x = 35y$; whence $\frac{x}{y} = \frac{35}{24}$. f. $\frac{x}{y} = \frac{b}{a}$. g. $\frac{x}{y} = \frac{c + d}{a + b}$.

h. $mx - rx = sy - ny$; whence $(m - r)x = (s - n)y$, and $\frac{x}{y} = \frac{s - n}{m - r}$.

2. a. $\frac{x}{y} = \frac{4}{3}$. b. $\frac{x}{y} = \frac{5}{m}$. c. $\frac{x}{y} = \frac{4}{1}$. d. $\frac{x}{y} = \frac{1}{7}$.

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1. Possible answers are: a. 1 ft., 2 ft., 6 in. b. 2 cm., 1 cm., $\frac{1}{2}$ cm. c. $\frac{1}{6}$ ft., $\frac{1}{12}$ ft., $\frac{1}{24}$ ft. d. 0.1 in., 0.05 in., 0.02 in. 2. a. 0.1 in. b. 0.01 in. c. 0.001 in. d. 0.0001 in. 3. No. 4. No. 5. Yes. 6. (1) $\frac{CD}{CE} = \frac{DA}{EB}$. (2) $\frac{DA}{CD} = \frac{EB}{CE}$. (3) $\frac{CD + DA}{DA} = \frac{CE + EB}{EB}$. (4) $\frac{CA}{DA} = \frac{CB}{EB}$. 7. (1) $\frac{4}{3}$. (2) Yes. (3) §160e. (4) Yes. (5) §217d. (6) $\frac{4}{3}$. (7) Yes. (8) Asmt. 6. (9) Not necessarily. 8. (1) $\frac{4}{7}$. (2) $\frac{4}{7}$. (3) $\frac{3}{7}$. (4) $\frac{3}{7}$.

9. $\frac{CM}{CA} = \frac{4}{7}$ and $\frac{CN}{CB} = \frac{4}{7}$ (§395); whence $\frac{CM}{CA} = \frac{CN}{CB}$ (Asmt. 7).

10. Asmt. 1.

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1. $CE = 2EB$. 2. $CE = \frac{2}{3}CB$.

3. $\frac{6}{3} = \frac{8}{x}$. Solving, $6x = 24$; whence $x = 4$. 4 in. Ans.

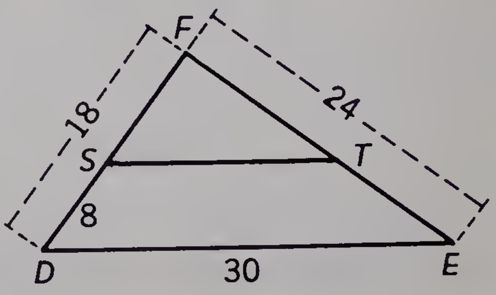
4. $\frac{10}{6} = \frac{x}{7\frac{1}{2}}$. Solving, $6x = 75$; whence $x = 12\frac{1}{2}$. $12\frac{1}{2}$ ft. Ans.

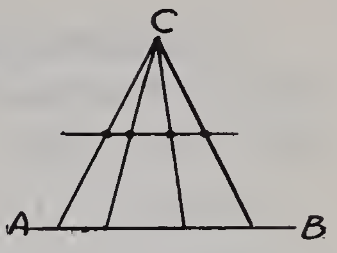
5. (1) $\frac{CD}{15 - CD} = \frac{12}{8}$. Solving, $8CD = 180 - 12CD$; whence $20CD = 180$, and $CD = 9$.

(2) $DA = CA - CD = 15 - 9 = 6$.

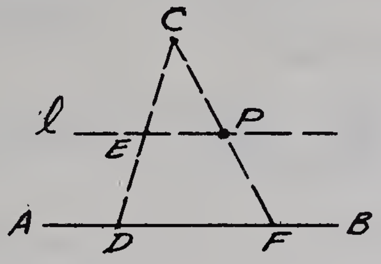
6. $\frac{5}{8} = \frac{12}{DF}$. Solving, $5DF = 96$; whence $DF = 19\frac{1}{5}$.

7. Let x = the length of ET. Then $\frac{18}{8} = \frac{24}{x}$ (§398). $18x = 192$; and $x = 10\frac{2}{3}$. ET = $10\frac{2}{3}$ in. Ans.





8. The locus of the midpoints of line segments drawn from a given point to a given line is the line which is parallel to the given line and bisects one of the line segments drawn from the point to the line.



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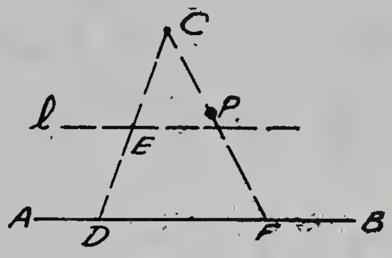


II

PART I.

Any point on the line parallel to the given line bisects the line segment joining the external point and any point in the line.

Given the line AB, the external point C, the line $\ell \parallel$ AB and bisecting CD, and P any point in ℓ.

To prove that CP = PF.

Planning the Proof: 1. We can prove line segments = by §§ 97, 157, 217, 308, 309, 378. 2. We shall use §217e.

Proof: 1. $\ell \parallel$ AB and bisects CD (Given). 2. ∴ CP = PF (§ 217e).

PART II.

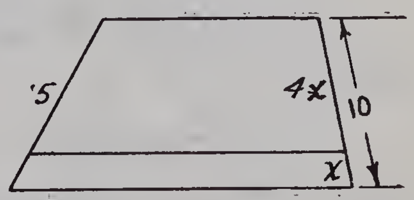
Any point bisecting the line segment joining the external point to the given line lies on the line parallel to the given line.

Given the line AB, the external point C, the line $\ell \parallel$ AB and bisecting CD, and CP = PF.

To prove that P is on ℓ.

Planning the Proof: 1. We can prove that a point is on a line by Asmt. 17, §§139, 296, 297, 301, 379. 2. We shall use Asmt. 17.

Proof: 1. CE = ED and $\ell \parallel$ AB (Given). 2. ∴ ℓ bisects CF (§217e). 3. CP = PF (Part I). 4. ∴ P lies on ℓ (Asmt. 17).

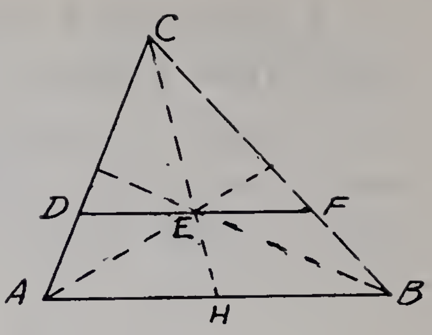


9. x + 4x = 15 (§399). Solving, 5x = 15; whence x = 3. Then 4x = 12.

3 in., 12 in. Ans.

10. Given the △ABC, E its centroid, and DEF $\parallel$ AB.

To prove that $\frac{CD}{DA} = \frac{CF}{FB} = \frac{2}{1}$.

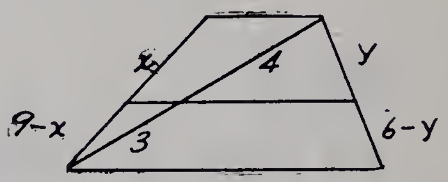


Planning the Proof: 1. We can prove line segments proportional by §§396-399. 2. We shall use §397, Asmt. 6.

Proof: 1. Draw CE intersecting AB in H (Asmt. 11). 2. CH is a median (§363). 3. $\frac{CE}{EH} = \frac{2}{1}$ (§379d). 4. DF ∥ AB (Given). 5. $\therefore \frac{CD}{DA} = \frac{CE}{EH} = \frac{CF}{FB}$ (§397). 6. $\therefore \frac{CD}{DA} = \frac{CF}{FB} = \frac{2}{1}$ (Asmt. 6).

11. (1) $\frac{x}{9 - x} = \frac{4}{3}$. Solving, $3x = 36 - 4x$; whence $7x = 36$, and $x = 5\frac{1}{7}$. Then $9 - x = 3\frac{6}{7}$.

$5\frac{1}{7}$ in., $3\frac{6}{7}$ in. Ans.



(2) $\frac{y}{6 - y} = \frac{4}{3}$. Solving $3y = 24 - 4y$; whence $7y = 24$, and $y = 3\frac{3}{7}$. Then $6 - y = 2\frac{4}{7}$.

$3\frac{3}{7}$ in., $2\frac{4}{7}$ in. Ans.

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1. Direct application of §400. (2) $\frac{2}{5} = \frac{3}{x}$. Solving, $2x = 15$, and $x = 7\frac{1}{2}$. $7\frac{1}{2}$ in. Ans.

2. Since $bx = ac$, $\frac{b}{a} = \frac{c}{x}$ (§387). Use a direct application of §400 for the construction.

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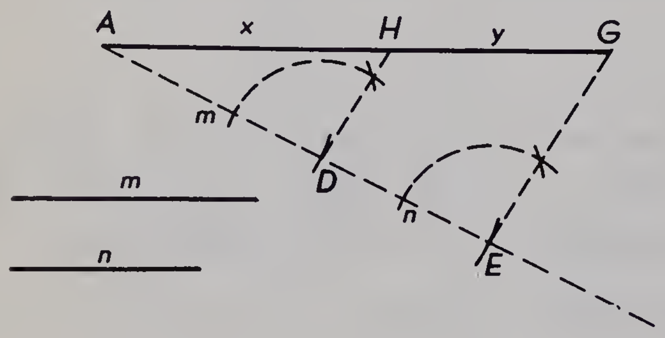
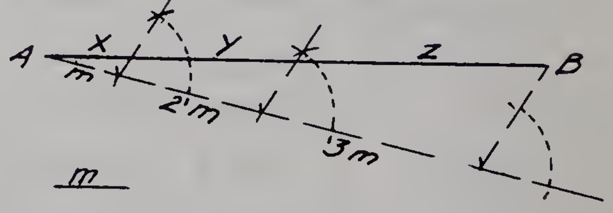
1. Direct application of §401.



2. Direct application of §401.

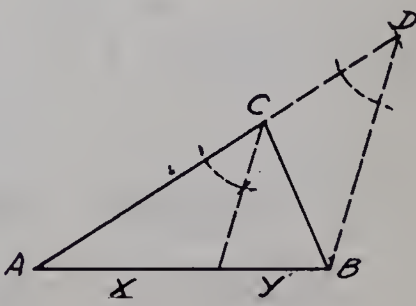


3. Given the △ABC with base AB.

To divide AB into segments x and y so that $\frac{x}{AC} = \frac{y}{BC}$.

Construction: 1. Extend AC to D so that CD = BC (Asmt. 13, §9). 2. Divide AB into 2 segments, x and y, which are proportional to AC and CD (§401).

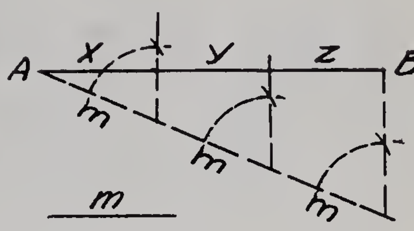
Then x and y are the required segments.



Proof: 1. $\frac{x}{AC} = \frac{y}{CD}$ (Const.). 2. But CD = BC (Const.). 3. $\therefore \frac{x}{AC} = \frac{y}{BC}$ (Asmt. 6).

4. Given the line segment AB.

To divide AB into segments x, y, and z so that x = y = z.



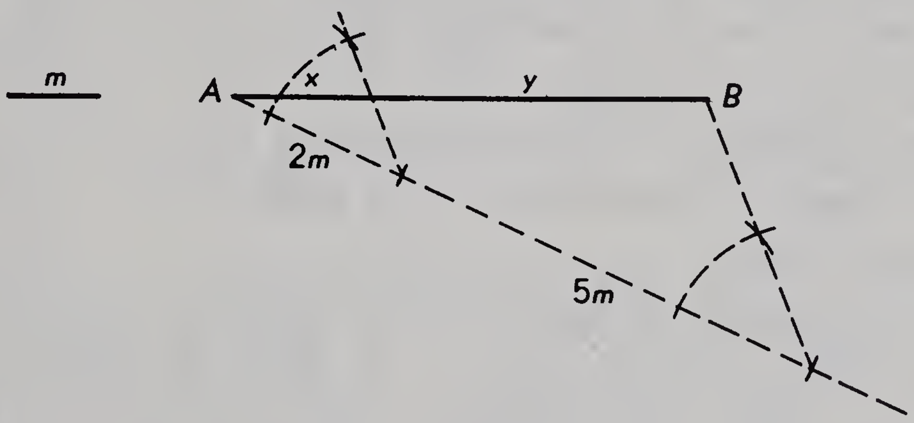
Construction: 1. Draw any line segment m (Asmt. 11). 2. Divide AB into three segments x, y, and z so that $\frac{x}{m} = \frac{y}{m} = \frac{z}{m}$ (§401).

Then x, y, and z are the required segments.

Proof: 1. $\frac{x}{m} = \frac{y}{m} = \frac{z}{m}$ (Const.). 2. $\therefore$ x = y = z (§388).

Note: Ex. 4 may also be solved by the direct application of §212.

5. Direct application of §401.



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(1) If a line which divides two sides of a triangle is not parallel to the third side, the two sides are not divided proportionally. (2) Asmt. 38.

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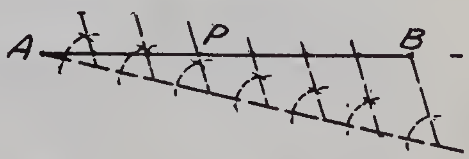
1. (1) $\frac{2}{5}$. (2) $\frac{3}{5}$. (3) 3x.

2. Given the line segment AB.



To divide AB internally into segments AP and PB so that $\frac{AP}{PB} = \frac{3}{4}$.

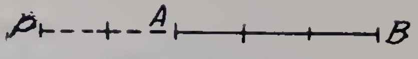
Construction: 1. Divide AB into 7 = parts and let P be the third point of division from A (§224).

Then AP and PB are the required segments.

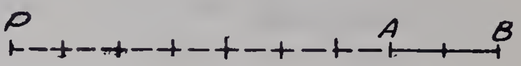
Proof: 1. AB is divided internally (§404). 2. AP = 3 units of measure and PB = 4 units of measure (Const.). 3. Then $\frac{AP}{PB} = \frac{3}{4}$ (§395).

3. $3x + 5x = 16$. Solving, $8x = 16$; whence $x = 2$. Then $3x = 6$, and $5x = 10$. 6 in., 10 in. Ans.

4. (1) $\frac{2}{3}$. (2) $\frac{5}{3}$. (3) $5x$. (4) $3x$.



5. $AP = 7x$, $PB = 9x$, and $AB = 9x - 7x = 2x$. But $2x = 14$; whence $x = 7$. Then $7x = 49$ and $9x = 63$. 49 in., 63 in. Ans.



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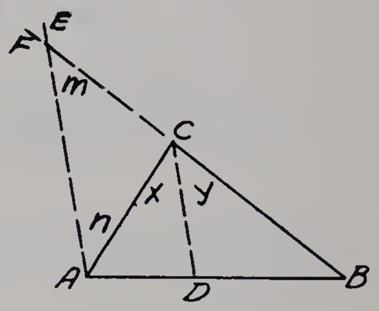
1. $\frac{AD}{6} = \frac{8}{10}$. Solving, $10AD = 48$; whence $AD = 4.8$.

2. $\frac{AC}{12} = \frac{4}{6}$. Solving, $6AC = 48$; whence $AC = 8$.

3. If a line through the vertex of an angle of a triangle divides the opposite side internally into segments which are proportional to the adjacent sides, the line bisects the angle.



Given the $\triangle ABC$, CD drawn from the vertex of $\angle ACB$ to AB so that $\frac{AD}{AC} = \frac{DB}{CB}$.

To prove that CD bisects $\angle ACB$.

Planning the Proof: 1. We can prove that a line segment bisects an $\angle$ by §22. 2. We shall use §22.

Proof: 1. Through A draw $AE \parallel DC$ (§180). 2. Extend BC to meet AE in F (Asmt. 13, 35, 37). 3. $\frac{AD}{AC} = \frac{DB}{CB}$ (Given). 4. $\frac{AD}{FC} = \frac{DB}{CB}$ (§397). 5. $\therefore AC = FC$ (§389). 6. $\therefore \angle m = \angle n$ (§98a). 7. $\angle m = \angle y$ and $\angle n = \angle x$ (§158a). 8. $\angle x = \angle y$ (Asmt. 7). 9. $\therefore$ CD bisects $\angle ACB$ (§22).

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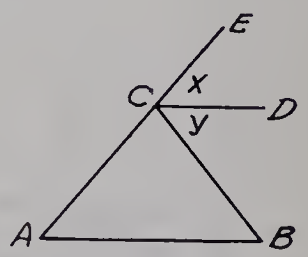
1. (1) Isos.



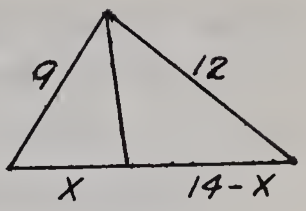
(2) Given the $\triangle ABC$ with CD, the bisector of ext. $\angle BCE$, $\parallel$ to AB.

To prove that $\triangle ABC$ is isos.

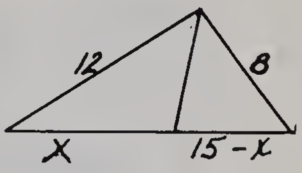
Planning the Proof: 1. We can prove that a $\triangle$ is isos. by §56. 2. We shall use §56.

Proof: 1. $\angle x = \angle y$ (Given). 2. $\angle x = \angle A$ and $\angle y = \angle B$ (§158a). 3. $\therefore \angle A = \angle B$ (Asmt. 7). 4. $\therefore AC = BC$ (§157a). 5. $\therefore \triangle ABC$ is isos. (§56).

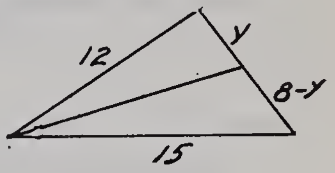
2. Let $x = AB$. Then $\frac{x + 12}{12} = \frac{14}{7}$. $7x + 84 = 168$. $7x = 84$. $x = 12$. Then $AD = 12 + 12 = 24$.



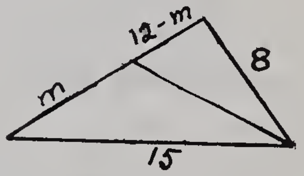
3. Let x and 14 - x represent the 2 segments. Then $\frac{x}{9} = \frac{14 - x}{12}$ (§405). Solving, $12x = 126 - 9x$; whence $21x = 126$, and $x = 6$. Then $14 - x = 8$. 6 in., 8 in. Ans.



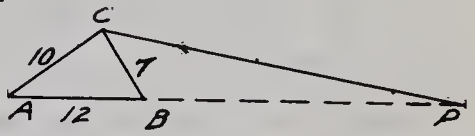
4. (1) $\frac{12}{x} = \frac{8}{15 - x}$. Solving, $8x = 180 - 12x$; whence $20x = 180$, and $x = 9$. Then $15 - x = 6$. 9 in., 6 in. Ans.



(2) $\frac{12}{y} = \frac{15}{8 - y}$. Solving, $15y = 96 - 12y$; whence $27y = 96$, and $y = 3\frac{5}{9}$. Then $8 - y = 4\frac{4}{9}$. $3\frac{5}{9}$ in., $4\frac{4}{9}$ in. Ans.



(3) $\frac{m}{15} = \frac{12 - m}{8}$. Solving, $8m = 180 - 15m$; whence $23m = 180$, and $m = 7\frac{19}{23}$. Then $12 - m = 4\frac{4}{23}$. $7\frac{19}{23}$ in., $4\frac{4}{23}$ in. Ans.

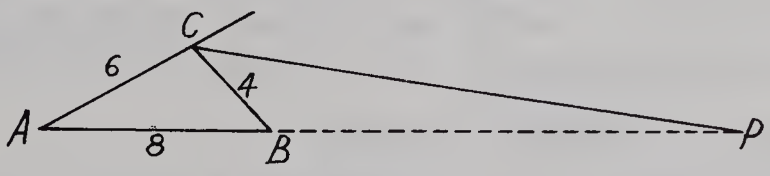


5. (1) The bisector will meet AB produced through B in a point P. For $\frac{AP}{PB} = \frac{10}{7}$ (§406). Since $10 > 7$, $AP > BP$ and BP is a part of AP.

(2) $AP = BP + 12$ (Asmt. 9).

Then $\frac{10}{BP + 12} = \frac{7}{BP}$. Solving, $10BP = 7BP + 84$; whence $3BP = 84$, and $BP = 28$. Then $AP = 28 + 12 = 40$.

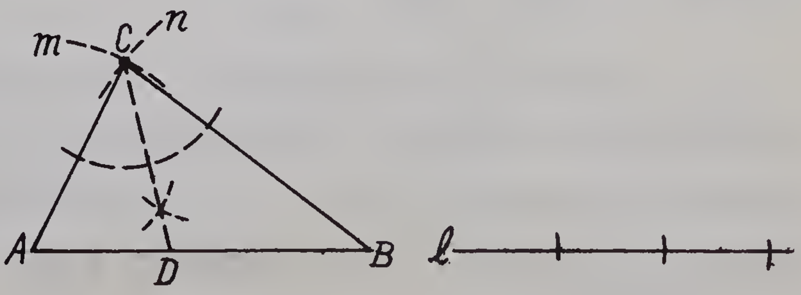
6. The largest int. ∠ is opp. the side = 8. Then the smallest ext. ∠ is opp. the side = 8. Let AB = the side = 8. Then AP and BP are the segments into which the bisector divides AB, and $AP = BP + 8$. Then $\frac{6}{BP + 8} = \frac{4}{BP}$ (§406). Solving, $6BP = 4BP + 32$; whence $2BP = 32$, and $BP = 16$. Then $AP = BP + 8 = 24$.



7. Given line segment AB.

To divide AB into two segments having the ratio 2 : 3.

Construction: 1. Draw any line ℓ and on it lay off 3 equal line segments (§9). 2. With A as a center and a radius equal to two of these equal line segments draw arc m (Asmt. 15). 3. With B as a center and a radius equal to three of these equal

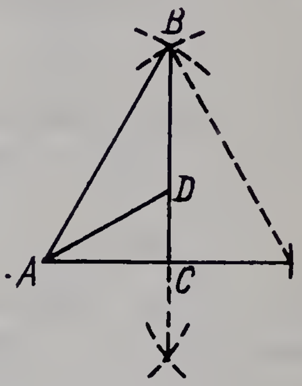


line segments draw arc n intersecting arc m in C (Asmt. 15). 4. Draw AC and BC (Asmt. 11). 5. Construct CD, the bisector of ∠C (§177a).

Then AD and DB are the required segments.

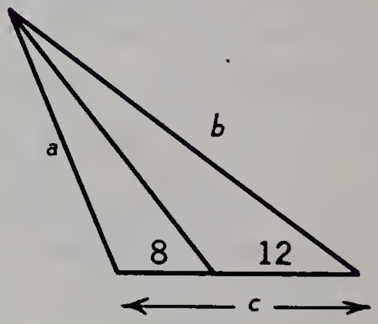
Proof: 1. AC : CB = 2 : 3 (Const.). 2. AC : CB = AD : DB (§405). 3. AD : DB = 2 : 3 (Asmt. 7).

8. Construction: 1. With AB as the hypotenuse construct the 30°-60° rt. △ACB (Ex. 12, p. 195). 2. Let AD bisect the 60° ∠, ∠A.



To prove that BD : DC = 2 : 1.

Proof: 1. BD : DC = AB : AC (§405). 2. AB : AC = 2 : 1 (§136). 3. ∴ BD : DC = 2 : 1 (Asmt. 7).



9. Let c be the length of the side divided into the 8" and 12" segments, let a be the length of the side adjacent to the 8" segment, and let b be the length of the side adjacent to the 12" segment. Then a + b + c = 80" and c = 8" + 12" = 20", whence a + b = 60" (Asmt. 2). Now $\frac{a}{b} = \frac{8}{12}$ (§405). ∴ $a = \frac{8b}{12}$. Substituting this value of a in a + b = 60 gives $\frac{8b}{12} + b = 60$, 20b = 720, and b = 36. ∴ $a = \frac{8}{12}(36) = 24$. The other two sides have the lengths 24" and 36".

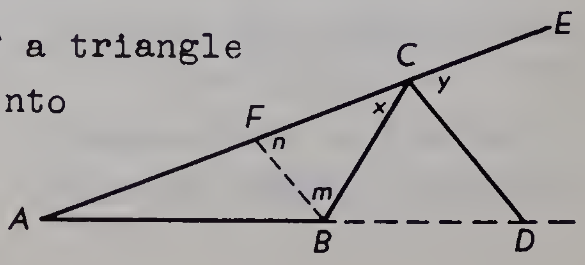
10. Given Ⓢ O and O' tangent internally at A, BC a chord of ⊙O and tangent to ⊙O' at D, and chords AB and AC intersecting ⊙O' in E and F respectively.

To prove that AD bisects ∠CAB.

Planning the Proof: 1. We can prove that a line bisects an ∠ by §22. 2. We shall use §22, Asmt.

Proof: 1. Draw EF (Asmt. 11). 2. EF ∥ BC (Ex. 11, p. 327). 3. ∴ ED = $\overset{\frown}{DF}$ (§337b). 4. ∠EAD $\overset{\circ}{=}$ $\frac{1}{2}\overset{\frown}{ED}$ and ∠DAF $\overset{\circ}{=}$ $\frac{1}{2}\overset{\frown}{DF}$ (§333b). 5. $\frac{1}{2}\overset{\frown}{ED} = \frac{1}{2}\overset{\frown}{DF}$ (Asmt. 4). 6. ∴ ∠EAD = ∠DAF (Asmt. 7). 7. ∴ AD bisects ∠CAB (§22).

11. If a line through the vertex of a triangle divides the opposite side externally into segments proportional to the adjacent sides, it bisects the exterior angle at that vertex.



Given $\triangle ABC$, AC extended to E and forming exterior angle $\angle BCE$, and CD so drawn that $\frac{AD}{DB} = \frac{AC}{CB}$.

To prove CD bisects $\angle BCE$.

Planning the Proof: 1. We can prove that a line bisects an angle by §22. 2. We shall use §22.

Proof: 1. Through B draw $BF \parallel DC$ (§179). 2. In $\triangle ADC$, $\frac{AD}{DB} = \frac{AC}{CF}$ (§398). 3. But $\frac{AD}{DB} = \frac{AC}{CB}$ (Given). 4. $\therefore CF = CB$ (§389). 5. $\angle n = \angle m$ (§98a). 6. $\angle m = \angle x$ and $\angle n = \angle y$ (§158a). 7. $\therefore \angle x = \angle y$ (Asmt. 7). 8. CD bisects $\angle BCE$ (§22).

12. *Planning the Proof*: 1. We can prove line segments proportional by §§396-399, 405, 406. 2. We shall use §§405, 406, Asmt. 7.

Proof: 1. CP bisects $\angle ACB$ (Given). 2. $\therefore \frac{AP}{PB} = \frac{AC}{BC}$ (§405). 3. CP' bisects $\angle BCE$ (Given). 4. $\therefore \frac{AP'}{P'B} = \frac{AC}{BC}$ (§406). 5. $\therefore \frac{AP}{PB} = \frac{AP'}{P'B}$ (Asmt. 7). 6. $\therefore$ AB is divided harmonically (§407).

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1. *Proof of Converse*: 1. $\frac{a - 1}{b} = \frac{bc - d}{bd}$ (Given). 2. $bd(a - 1) = b(bc - d)$ (§386). 3. $abd - bd = b^2c - bd$ (Asmt. 6). 4. $ad - d = bc - d$ (Asmt. 4). 5. $ad = bc$ (Asmt. 1). 6. $a : b = c : d$ (§387).

Proof: 1. $a : b = c : d$ (Given). 2. $ad = bc$ (§386). 3. $ad - d = bc - d$ (Asmt. 2). 4. $abd - bd = b^2c - bd$ (Asmt. 3). 5. $bd(a - 1) = b(bc - d)$ (Asmt. 6). 6. $(a - 1) : b = (bc - d) : bd$ (§387).

2. *Proof of Converse*: 1. $(a + 1) : 1 = (bc + d) : d$ (Given). 2. $d(a + 1) = 1(bc + d)$ (§386). 3. $ad + d = bc + d$ (Asmt. 6). 4. $ad = bc$ (Asmt. 2). 5. $a : b = c : d$ (§387).

Proof: 1. $a : b = c : d$ (Given). 2. $ad = bc$ (§386). 3. $ad + d = bc + d$ (Asmt. 1). 4. $d(a + 1) = 1(bc + d)$ (Asmt. 6). 5. $a + 1 : 1 = bc + d : d$ (§387).

3. *Proof of Converse*: 1. $a : b = (c - a) : (d - b)$ (Given). 2. $a(d - b) = b(c - a)$ (§386). 3. $ad - ab = bc - ab$ (Asmt. 6). 4. $ad = bc$ (Asmt. 1). 5. $a : b = c : d$ (§387).

Proof: 1. $a : b = c : d$ (Given). 2. $ad = bc$ (§386). 3. $ad - ab = bc - ab$ (Asmt. 2). 4. $a(d - b) = b(c - a)$ (Asmt. 6). 5. $a : b = (c - a) : (d - b)$ (§387).

4. *Proof of Converse*: 1. $(a - b) : b = (c - d) : d$ (Given).

2. $d(a - b) = b(c - d)$ (§386). 3. $ad - bd = bc - bd$ (Asmt. 6). 4. $ad = bc$ (Asmt. 1). 5. $a : b = c : d$ (§387).

Proof: 1. $a : b = c : d$ (Given). 2. $ad = bc$ (§386). 3. $ad - bd = bc - bd$ (Asmt. 2). 4. $d(a - b) = b(c - d)$ (Asmt. 6). 5. $(a - b) : b = (c - d) : d$ (§387).

5. Proof of Converse: 1. $(a + b) : (a - b) = (c + d) : (c - d)$ (Given). 2. $(a + b)(c - d) = (a - b)(c + d)$ (§386). 3. $ac - ad + bc - bd = ac + ad - bc - bd$ (Asmt. 6). 4. $-ad + bc = ad - bc$ (Asmt. 1, 2). 5. $2bc = 2ad$ (Asmt. 1). 6. $bc = ad$ (Asmt. 4). 7. $a : b = c : d$ (§387).

Proof: 1. $a : b = c : d$ (Given). 2. $bc = ad$ (§386). 3. $2bc = 2ad$ (Asmt. 3). 4. $-ad + bc = ad - bc$ (Asmt. 2). 5. $ac - ad + bc - bd = ac + ad - bc - bd$ (Asmt. 1, 2). 6. $(a + b)(c - d) = (a - b)(c + d)$ (Asmt. 6). 7. $(a + b) : (a - b) = (c + d) : (c - d)$ (§387).

6. Proof of Converse: 1. $(a^2 - b) : (ac - d) = a : c$ (Given). 2. $c(a^2 - b) = a(ac - d)$ (§386). 3. $a^2c - bc = a^2c - ad$ (Asmt. 6). 4. $-bc = -ad$ (Asmt. 2). 5. $bc = ad$ (Asmt. 4). 6. $a : b = c : d$ (§387).

Proof: 1. $a : b = c : d$ (Given). 2. $bc = ad$ (§386). 3. $-bc = -ad$ (Asmt. 4). 4. $a^2c - bc = a^2c - ad$ (Asmt. 1). 5. $c(a^2 - b) = a(ac - d)$ (Asmt. 6). 6. $(a^2 - b) : (ac - d) = a : c$ (§387).

PAGES 398-399 (Review Exercises)

1. (1) See §381. (2) See §382. 2. No. 3. (1) and (2). See §394. 4. (1) and (2). See §394. 5. a, b, and d. 6. Any 6 of the following: §§386, 387, 388, 389, 390, 391, 392, 393.

7. a. $\frac{x}{m} = \frac{y}{n}$. b. $\frac{y}{x} = \frac{n}{m}$. c. $\frac{x + y}{y} = \frac{m + n}{n}$. d. $\frac{x - y}{y} = \frac{m - n}{n}$.

8. Any 2 of the following: $\frac{a}{c} = \frac{d}{b}$, $\frac{a}{d} = \frac{c}{b}$, $\frac{c}{a} = \frac{b}{d}$, $\frac{d}{a} = \frac{b}{c}$.

9. a. §§402, 403. b. §§397, 398, 399, 405, 406.

10. §§400, 401.

11. (1) The mean proportional between x and z.
(2) The third proportional to x and y.

12. ±6. 13. 27. 14. 14.4. 15. 16 and 24. 16. $6\frac{2}{3}$. 17. $10\frac{2}{3}$. 18. $25\frac{1}{5}$. 19. 12 in. and 15 in.

20. $x = \frac{b^2}{a}$ (Given). $\therefore ax = b^2$ (Asmt. 3), and $\frac{a}{b} = \frac{b}{x}$ (§387). The construction is then a direct application of (§400).

PAGE 400 (Test 30)

1. F. 2. F. 3. F. 4. T. 5. F. 6. T. 7. T.
8. T. 9. F. 10. T. 11. F. 12. T. 13. T. 14. F.
15. T. 16. T. 17. T. 18. T. 19. F. 20. T.

PAGE 401 (Test 31)

1. 6. 2. $x = 3$. 3. $\frac{6}{m} = \frac{5}{2n}$. 4. 3 in. 5. EB = 12.
6. BE = 15. 7. BE = 6. 8. 36° and 54°. 9. SX = 9.6.
10. YS = 10.8. 11. MH = 8. 12. GK = $10\frac{2}{3}$. 13. 8 : c.
14. $b^2 - d^2$.

Chapter 13. Similar Polygons

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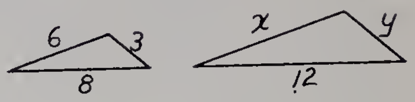
1. Let x = the no. of inches in the other leg. Then $\frac{5}{15} = \frac{7}{x}$. Solving, 5x = 105; whence x = 21. 21 in. Ans.



2. Let x = the no. of feet in the side corresponding to 6. Then $\frac{6}{x} = \frac{8}{12}$. Solving, 8x = 72; whence x = 9.

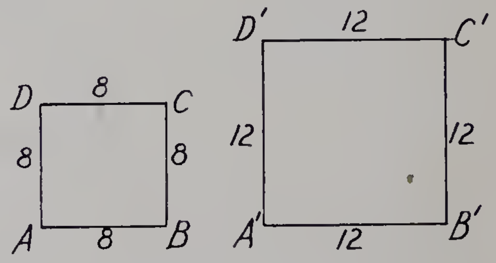
Let y = the no. of feet in the side corresponding to 3. Then $\frac{3}{y} = \frac{8}{12}$. Solving, 8y = 36; whence y = 4.5. 9 ft., 4.5 ft. Ans.

3. (1) $\frac{A'B'}{10} = \frac{6}{8}$. Solving, 8A'B' = 60; whence A'B' = 7.5

(2) $\frac{B'C'}{5} = \frac{6}{8}$. Solving, 8B'C' = 30; whence B'C' = 3.75.

(3) $\frac{C'D'}{6} = \frac{6}{8}$. Solving, 8C'D' = 36; whence C'D' = 4.5.

4. Since the sides of the polygons are proportional (§412), each side of the second polygon is to the corresponding side of the first as $\frac{18}{12}$. Then the remaining sides of the second polygon are 6, 7.5, 9, and 12, respectively. The ratio of similitude is $\frac{2}{3}$.



5. (1) AB = BC = CD = DA and A'B' = B'C' = C'D' = D'A' (§194).

Then $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DA}{D'A'}$ (Asmt. 4).

(2) Yes (§191). (3) Yes (§413).
(4) Yes.

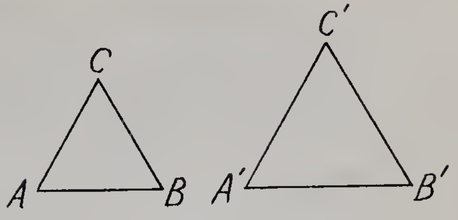
6. (1) Yes. (2) Their corresponding sides are proportional and their corresponding $\angle$s are =. (3) Not necessarily.
(4) Their corresponding sides may be proportional but not =.
(5) 1.

7. (1) Yes. (2) The students' figures should resemble those on page 403 of the text.

8. (1) No. (2) No. (3) Yes.

9. Given the equilateral $\triangle$s ABC and A'B'C' with $\angle$A corresponding to $\angle$A', $\angle$B to $\angle$B', and $\angle$C to $\angle$C'.

To prove that $\triangle$ABC $\sim$ $\triangle$A'B'C'.



Planning the Proof: 1. We can prove $\triangle$ $\sim$ by §413. 2. We shall use §413.

Proof: 1. $\triangle ABC$ and $\triangle A'B'C'$ are equilateral (Given). 2. $\therefore AB = BC = CA$ and $A'B' = B'C' = C'A'$ (§183b). 3. $\therefore \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$ (Asmt. 4). 4. $\angle A + \angle B + \angle C = 180°$ and $\angle A' + \angle B' + \angle C' = 180°$ (§120). 5. $\angle A = \angle B = \angle C$ and $\angle A' = \angle B' = \angle C'$ (§84). 6. $\therefore 3\angle A = 180°$ and $3\angle A' = 180°$ (Asmt. 6). 7. $\therefore \angle A = 60°$ and $\angle A' = 60°$ (Asmt. 4). 8. $\therefore \angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$ (Asmt. 7). 9. $\therefore \triangle ABC \sim \triangle A'B'C'$ (§413).

10. Given the equiangular $\triangle$ ABC and A'B'C' with $\angle A$ corresponding to $\angle A'$, $\angle B$ to $\angle B'$, and $\angle C$ to $\angle C'$. (See Fig. for Ex. 9.)

To prove that $\triangle ABC \sim \triangle A'B'C'$.

Planning the Proof: (Same as in Ex. 9).

Proof: 1. $\angle A + \angle B + \angle C = 180°$ and $\angle A' + \angle B' + \angle C' = 180°$ (§120). 2. But $\angle A = \angle B = \angle C$ and $\angle A' = \angle B' = \angle C'$ (Given). 3. $\therefore 3\angle A = 180°$ and $3\angle A' = 180°$ (Asmt. 6). 4. $\therefore \angle A = 60°$ and $\angle A' = 60°$ (Asmt. 4). 5. $\therefore \angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$ (Asmt. 7). 6. $AB = BC = AC$ and $A'B' = B'C' = A'C'$ (§129). 7. $\therefore \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$ (Asmt. 4). 8. $\therefore \triangle ABC \sim \triangle A'B'C'$ (§413).

11. $\frac{21.21}{AF} = \frac{15}{6}$ (§409b). Solving, $15AF = 127.26$; whence $AF = 8.484$. 8.484 in. Ans.

PAGES 408-409

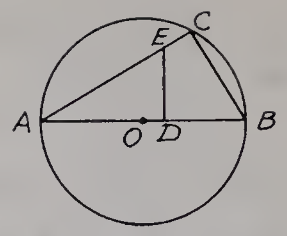
1. (1) Yes. (2) Yes. (3) Yes. (4) Yes (§415). (5) $\frac{1}{3}$. (6) $\frac{1}{3}$. 2. See §409a, b, c, d, and §420a, b, c.

3. Let x = the height of the tower. Then $\frac{x}{20} = \frac{120}{15}$. Solving, $15x = 2400$; whence $x = 160$. 160 ft. Ans.

4. Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 413, 417, and Asmt. 2. We shall use §412.

Proof: 1. $AC \perp AB$ and $BD \perp AB$ (Given). 2. $\angle CAO = \angle OBD$ (§50a). 3. $\angle AOC = \angle BOD$ (§50d). 4. $\therefore \triangle ACO \sim \triangle ODB$ (§415). 5. $\therefore \frac{AC}{BD} = \frac{CO}{DO}$ (§412). 6. $\therefore \frac{AC}{CO} = \frac{BD}{DO}$ (§391).

5. Given the $\triangle ABC$ inscribed in $\odot O$ with the diameter AB, and $ED \perp AB$, E being any point on AC.



To prove that $\triangle ABC \sim \triangle ADE$.

Planning the Proof: 1. We can prove $\triangle$s $\sim$ by §§412, 415, 416, 418. 2. We shall use §416.

Proof: 1. $ED \perp AB$ and $\angle ACB$ is inscribed in semicircle ACB (Given). 2. $\therefore$ $\angle$s ADE and ACB are rt. $\angle$s (§§14, 335). 3. $\therefore$ $\triangle$s ADE and ABC are rt. $\triangle$s (§57). 4. $\angle A = \angle A$ (Iden.). 5. $\therefore$ $\triangle ABC \sim \triangle ADE$ (§416).

6. Three relationships which can be proved are (a) $\triangle AEC \sim \triangle BED$, (b) $\frac{CE}{EB} = \frac{AE}{ED}$, and (c) $CE \times ED = AE \times EB$.

Proof: (a) 1. AB and CD are chords of $\odot O$ (Given). 2. Draw AC and BD (Asmt. 11). 3. $\angle CAB = \angle CDB$ and $\angle ACD = \angle DBA$ (§334). 4. $\therefore$ $\triangle AEC \sim \triangle BED$ (§415). (b) $\frac{CE}{EB} = \frac{AE}{ED}$ (§412). (c) $CE \times ED = AE \times EB$ (§386).

7. *Planning the Proof*: 1. We can prove the product of two quantities = the product of two other quantities by Asmt. and §386. 2. We shall use §386.

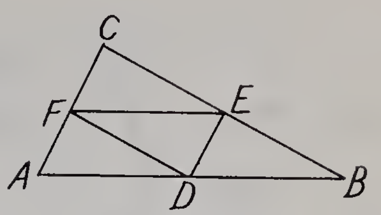
Proof: 1. Draw AE (Asmt. 11). 2. AB is a diameter (Given). 3. $\widehat{AEB}$ is a semicircle (§283). 4. $\angle E$ is a rt. $\angle$ (§319). 5. $DC \perp AB$ (Given). 6. $\angle C$ is a rt. $\angle$ (§14). 7. $\triangle AEB$ and $\triangle BCD$ are rt. $\triangle$s (§57). 8. $\angle ABE = \angle DBC$ (§50d). 9. $\triangle AEB \sim \triangle BCD$ (§416). 10. $\frac{AB}{BD} = \frac{EB}{BC}$ (§412). 11. $AB \times BC = EB \times BD$ (§386).

8. *Planning the Proof*: 1. We can prove (a) $\triangle$s$\sim$ as in Ex. 5; (b, c) that a line segment is the mean proportional between two other segments §420c. 2. We shall use (a) §415; (b, c) §420c.

Proof: 1. AB is tangent to $\odot O$ at B and AC is a secant from A (Given). 2. Draw BC and BD (Asmt. 11). 3. $\angle C \stackrel{\circ}{=} \frac{1}{2}\widehat{BD}$ (§333b). 4. $\angle ABD \stackrel{\circ}{=} \frac{1}{2}\widehat{BD}$ (§333c). 5. $\therefore$ $\angle ABD = \angle C$ (Asmt. 7). 6. $\angle CAB = \angle BAD$ (Iden.). 7. $\therefore$ $\triangle ABD \sim \triangle ABC$ (§415). 8. $\therefore$ $\frac{AC}{AB} = \frac{AB}{AD}$ (§412). 9. $\therefore$ $\overline{AB}^2 = AC \times AD$ (§386).

9. *Planning the Proof*: 1. We can prove (a, b, c) $\triangle$s $\sim$: (Same as in Ex. 5); (d) that a line segment is the mean proportional between 2 other line segments: §420c. 2. We shall use (a) §416, (b) §416, (c) §418, (d) §420c.

Proof: 1. In $\triangle$s ADC and ABC, $\angle A = \angle A$ (Iden.), and $\angle$s ADC and ACB are rt. $\angle$s (Given). 2. $\therefore$ $\triangle ADC \sim \triangle ABC$ (§416). 3. Similarly, $\angle B = \angle B$, $\angle$s ACB and BDC are rt. $\angle$s, and $\triangle BCD \sim \triangle ABC$ (Statements 1, 2). 4. $\therefore$ $\triangle ADC \sim \triangle BCD$ (§418). 5. $\therefore$ $\frac{AD}{DC} = \frac{DC}{DB}$ (§412). 6. $\overline{DC}^2 = AD \times DB$ (§386).



10. Given the $\triangle ABC$ with the $\triangle DEF$ formed by joining the midpoints of AB, BC, and CA.

To prove that $\triangle DEF \sim \triangle ABC$.

Planning the Proof: 1. We can prove $\triangle$s $\sim$ as in Ex. 5. 2. We shall use §415.

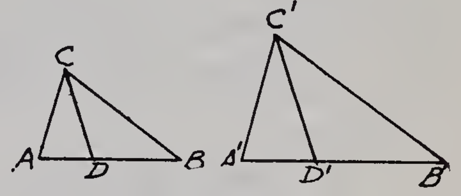
Proof: 1. $AF = FC$, $CE = EB$, and $AD = DB$ (Given). 2. $\therefore FE \parallel AB$, $DF \parallel BC$, and $DE \parallel AC$ (§221b). 3. $\therefore \angle C = \angle FDE$ and $\angle B = \angle DFE$ (§158b). 4. $\therefore \triangle DEF \sim \triangle ABC$ (§415).

11. Planning the Proof: (Same as in Ex. 5).

Proof: 1. $\angle$s x and y are inscribed $\angle$s intercepting $\overset{\frown}{BC}$ (Given). 2. $\therefore \angle x = \angle y$ (§334). 3. $\angle E = \angle E$ (Iden.). 4. $\therefore \triangle BDE \sim \triangle ACE$ (§415). 5. $\frac{BD}{CA} = \frac{DE}{AE}$ (§412).

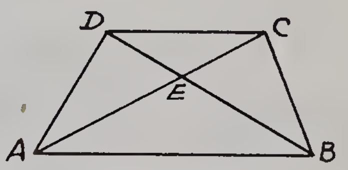
12. Given $\triangle ABC \sim \triangle A'B'C'$, with AC and A'C' corresponding sides, $\angle C$ corresponding to $\angle C'$, CD bisecting $\angle C$, and C'D' bisecting $\angle C'$.

To prove that $\frac{CD}{C'D'} = \frac{AC}{A'C'}$.



Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 417, and Asmt. 2. We shall use §412.

Proof: 1. $\triangle ABC \sim \triangle A'B'C'$ (Given). 2. $\therefore \angle A = \angle A'$ and $\angle ACB = \angle A'C'B'$ (§412). 3. CD bisects $\angle C$ and C'D' bisects $\angle C'$ (Given). 4. $\therefore \angle ACD = \angle A'C'D'$ (Asmt. 4). 5. $\therefore \triangle ACD \sim \triangle A'C'D'$ (§415). 6. $\therefore \frac{CD}{C'D'} = \frac{AC}{A'C'}$ (§412).



13. Given the trapezoid ABCD with $AB \parallel DC$ and diagonals AC and BD intersecting in E.

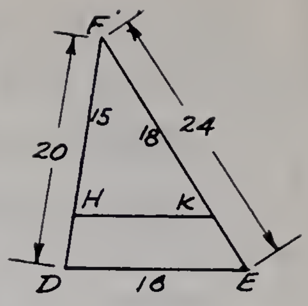
To prove that $\frac{DE}{EB} = \frac{CE}{EA}$.

Planning the Proof: (Same as in Ex. 12.)

Proof: 1. $AB \parallel DC$ (Given). 2. $\therefore \angle ECD = \angle EAB$ and $\angle EDC = \angle EBA$ (§158a). 3. $\therefore \triangle EDC \sim \triangle EBA$ (§415). 4. $\therefore \frac{ED}{EB} = \frac{CE}{EA}$ (§412).

14. Planning the Proof: (Same as in Ex. 12.)

Proof: 1. $AD \perp BC$ and $BE \perp AC$ (Given). 2. $\angle AEB = \angle BDA$ (§50a). 3. $\angle AFE = \angle BFD$ (§50d). 4. $\therefore \triangle AFE \sim \triangle BFD$ (§415). 5. $\therefore \frac{AF}{BF} = \frac{EF}{FD}$ (§412). 6. In rt. $\triangle$s ADC and BEC, $\angle C = \angle C$ (Iden.). 7. $\therefore \triangle ADC \sim \triangle BEC$ (§416). 8. $\therefore \frac{AD}{BE} = \frac{AC}{BC}$ (§412).



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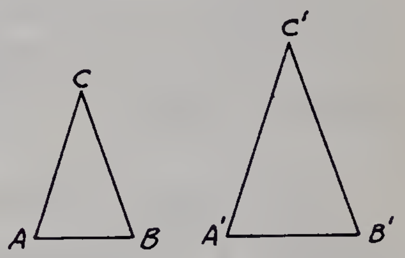
1. (1) DF = 20, DE = 16, EF = 24, FH = 15, and FK = 18 (Given).

Then $\frac{FH}{FD} = \frac{15}{20}$ and $\frac{FK}{FE} = \frac{18}{24}$ (Asmt. 4); whence $\frac{FH}{FD} = \frac{FK}{FE}$ (Asmt. 7). But ∠HFK = ∠DFE (Iden.); whence △HKF ~ △DEF (§422).

(2) $\frac{HK}{DE} = \frac{HF}{DF}$ (§412), or $\frac{HK}{16} = \frac{15}{20}$ (Asmt. 6). Solving, 20HK = 240; whence HK = 12.



2. Given the isos. △s ABC and A'B'C' with AC = BC, A'C' = B'C', and ∠A = ∠A'.

To prove that △ABC ~ △A'B'C'.

Planning the Proof: 1. We can prove △s ~ by §§412, 415, 416, 418, 422. 2. We shall use §415.

Proof: 1. AC = BC, A'C' = B'C', and ∠A = ∠A' (Given). 2. ∠B = ∠A and ∠B' = ∠A' (§98a). 3. ∴ ∠B = ∠B' (Asmt. 7). 4. ∴ △ABC ~ △A'B'C' (§415).

3. Yes (§422). 4. No.

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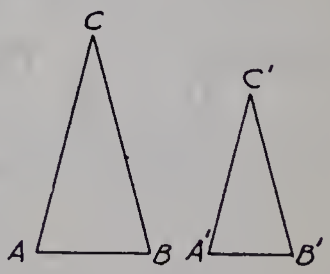
Part of a definition.

PAGE 412

1. (1) §§412, 415, 416, 418, 422, 423. (2) No. (3) Triangles. (4) §423. 2. Yes (§410a).

3. Planning the Proof: 1. We can prove △s ~ by §§412, 415, 416, 418, 422, 423. 2. We shall use §422.

Proof: 1. Polygon ABCDE ~ polygon A'B'C'D'E' (Given). 2. ∴ ∠C = ∠C' and $\frac{DC}{D'C'} = \frac{CB}{C'B'}$ (§412). 3. ∴ △BCD ~ △B'C'D' (§422).



4. Given the isos. △s ABC and A'B'C' with AC = BC, A'C' = B'C', and ∠C = ∠C'.

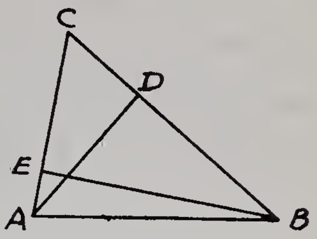
To prove that △ABC ~ △A'B'C'.

Planning the Proof: (Same as in Ex. 3).

Proof: 1. AC = BC and A'C' = B'C' (Given). 2. ∴ $\frac{AC}{A'C'} = \frac{BC}{B'C'}$ (Asmt. 4). 3. ∠C = ∠C' (Given). 4. △ABC ~ △A'B'C' (§422).

5. (1) Given $\triangle ABC$ with $\angle C$ acute, $AD \perp BC$, and $BE \perp AC$.
To prove (a) that $\triangle CEB \sim \triangle CDA$; (b) that $\frac{AD}{BE} = \frac{AC}{BC}$.

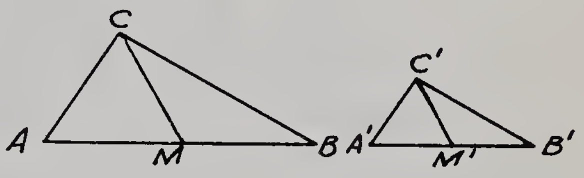
Planning the Proof: 1. We can prove (a) $\triangle$s $\sim$ as in Ex. 3; (b) line segments proportional by §§396, 409, 412, 417, and Asmt. 2. We shall use (a) §416; (b) §412.



Proof: 1. $AD \perp BC$ and $BE \perp AC$ (Given). 2. $\therefore$ $\triangle$s CEB and CDA are rt. $\triangle$s (§§14, 57). 3. $\angle C = \angle C$ (Iden.). 4. $\therefore \triangle CEB \sim \triangle CDA$ (§416). 5. $\therefore \frac{AD}{BE} = \frac{AC}{BC}$ (§412).

(2) The corresponding altitudes of two similar triangles have the same ratio as any two corresponding sides (§417).

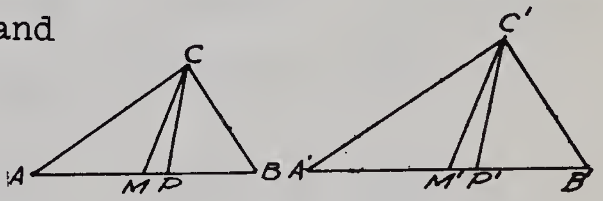
6. Given $\triangle ABC \sim \triangle A'B'C'$ with AC and $A'C'$ corresponding sides, and CM and $C'M'$ corresponding medians.
To prove that $\frac{CM}{C'M'} = \frac{AC}{A'C'}$.



Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 417, and Asmt. 2. We shall use §412.

Proof: 1. $\triangle ABC \sim \triangle A'B'C'$ (Given). 2. $\therefore \angle A = \angle A'$ and $\frac{AC}{A'C'} = \frac{AB}{A'B'}$ (§412). 3. $AB = 2AM$ and $A'B' = 2A'M'$ (Given). 4. $\therefore \frac{AC}{A'C'} = \frac{2AM}{2A'M'} = \frac{AM}{A'M'}$ (Asmt. 6). 5. $\therefore \triangle AMC \sim \triangle A'M'C'$ (§422). 6. $\therefore \frac{CM}{C'M'} = \frac{AC}{A'C'}$ (§412).

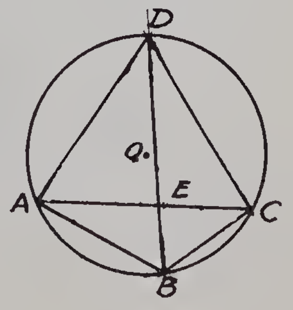
7. Given $\triangle ABC \sim \triangle A'B'C'$ with CM and $C'M'$ corresponding medians, and CP and $C'P'$ bisectors of corresponding $\angle$s.
To prove that $\frac{CM}{C'M'} = \frac{CP}{C'P'}$.



Planning the Proof: 1. (Same as in Ex. 6). 2. We shall use Asmt. 7.

Proof: 1. $\triangle ABC \sim \triangle A'B'C'$ and CM and $C'M'$ are corresponding medians (Given). 2. $\therefore \frac{CM}{C'M'} = \frac{AC}{A'C'}$ (Ex. 6). 3. CP and $C'P'$ are bisectors of corresponding $\angle$s (Given). 4. $\therefore \frac{CP}{C'P'} = \frac{AC}{A'C'}$ (Ex. 12, p. 409). 5. $\therefore \frac{CM}{C'M'} = \frac{CP}{C'P'}$ (Asmt. 7).

8. Planning the Proof: 1. We can prove $\triangle$s $\sim$ as in Ex. 3. 2. We shall use §415.



Proof: 1. $\angle ABD = \angle DBC$ (Given). 2. $\angle ADB = \angle ACB$ (§334). 3. $\therefore \triangle BEC \sim \triangle BDA$ (§415). 4. $\angle BAE = \angle BDC$ (§334). 5. $\therefore$ From 1 and 4, $\triangle BEA \sim \triangle BDC$ (§415).

9. Set stakes at A and B. Set a stake at O from which A and B are visible. Set a fourth stake at D which is in line with A and O. Measure AO, BO, and OD. Compute the value of x in the proportion, $\frac{x}{OD} = \frac{BO}{AO}$. Set a fifth stake at E in line with B and O making OE = x. Then measure the length of DE. Finally, find the value of AB in the proportion, $\frac{AB}{DE} = \frac{BO}{OE}$.

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1. The student might choose any 2 of the following: §§420, 425, 427, 428.

2. a. $\frac{AD}{CD} = \frac{CD}{DB}$; $\frac{5}{10} = \frac{10}{DB}$; DB = 20.

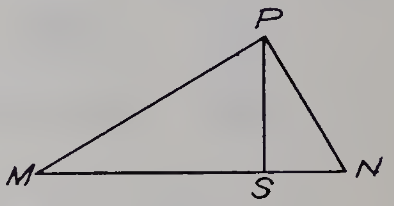
b. $\frac{AD}{CD} = \frac{CD}{DB}$; $\frac{AD}{8} = \frac{8}{4}$; AD = 16. AB = AD + DB = 20.

c. AD = 35 - 7 = 28. $\frac{AD}{CD} = \frac{CD}{DB}$; $\frac{28}{CD} = \frac{CD}{7}$; $\overline{CD}^2 = 196$; CD = ±14; CD = 14.

d. Let x = DB. $\frac{AB}{BC} = \frac{BC}{BD}$. $\frac{x + 20}{24} = \frac{24}{x}$; $x^2 + 20x = 576$; $x^2 + 20x + 100 = 676$; x + 10 = ±26; x = 16, or -36. BD = 16.

e. $\frac{AD}{AC} = \frac{AC}{AB}$; $\frac{15}{AC} = \frac{AC}{40}$; $\overline{AC}^2 = 600$; AC = $10\sqrt{6}$.

3. a. $\frac{9}{PS} = \frac{PS}{4}$ (§425). Solving, $\overline{PS}^2 = 36$; whence PS = 6.



b. $\frac{21}{56} = \frac{56}{SN}$ (§425). Solving, 21SN = 3136; whence SN = 149.33+. SN = 149.3. Ans.

c. $\frac{MS}{24} = \frac{24}{18}$ (§425). Solving, 18MS = 576; whence MS = 32.

d. $\frac{18}{PS} = \frac{PS}{20}$ (§425). Solving, $\overline{PS}^2 = 360$; whence PS = 18.97+. PS = 19.0. Ans.

e. $\frac{MS}{12.3} = \frac{12.3}{5.4}$ (§425). Solving, 5.4MS = 151.29; whence MS = 28.01+. MS = 28.0. Ans.

4. a. $\frac{3}{CD} = \frac{CD}{27}$ (§428). Solving, $\overline{CD}^2 = 81$; whence CD = 9.

b. $\frac{4}{CD} = \frac{CD}{20 - 4}$ (§428). Solving, $\overline{CD}^2 = 64$; whence CD = 8.

c. $\frac{AD}{9} = \frac{9}{16.2}$ (§428). Solving, 16.2AD = 81; whence AD = 5.

d. $\frac{8.2}{CD} = \frac{CD}{24.6}$ (§428). Solving, $\overline{CD}^2 = 201.72$; whence CD = 14.20+. CD = 14.2. Ans.

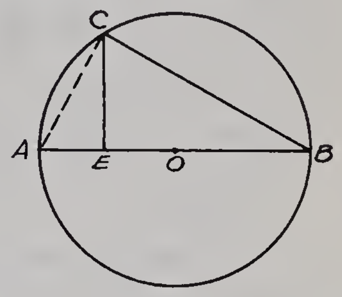
5. a. $\frac{4}{BC} = \frac{BC}{9}$ (§427). Solving, $\overline{BC}^2 = 36$; whence BC = 6.

b. $\frac{4}{8} = \frac{8}{DA + 4}$ (§427). Solving, 4DA + 16 = 64; whence DA = 12.

c. $\frac{AB - 10}{12} = \frac{12}{AB}$ (§427). Solving, $\overline{AB}^2 - 10AB = 144$, or $(AB - 5)^2 = 169$; whence AB - 5 = 13, and AB = 18.

6. Given the $\odot O$ with C any point on $\odot O$, CE $\perp$ to a diameter AB, and BC drawn.

To prove that $\overline{BC}^2 = 2AO \times BE$.

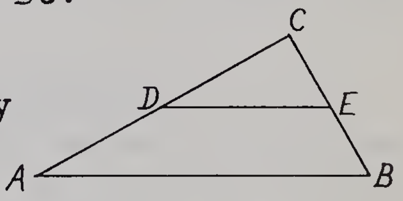


Planning the Proof: 1. We can prove that a line segment is the mean proportional between two other line segments by §§420, 425, 427, 428, and Asmt. 2. We shall use §427, Asmt. 6.

Proof: 1. Draw AC (Asmt. 11). 2. AB is a diameter of $\odot O$ (Given). 3. $\therefore \angle ACB$ is a rt. $\angle$ (§§335, 14). 4. $\therefore \triangle ACB$ is a rt. $\triangle$ (§57). 5. CE $\perp$ AB (Given). 6. $\therefore \overline{BC}^2 = AB \times BE$ (§427). 7. AB = 2AO (§§23, 277). 8. $\therefore \overline{BC}^2 = 2AO \times BE$ (Asmt. 6).

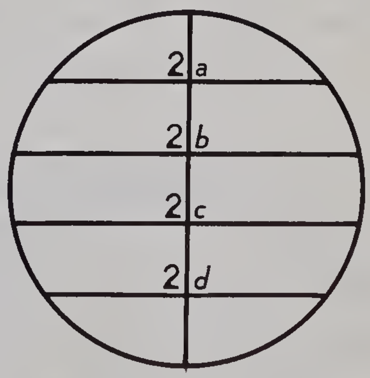
7. Given the $\triangle ABC$ with DE bisecting AC and BC.

To prove that $\triangle DEC \sim \triangle ABC$.



Planning the Proof: 1. We can prove $\triangle$s $\sim$ by §§412, 415, 416, 418, 422, 423, 424. 2. We shall use §412.

Proof: 1. DC = $\frac{1}{2}$AC and EC = $\frac{1}{2}$BC (Given). 2. $\therefore$ DE $\parallel$ AB (§410b). 3. $\therefore \angle CDE = \angle A$ and $\angle CED = \angle B$ (§158a). 4. $\angle C = \angle C$ (Iden.). 5. DE = $\frac{1}{2}$AB (§218a). 6. From 1 and 5, $\frac{CD}{CA} = \frac{CE}{CB} = \frac{DE}{AB}$ (Asmt. 4, 7). 7. $\therefore \triangle DEC \sim \triangle ABC$ (§412).



8. Each segment of the diameter = 15 ÷ 5 = 3 in. Let 2a, 2b, 2c, and 2d be the lengths of the chords $\perp$ diameter at the end of 3 in., 6 in., 9 in., and 12 in., respectively. The diameter bisects these chords (§309a). Then $\frac{3}{a} = \frac{a}{12}$ (§428), whence $a^2 = 36$, and a = 6. $\therefore$ The chord of length 2a is 12 in. long. $\frac{6}{b} = \frac{b}{9}$ (§428), whence $b^2 = 54$, and $b = 3\sqrt{6}$. $\therefore$ The chord of length 2b is $6\sqrt{6}$ in. long. The chord of length 2c = $6\sqrt{6}$ in. and the chord of length 2d = 12 in.

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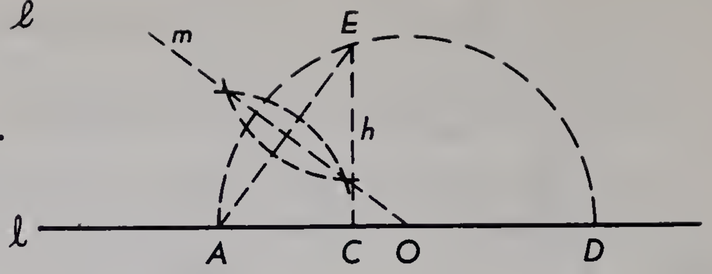
1. Direct application of §429, with m = 2 in., and n = $4\frac{1}{2}$ in.

2. Direct application of §429.



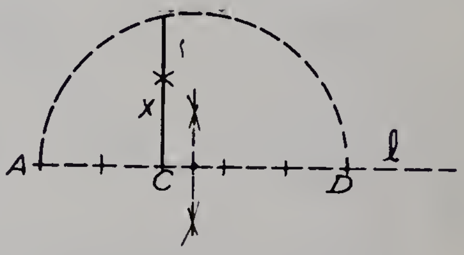
_3_. Construction: 1. On line ℓ construct $AC = 3$ units (§9). 2. At C construct $h \perp \ell$ (§178b). 3. On h construct $CE = 4$ units (§9). 4. Draw AE (Asmt. 11). 5. Construct m the $\perp$ bisector of AE and extend it to intersect ℓ in O (§177b). 6. With O as center and OA as radius construct a circle (Asmt. 15). Extend ℓ to intersect the circle at D (Asmt. 13).

CD is the required third proportional.

Proof: 1. $EC \perp \ell$ (Const.). 2. AE is not $\perp \ell$ (§94). 3. $m \perp AE$ (Const.). 4. m is either $\parallel \ell$ or intersects ℓ (§100). 5. If $m \parallel \ell$, then $AE \perp \ell$ (§114). 6. This is impossible (Statement 2). 7. ∴ the assumption $m \parallel \ell$ is false (§103). 8. ∴ the statement m intersects ℓ is true, that is, the point O exists (§100). 9. $AO = OE$ (§97b). 10. ∴ E lies on the circle with O as center and OA as radius (Asmt. 42). 11. Then $\frac{AC}{EC} = \frac{EC}{CD}$ (§428). 12. $AC = 3$ and $EC = 4$ (Const.). 13. $\therefore \frac{3}{4} = \frac{4}{CD}$ (Asmt. 6). 14. ∴ CO is the third proportional to 3 and 4 (§385).

_4_. (1) Construction: 1. On any line ℓ lay off $AC = 2$ units and $CD = 3$ units (§9). 2. Construct x, the mean proportional between AC and CD (§429). 3. Measure x.

Then $x = \sqrt{6} = 2.4$

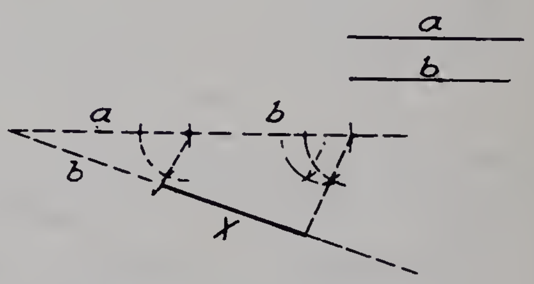


Proof: 1. $x^2 = AC \times CD$, $AC = 2$, and $CD = 3$ (Const.). 2. $\therefore x^2 = 2 \times 3 = 6$ (Asmt. 6). 3. $\therefore x = \sqrt{6} = 2.4$ (Asmt. 8).

(2) Arithmetically, the student should find that $\sqrt{6} = 2.4$ to the nearest tenth.

(3) Geometric.

_5_. Construction: 1. In $x = \frac{b^2}{a}$, b is the mean proportional between a and x (§385). 2. Construct x so that $\frac{a}{b} = \frac{b}{x}$ (§411a).

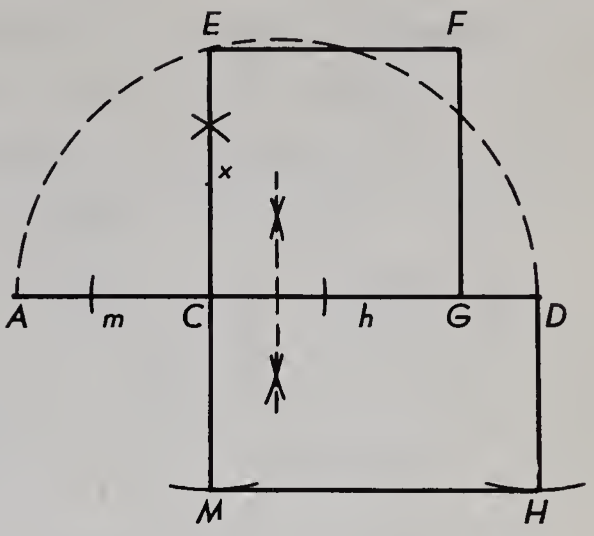


Then x is the required segment.

Proof: 1. $\frac{a}{b} = \frac{b}{x}$ (Const.). 2. $\therefore ax = b^2$ (§386). 3. $\therefore x = \frac{b^2}{a}$ (Asmt. 4).

6. Construction: 1. Construct x so that $\frac{m}{x} = \frac{x}{n}$ (Direct application of §429). 2. Upon x construct square CEFG (Ex. 1, p. 224).

Square CEFG is the required square.



Proof: 1. Extend CE to m so that CM = m (§9). 2. With CM and CD as adjacent sides construct rectangle MHDC (Ex. 5, p. 224). 3. CD = n (Const.). 4. Area CMHD = mn (§228). 5. Area square CEFG = x^2 (§233). 6. But $\frac{m}{x} = \frac{x}{n}$ (Const.). 7. $x^2 = mn$ (§386). 8. Area square CEFG = area CMHD (Asmt. 6).

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1. Let the side of inscribed square = c. Area large square = $(a + b)^2 = a^2 + 2ab + b^2$. Area inscribed square = c^2. Area of the four △s = $4(\frac{1}{2}ab) = 2ab$. Then $a^2 + 2ab + b^2 = c^2 + 2ab$ (Asmt. 7, 9). Then $a^2 + b^2 = c^2$ (Asmt. 2).

2. Planning the Proof: 1. We can show that a △ is a rt. △ by §255. 2. We shall use §255.

Proof: 1. $n^2 + (\frac{n^2 - 4}{4})^2 = n^4 + \frac{n^4 - 8n^2 + 16}{16} = \frac{n^4 + 8n^2 + 16}{16}$ (Iden.). 2. $(\frac{n^2 + 4}{4})^2 = \frac{n^4 + 8n^2 + 16}{16}$ (Iden.). 3. $n^2 + (\frac{n^2 - 4}{4})^2 = (\frac{n^2 + 4}{4})^2$ (Asmt. 7). 4. ∴ If n, $\frac{n^2 - 4}{4}$, and $\frac{n^2 + 4}{4}$ are the sides of a △, the △ is a rt. △ (§255).

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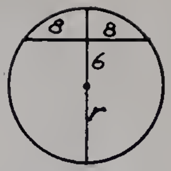
1. Let x = the length of the fourth segment. Then 6x = 4 x 8 (§431); whence 6x = 32, and x = $5\frac{1}{3}$. $5\frac{1}{3}$ in. Ans.

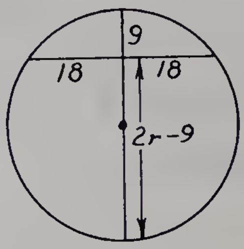
2. Let x = the length of 1 segment. Then 23 - x = the length of the other segment, and x(23 - x) = 6 x 15 (§431), or $x^2 - 23x + 90 = 0$. Solving by formula, $x = \frac{23 \pm \sqrt{529 - 360}}{2} = \frac{23 \pm 13}{2}$ = 18 or 5. Then 23 - x = 5 or 18. 18 in., 5 in. Ans.

3. Let 2r = the diameter of the ⊙ drawn ⊥ to the given chord.



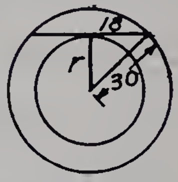
Then $(r + 6)$ and $(r - 6)$ are the segments of the diameter formed by the intersection of the chord, and $(r + 6)(r - 6) = 8 \times 8$ (§431). Solving, $r^2 - 36 = 64$; whence $r^2 = 100$, and $r = 10$. Then the diameter $= 2r = 20$. 20 in. Ans.

4. Let x = the length of 1 segment. Then $25 - x$ = the length of the other, and $x(25 - x) = 6 \times 14$ (§431). Solving, $25x - x^2 = 84$, or $x^2 - 25x + 84 = 0$; whence $(x - 21)(x - 4) = 0$, and $x = 21$ or 4. Then $25 - x = 4$ or 21. 4 in., 21 in. Ans.



5. Let r = the radius of the ⊙. Then 9 and $2r - 9$ are the segments of the diameter intersecting the given chord in its midpoint, and $9(2r - 9) = 18 \times 18$ (§431). Solving, $18r - 81 = 324$; whence $18r = 405$, and $r = 22\frac{1}{2}$. $22\frac{1}{2}$ in. Ans.

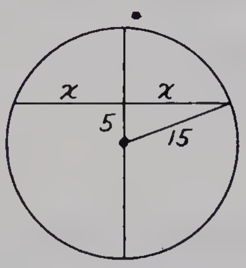
6. $EH \times HM = RH \times HS$ (§431). $6HM = 56$ (Asmt. 6). $HM = 9\frac{1}{3}$.



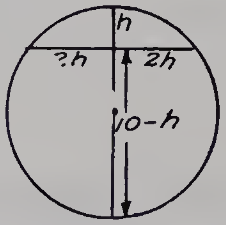
7. Let r = the radius of the smaller ⊙. Then $r^2 + 18^2 = 30^2$ (§§311b, 309a, 430); whence $r^2 = 576$, and $r = 24$. 24 in. Ans.

8. Let r = the radius of the arc. Then $9(2r - 9) = 12 \times 12$ (Ex. 5, above). Solving, $18r - 81 = 144$; whence $18r = 225$, and $r = 12.5$. 12.5 ft. Ans.

9. $x^2 + 25 = 225$ (§430). Then $x^2 = 200$.



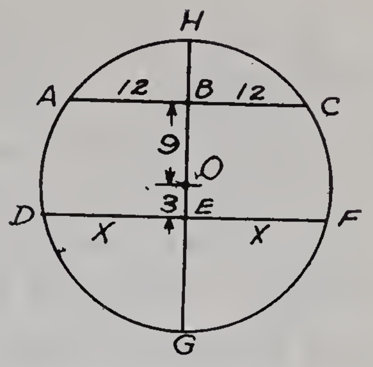
(Ex. 9)



(Ex. 10)

10. Let h = the height of the arch. Then $h(10 - h) = 2h \times 2h$ (Ex. 5, above). Solving, $10h - h^2 = 4h^2$, or $5h^2 - 10h = 0$; whence $5h(h - 2) = 0$, and $h = 0$ or 2. 2 ft. Ans.

11. $HG \perp AC$ and $HG \perp DF$ (Asmt. 21). Then $AB = BC = 12$ and $DE = EF$ (§309a). Let r = the radius. Then $HB = r - 9$ and



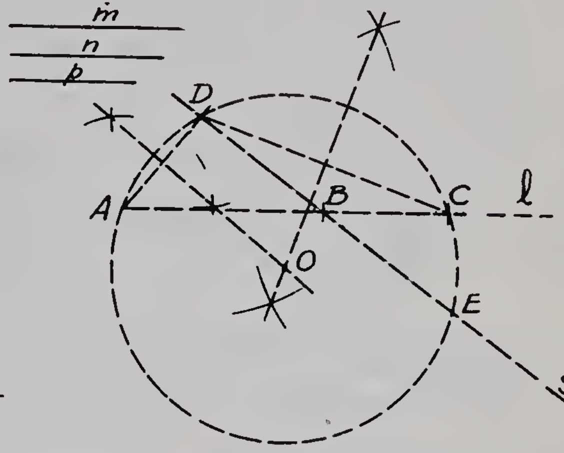
BG = r + 9 (Asmt. 6), and $(r - 9)(r + 9) = 12 \times 12$ (§431). Solving, $r^2 - 81 = 144$; whence $r^2 = 225$, and r = 15. Then HE = 18 and EG = 12 (Asmt. 6). Let 2x = the length of the second chord. Then x = DE = EF, and $x^2 = 18 \times 12$; whence $x = 6\sqrt{6}$ and $2x = 12\sqrt{6} = 12 \times 2.449 = 29.388$. 29.39 in. Ans.

12. Given 3 line segments, m, n, and p.

To construct the fourth proportional to m, n, and p.

Construction: 1. On any line ℓ construct AB = m and BC = p (§9). 2. Through B draw any other line s (Asmt. 12). 3. On s construct BD = n (§9). 4. Construct a ⊙O through A, C, and D intersecting s in point E (§366).

Then BE is the fourth proportional to m, n, and p.



Proof: 1. $AB \times BC = DB \times BE$ (§431). 2. $\therefore mp = n(BE)$ (Asmt. 6). 3. $\therefore \frac{n}{m} = \frac{p}{BE}$ (§387). 4. ∴ BE is the fourth proportional to m, n, and p (§384).

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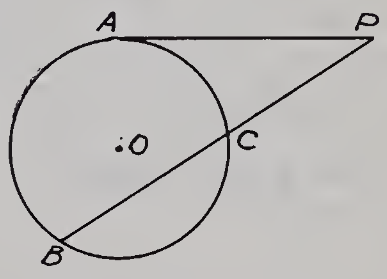


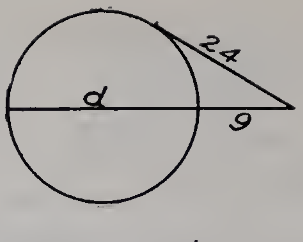
1. (1) PA = 7 in. and PB = 10 in. Then 10PC = 49 (§433), whence PC = 4.9.

(2) BC = PB - PC = 10 in. 4.9 in. = 5.1 in. 4.9 in., 5.1 in. Ans.

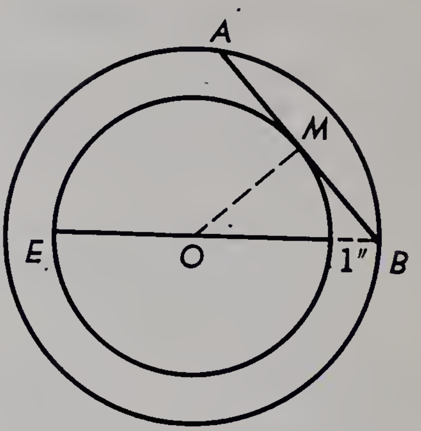
2. In the figure for Ex. 1, PB = 18 in. and PC = 8 in. Then $\overline{PA}^2 = 18 \times 8 = 144$ (§433); whence PA = 12.
12 in. Ans.

3. In the figure for Ex. 1, PB = 36 in. and BC = 10 in. Then $\overline{PA}^2 = 36(36 - 10) = 936$ (§433); whence $PA = \sqrt{936} = 30.594+$.
30.59 in. Ans.

4. In the figure for Ex. 1, PA = 24 in. and PB = (PC + 20) in. Then PC = PB - 20, and $PB(PB - 20) = 24^2$ (§433). Solving, $\overline{PB}^2 - 20PB = 576$, or $\overline{PB}^2 - 20PB + 100 = 676$; whence PB - 10 = 26, and PB = 36.
36 in. Ans.



5. Let d = the diameter. Then 9(d + 9) = 24^2 (§433). Solving, 9d + 81 = 576; whence 9d = 495, and d = 55. 55 in. Ans.

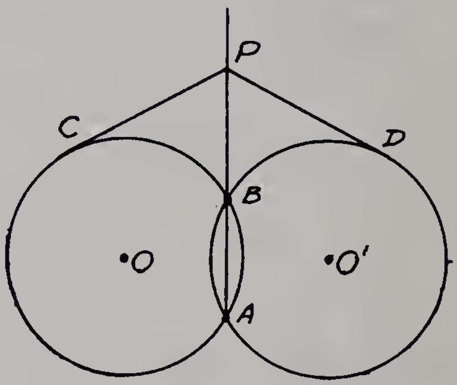


6. Let O be the common center of the circles. Draw OM to M the point of contact of AB with the small circle. OM ⊥ AB (§295). MB = $\frac{1}{2}$AB = $\frac{1}{2}$(10) = 5 (§290, Asmt. 6). MB is the tangent from B to the small circle (§299). Draw BO and extend it to intersect the small circle at E (beyond O). Let d = the diameter of the small circle. Then EB = d + 1. $\therefore 5^2 = (d + 1) \cdot 1$ (§433), d + 1 = 25, and d = 24. The diameter of the small circle is 24 in. and the diameter of the large circle is 24 + 2 = 26 (in.).

24 in., 26 in. Ans.

7. Given the Ⓢ O and O' intersecting in A and B, P any point in their common chord AB produced, and PC and PD tangents to Ⓢ O and O' respectively.

To prove that PC = PD.

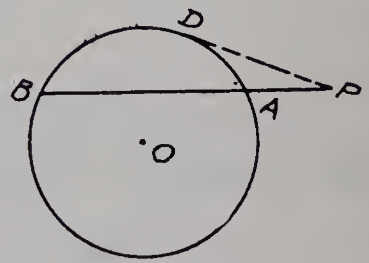


Planning the Proof: 1. We can prove two line segments = by §§97, 157, 217, 308, 309. 2. We shall use Asmt. 7, 8.

Proof: 1. PC is tangent to ⊙O, PD is tangent to ⊙O', and PA is a common secant (Given). 2. ∴ $\overline{PC}^2$ = PA x PB and $\overline{PD}^2$ = PA x PB (§433). 3. ∴ $\overline{PC}^2 = \overline{PD}^2$ (Asmt. 7). 4. ∴ PC = PD (Asmt. 8).

8. Given the ⊙O, P any point without ⊙O, and PB any secant to ⊙O from P intersecting ⊙O in A.

To prove that PB x PA = a constant.



Planning the Proof: 1. We can find the product of line segments by §§421, 425, 427, 428, 431, 433. 2. We shall use §433, Asmt. 7.

Proof: 1. Draw PD tangent to ⊙O at D (§336c). 2. PB is a secant intersecting ⊙O in A (Given). 3. ∴ PB x PA = $\overline{PD}^2$ (§433). 4. The secant from P may assume any one of several positions in the ⊙, but always PB x PA = $\overline{PD}^2$, which remains a constant.* 5. ∴ PB x PA = a constant (Asmt. 7).

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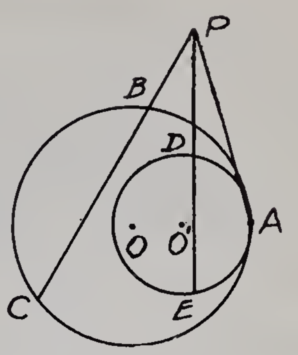
*For a discussion of contents and variables see §473, p. 464.

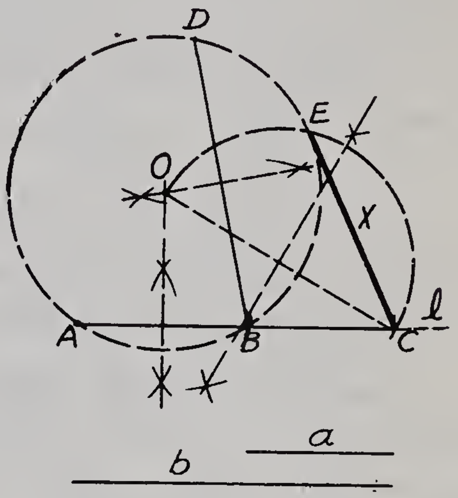


9. Given the $\circledS$ O and O' tangent internally, PA their common tangent, PC a secant to $\odot$O, and PE a secant to $\odot$O'.

To prove that PC x PB = PE x PD.

Planning the Proof: (Same as in Ex. 8.)

Proof: 1. PA is a common tangent, and PE is a secant to $\odot$O' and PC is a secant to $\odot$O (Given). 2. ∴ PC x PB = $\overline{PA}^2$ and PE x PD = $\overline{PA}^2$ (§433). 3. ∴ PC x PB = PE x PD (Asmt. 7).



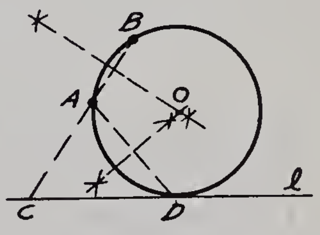
10. Given the 2 line segments a and b.

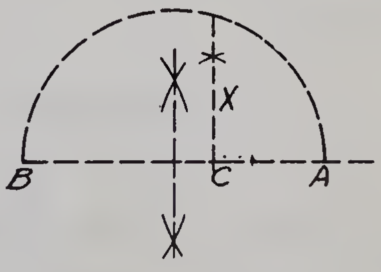
To construct a line segment x, so that $\frac{a}{x} = \frac{x}{b}$.

Construction: 1. On any line ℓ construct AC = b and CB = a (§9). 2. Construct $\odot$O through A and B and some other point as D (§366). 3. From C construct x, or CE, tangent to $\odot$O (§336c).

Then x is the required segment.

Proof: 1. AC = b, CB = a, CE is a tangent to $\odot$O, and CA is a secant to $\odot$O (Const.). 2. ∴ $\frac{CB}{CE} = \frac{CE}{CA}$ (§433). 3. ∴ $\frac{a}{x} = \frac{x}{b}$ (Asmt. 6).





11. Given 2 points A and B, and a line ℓ.

To construct a $\odot$ tangent to line ℓ and passing through A and B.

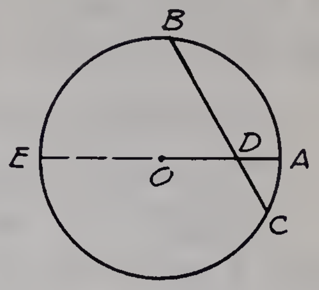
Construction: 1. Draw line AB meeting line ℓ in C (Asmt. 11, 13). 2. Construct x the mean proportional between CB and CA (§429). 3. On line ℓ construct CD = x (§9). 4. Construct a $\odot$O through A, B, and D (§366).

Then $\odot$O is the required $\odot$.

Proof: 1. $\odot$O passes through A, B, and D (Const.). 2. x^2 = CA x CB and x = CD (Const.). 3. ∴ $\overline{CD}^2$ = CA x CB (Asmt. 6). 4. If CD is not a tangent to the $\odot$ it is a secant and intersects the $\odot$ in another point as F (§277). 5. Then CD x CF = CA x CB (Ex. 8, p. 421). 6. ∴ CD x CF = $\overline{CD}^2$ (Asmt. 7). 7. ∴ CD = CF (Asmt. 4) which is impossible (Statement 4). 8. ∴ CD is tangent to $\odot$O. 9. D is in line ℓ (Const.). 10. ∴ $\odot$O is tangent to line ℓ (§277).

12. Given the $\odot O$ with the radius OA and a chord BC intersecting in D.

To prove that $BD \times DC = \overline{OA}^2 - \overline{OD}^2$.

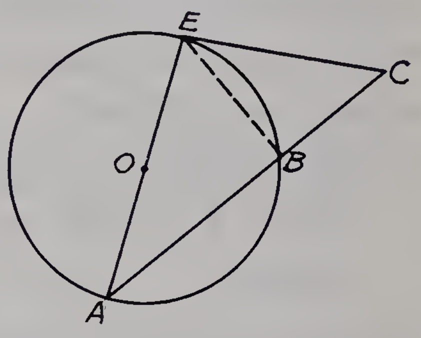


Planning the Proof: 1. We can find products of line segments by §§421, 425, 427, 428, 431, 433. 2. We shall use §431, Asmt. 6.

Proof: 1. BC is a chord of the $\odot$ (Given). 2. Extend AO to meet the $\odot$ in E (Asmt. 13). 3. Then EA is a diameter and a chord of the $\odot$ (§277). 4. Then $BD \times DC = ED \times DA$ (§431). 5. Then $BD \times DC = (AO + OD)(AO - OD)$, or $BD \times DC = \overline{AO}^2 - \overline{OD}^2$ (Asmt. 6).

13. Given the $\odot O$ with the diameter AE, the tangent CE, and the secant CA intersecting the $\odot$ in B.

To prove that $\overline{AE}^2 = AC \cdot AB$.



Planning the Proof: 1. We can find products of line segments by §§421, 425, 427, 428, 431, 433. 2. We shall use §427.

Proof: 1. Draw BE (Asmt. 11). 2. AE is a diameter of the $\odot$ (Given). 3. $\therefore EB \perp AB$ (§335). 4. $\angle AEC$ is a rt. $\angle$ (§311b). 5. $\therefore \triangle AEC$ is a rt. $\triangle$ (§57). 6. $\therefore \overline{AE}^2 = AC \cdot AB$ (§427).

PAGE 422

1. Yes, because a $\triangle$ is a polygon (§183b).

2. The perimeter of one polygon $= 3 + 7 + 6 + 8 + 15 = 39$ (Asmt. 9). Let p = the perimeter of the similar polygon whose longest side = 20. Then $\frac{p}{39} = \frac{20}{15}$ (§434). Solving, $15p = 780$; whence $p = 52$.

3. The perimeter of the first polygon $= (3 + 4 + 5 + 6 + 7)$ in. = 25 in. (Asmt. 9). The ratio of the perimeter of the second polygon to the perimeter of the first $= \frac{45}{25} = \frac{9}{5}$ (§381). Each side of the second polygon $= \frac{9}{5}$ of the corresponding side of the first (§434). Then $\frac{9}{5} \times 3 = 5\frac{2}{5}$; $\frac{9}{5} \times 4 = 7\frac{1}{5}$; $\frac{9}{5} \times 5 = 9$; $\frac{9}{5} \times 6 = 10\frac{4}{5}$; and $\frac{9}{5} \times 7 = 12\frac{3}{5}$.

$5\frac{2}{5}$ in., $7\frac{1}{5}$ in., 9 in., $10\frac{4}{5}$ in., $12\frac{3}{5}$ in. Ans.

PAGE 425

1. The perimeter of the first polygon = (6 + 10 + 12 + 14) in. = 42 in. (Asmt. 9). Let p = the perimeter of the similar polygon having its shortest side = 9 in. Then $\frac{p}{42} = \frac{9}{6}$ (§434). Solving, 6p = 378; whence p = 63. 63 in. Ans.

2. The perimeter of 1 quad. = (9 + 10 + 12 + 15) in. = 46 in. (Asmt. 9). The ratio of the perimeter of the second quad. to the perimeter of the first = $\frac{61\frac{1}{3}}{46} = \frac{184}{138}$ (§381, Asmt. 3). Each side of the second quad. = $\frac{184}{138}$ of the corresponding side of the first (§434). Then $\frac{184}{138} \times 9 = \frac{552}{46} = 12$; $\frac{184}{138} \times 10 = \frac{1840}{138} = 13\frac{1}{3}$; $\frac{184}{138} \times 12 = \frac{736}{46} = 16$; and $\frac{184}{138} \times 15 = \frac{920}{46} = 20$.

12 in., $13\frac{1}{3}$ in., 16 in., 20 in. Ans.

3. §§253, 415, 416, 417, 422, 423, 424, 430, 434, 435, 436.

4. 6 mi. = 380,160 in. $\frac{380,160}{100,000} \times 1 = 3.8016$. The map is a square 3.8 in. on a side.

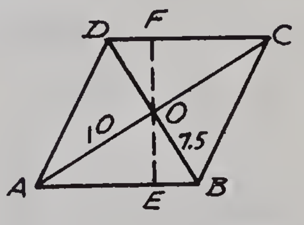
(1) The ratio of 4 mi. to 6 mi. = $\frac{2}{3}$. Then the distance between the post offices on the map = $\frac{2}{3} \times 3.8 = 2.53+$.

2.5 in. Ans.

(2) $d = s\sqrt{2}$ (§253). Then $d = 3.8\sqrt{2} = 3.8 \times 1.414 = 5.3732$.

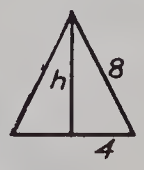
5.4 in. Ans.

5. (1) △AOB is a rt. △ (§§220a, 57). $OB = \frac{1}{2}DB = 7.5$ and $OA = \frac{1}{2}AC = 10$ (§217c, Asmt. 4, 6). Then $\overline{AB}^2 = 7.5^2 + 10^2$ (§430); whence $\overline{AB}^2 = 156.25$, and AB = 12.5. The perimeter = 4 × 12.5 = 50. 50 in. Ans.

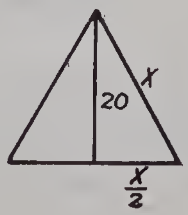


6. 1. The area of first rectangle = (a + 3)(2a − 1) (§273a).
2. The area of second rectangle = (3a + 2)(2a − 1) (§273a).
3. The required ratio = $\frac{(a + 3)(2a - 1)}{(3a + 2)(2a - 1)} = \frac{a + 3}{3a + 2}$.

7. $h^2 + 16 = 64$ (§430). Solving, $h^2 = 48$; whence h = 6.928. 6.93 in. Ans.



8. Let x = the length of each side. Then $x^2 = \frac{x^2}{4} + 20^2$ (§430). Solving, $\frac{3}{4}x^2 = 400$; whence $x^2 = 533.33+$, and $x = 23.093+$. 23.09 in. Ans.

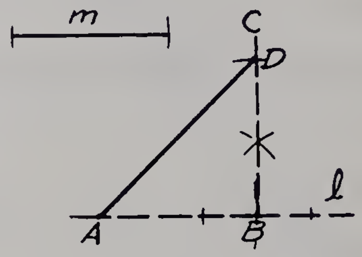


9. (1) $\overline{AC}^2 = 1^2 + 1^2 = 2$ (§430); whence $AC = \sqrt{2}$. (2) $\overline{AD}^2 = \overline{AC}^2 + 1^2 = (\sqrt{2})^2 + 1 = 3$ (§430); whence $AD = \sqrt{3}$. (3) Construct a rt. $\triangle ABC$ with AB = BC = 1 in. and $\angle B = 90°$. At C construct $\angle ACD = 90°$ and CD = 1 in. Draw AD. Then $AD = \sqrt{3}$ in.

10. 1. The area of each of the six $\triangle$ of the hexagon $= \frac{s^2}{4}\sqrt{3}$. 2. Then the area of the hexagon $= 6(\frac{s^2}{4}\sqrt{3}) = \frac{3s^2}{2}\sqrt{3}$. 3. Then $\frac{3s^2}{2}\sqrt{3} = 150\sqrt{3}$. 4. $3s^2\sqrt{3} = 300\sqrt{3}$. 5. $s^2 = 100$. 6. $s = 10$, each side.

11. Given the line segment m.

To construct a line segment $= m\sqrt{2}$.



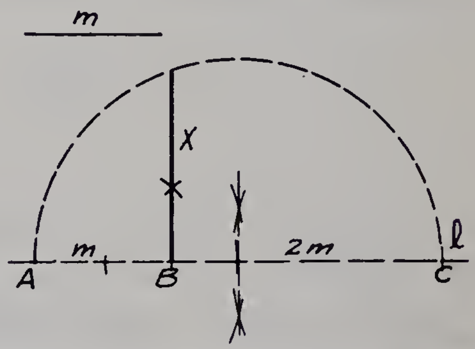
Construction: 1. On any line ℓ construct AB = m (§9). 2. At B construct BC $\perp$ ℓ (§178b). 3. Construct BD = m (§9). 4. Draw AD (Asmt. 11).

Then AD is the required segment.

Proof: 1. BD $\perp$ AB (Const.). 2. $\therefore \overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2$ (§430). 3. AB = m and BD = m (Const.). 4. $\therefore \overline{AD}^2 = 2m^2$ (Asmt. 6). 5. $\therefore AD = m\sqrt{2}$ (Asmt. 8).

12. Given the line segment m.

To construct a line segment $= m\sqrt{2}$.



Construction: 1. On any line ℓ construct AB = m and BC = 2m (§9). 2. Construct x the mean proportional between AB and BC (§429).

Then x is the required segment.

Proof: 1. AB = m, BC = 2m, and $\frac{AB}{x} = \frac{x}{BC}$ (Const.). 2. $\therefore \frac{m}{x} = \frac{x}{2m}$ (Asmt. 6). 3. $\therefore x^2 = 2m^2$ (§386). 4. $\therefore x = m\sqrt{2}$ (Asmt. 8).

13. Through O, draw EF $\perp$ AB (§178a, Asmt. 21). $\triangle BEO \sim \triangle AOB$ (§424). Then $\frac{OE}{AO} = \frac{OB}{AB}$, or $\frac{OE}{10} = \frac{7.5}{12.5}$ (§412, Asmt. 6). Solving, 12.5 OE = 75; whence OE = 6. $\triangle AOB \cong \triangle DOC$ (§95a). Then OF = OE (§97a); whence EF = 2 OE = 2 × 6 = 12. 12 in. Ans.

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1. Direct application of §437.

2. Let s be one side of the given hexagon, then construct a segment 2s and upon that segment construct a hexagon $\sim$ to the given hexagon as shown in §437.

PAGE 427

1. $\frac{16}{25}$. 2. $\frac{4}{5}$.

PAGE 428

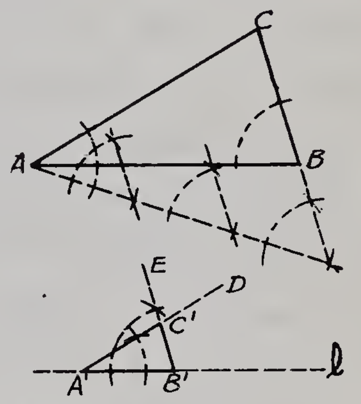
The area of A'B'C'D'E' = 9 times the area of ABCDE.

PAGE 429

1. (1) 1 : 2 (§417). (2) 1 : 2 (§434). (3) 1 : 4 (§438).

2. (1) By §440, $\frac{A}{A_1} = \frac{s^2}{s_1^2} = \frac{6^2}{18^2} = \frac{1}{9}$. (2) By §434, $\frac{P}{P_1} = \frac{6}{18} = \frac{1}{3}$.

3. By §440, $\frac{s^2}{s_1^2} = \frac{16}{1}$. Then $\frac{s}{s_1} = \frac{4}{1}$.



4. Given $\triangle$ABC.

To construct a $\triangle$A'B'C' $\sim$ $\triangle$ABC and $= \frac{1}{9}$ of it.

Construction: 1. Divide AB into 3 = parts (§224). 2. On any line ℓ take A'B' = $\frac{1}{3}$AB (§9). 3. At A' construct $\angle$B'A'D = $\angle$A and at B' construct $\angle$A'B'E = $\angle$B and let B'E intersect A'D in C' (§180).

Then $\triangle$A'B'C' is the required $\triangle$.

Proof: 1. A'B' = $\frac{1}{3}$AB, $\angle$A' = $\angle$A, and $\angle$B' = $\angle$B (Const.). 2. $\therefore$ $\triangle$A'B'C' $\sim$ $\triangle$ABC (§415). 3. $\frac{\triangle A'B'C'}{\triangle ABC} = \frac{\overline{A'B'}^2}{\overline{AB}^2}$ (§438). 4. But $\frac{A'B'}{AB} = \frac{1}{3}$ (Asmt. 4). 5. $\therefore$ $\frac{\triangle A'B'C'}{\triangle ABC} = \frac{1}{9}$ (Asmt. 6), or $\triangle$A'B'C' = $\frac{1}{9}\triangle$ABC (Asmt. 3).

5. 1. Let x = the corresponding side of the second polygon. 2. $\frac{64}{100} = \frac{6^2}{x^2}$ (§440). 3. $\frac{64}{100} = \frac{36}{x^2}$. 4. $64x^2 = 3600$. 5. $x^2 = 56.25$. 6. $x = 7.5$. The side is 7.5 inches.

6. By §440, $\frac{s^2}{20^2} = \frac{1152}{200}$. Solving, s = 48 (in.).

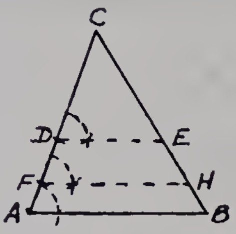
7. Let A = the area of the smaller square. Then A + 405 = the area of the larger square. By §440, $\frac{A}{A + 405} = \frac{(2)^2}{(3)^2}$. Solving, A = 324 and A + 405 = 729.

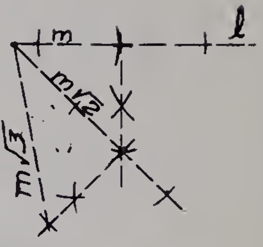
8. Let s = the side of the larger triangle. Then by §254,

$\frac{s}{2}\sqrt{3}$ = its altitude or the side of the smaller triangle. By §438, $\frac{\text{Area of smaller }\triangle}{\text{Area of larger }\triangle} = \frac{(\frac{s}{2}\sqrt{3})^2}{s^2} = \frac{3}{4}$.

9. The $\triangle$s are $\sim$ by §422. 10. $\frac{1}{4}$.

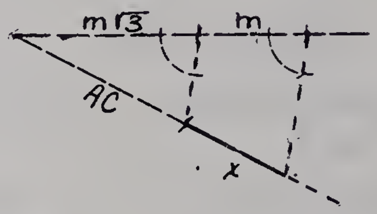
11. Given $\triangle ABC$.

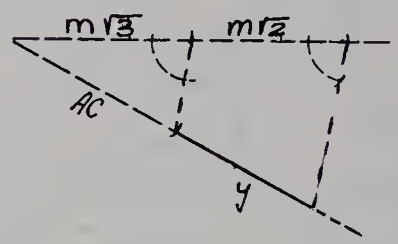




To divide $\triangle ABC$ into 3 = parts by drawing lines $\parallel$ to the base AB.

Construction: 1. On any line ℓ mark off a segment m (§9). 2. Construct $m\sqrt{2}$ and $m\sqrt{3}$ (Ex. 9, Ex. 12, page 425). 3. Find x, the 4th proportional to $m\sqrt{3}$, m, and AC (§411a). 4. On CA take CD = x (§9). 5. Construct a line through D $\parallel$ AB intersecting BC in E (§179). 6. Find y, the 4th proportional to $m\sqrt{3}$, $m\sqrt{2}$, and AC (§411a). 7. On CA take CF = y (§9). 8. Through F construct a line $\parallel$ AB intersecting BC in H (§179).





Then DE and FH divide $\triangle ABC$ into 3 = parts.

Proof: 1. DC = x and FC = y (Const.). 2. $\frac{m\sqrt{3}}{AC} = \frac{m}{x}$ and $\frac{m\sqrt{3}}{AC} = \frac{m\sqrt{2}}{y}$ (Const.). 3. $\therefore \frac{DC}{AC} = \frac{1}{\sqrt{3}}$ and $\frac{FC}{AC} = \frac{\sqrt{2}}{\sqrt{3}}$ (Asmt. 4, 3, 6). 4. DE $\parallel$ AB and FH $\parallel$ AB (Const.). 5. $\angle CDE = \angle A$ and $\angle CFH = \angle A$ (§158a). 6. $\therefore \triangle DEC \sim \triangle ABC$ and $\triangle FHC \sim \triangle ABC$ (§415). 7. $\frac{\triangle DEC}{\triangle ABC} = \frac{\overline{DC}^2}{\overline{AC}^2}$ and $\frac{\triangle FHC}{\triangle ABC} = \frac{\overline{FC}^2}{\overline{AC}^2}$ (§438). 8. From 3, $\frac{\overline{DC}^2}{\overline{AC}^2} = \frac{1}{3}$ and $\frac{\overline{FC}^2}{\overline{AC}^2} = \frac{2}{3}$ (Asmt. 8). 9. $\therefore \frac{\triangle DEC}{\triangle ABC} = \frac{1}{3}$ and $\frac{\triangle FHC}{\triangle ABC} = \frac{2}{3}$ (Asmt. 6). 10. $\triangle DEC = \frac{1}{3}\triangle ABC$ and $\triangle FHC = \frac{2}{3}\triangle ABC$ (Asmt. 3). 11. FHED = $\frac{1}{3}\triangle ABC$ (Asmt. 2). 12. ABHF = $\frac{1}{3}\triangle ABC$ (Asmt. 2). 13. $\therefore \triangle DEC$ = FHED = ABHF (Asmt. 7).

12. Given $\triangle$s ABC and AED with $\angle BAC = \angle EAD$.

To prove that $\frac{\triangle ABC}{\triangle AED} = \frac{AB \times AC}{AE \times AD}$.

Planning the Proof: 1. We can compare areas of polygons by §§272, 274, 438, 439, 440. 2. We shall use §272b, Asmt.

Proof: 1. $\triangle ABC$ and $\triangle AEC$ have the same altitude (§90). 2. $\frac{\triangle ABC}{\triangle AEC} = \frac{AB}{AE}$ (§272b). 3. In like manner, $\frac{\triangle AEC}{\triangle AED} = \frac{AC}{AD}$ (Statements

1-2). 4. $\therefore \frac{\triangle ABC}{\triangle AEC} \times \frac{\triangle AEC}{\triangle AED} = \frac{AB}{AE} \times \frac{AC}{AD}$ (Asmt. 3). 5. $\therefore \frac{\triangle ABC}{\triangle AED} = \frac{AB \times AC}{AE \times AD}$ (Asmt. 6).

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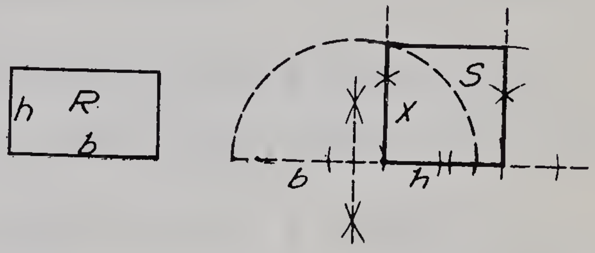
1. One method of solving this problem is to apply §275a directly.

Another method is as follows: 1. Denoting the given squares by S and T respectively, we have S = 9 and T = 16 (§273h). 2. S + T = 9 + 16 = 25 (Asmt. 1). 3. Since $25 = 5^2$, the square having a side of 5 in. is the required square (§273h). The square may then be constructed as described in the solution of Ex. 9, page 192.

2. Given rect. R with base b and height h.

To construct a square S = rect. R.



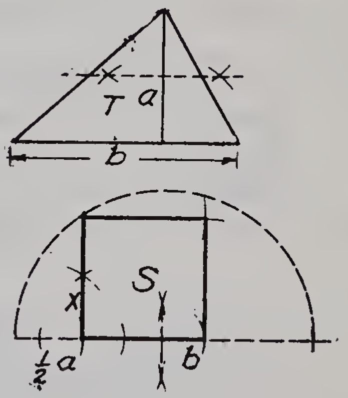
Construction: 1. Construct x, the mean proportional between h and b (§429). 2. Construct a square S having x as a side (Ex. 1, page 224).

Then S is the required square.

Proof: 1. $x^2 = bh$ (Const.). 2. But $x^2 = S$ (§273h). 3. R = bh (§273a). 4. ∴ S = R (Asmt. 6).

3. Given △T with base b and alt. a.

To construct a square S = T.



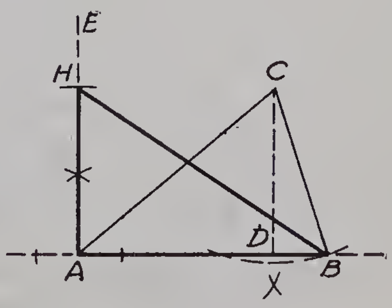
Construction: 1. Construct x, the mean proportional between $\frac{1}{2}a$ and b (§429). 2. Construct a square S having x as a side (§§185, 178b).

Then S = T.

Proof: 1. $x^2 = \frac{1}{2}ab$ (Const.). 2. $x^2 = S$ (§273h). 3. $\frac{1}{2}ab = T$ (§273c). 4. ∴ S = T (Asmt. 7).

4. Given △ABC with base AB.

To construct a rt. △ = △ABC and having AB as a base.



Construction: 1. From C, construct CD ⊥ AB (§178a). 2. At A, construct AE ⊥ AB (§178b). 3. On AE lay off AH = DC (§9). 4. Draw BH (Asmt. 11).

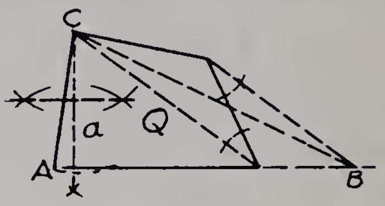
Then △ABH is the required △.

Proof: 1. AH ⊥ AB and CD ⊥ AB (Const.). 2. △ABH is a rt. △ (Const., §57). 3. AH is the altitude of △ABH on base AB and CD is the altitude of △ABC on base AB (§90). 4. AH = CD (Const.). 5. △ABC = $\frac{1}{2}$CD x AB and △ABH = $\frac{1}{2}$AH x AB (§273c). 6. $\frac{1}{2}$CD x AB = $\frac{1}{2}$AH x AB (Asmt. 6). 7. ∴ △ABC = △ABH (Asmt. 7).

5. Given a quad. Q.

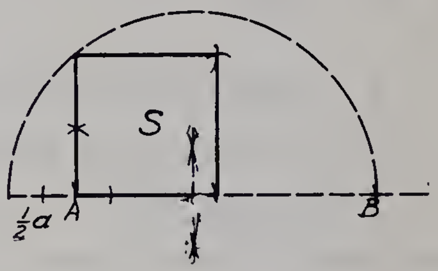
To construct a square S = Q.

Construction: 1. Construct a △ABC = Q (§275b). 2. Construct a square S = △ABC (Ex. 3, above).



Then S is the required square.

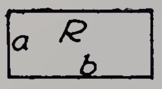
Proof: 1. S = △ABC and △ABC = Q (Const.). 2. ∴ S = Q (Asmt. 6).



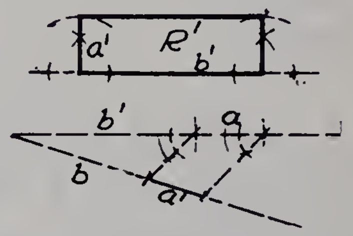
6. Same as Ex. 5 except that Q may have any number of sides.

7. Given a rect. R with base b and altitude a and a line segment b'.

To construct a rect. R' with base b' and = R.



Construction: 1. Construct a' the fourth proportional to b', a, and b (§411a). 2. Construct a rect. R' having b' as a base and a' as the altitude (§§185, 178b, 9).

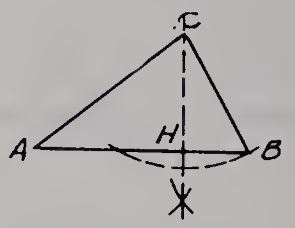


Then R' is the required rect.

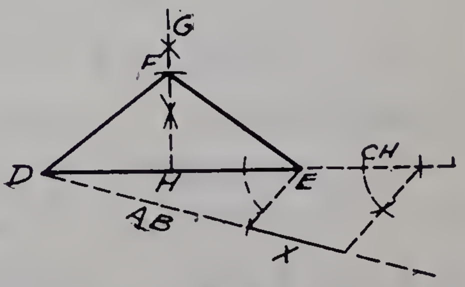
Proof: 1. R = ab and R' = a'b' (§273a). 2. $\frac{b'}{b} = \frac{a}{a'}$ (Const.). 3. ∴ a'b' = ab (§386). 4. ∴ R = R' (Asmt. 7).

8. Given △ABC and line segment DE.

To construct an isos. △DEF = △ABC and having DE as the base.



Construction: 1. From C, construct CH, the altitude of △ABC (§178a). 2. Construct x, the fourth proportional to DE, CH, and AB (§411a). 3. Construct HG the ⊥ bisector of DE (§177b). 4. On HG take HF = x (§9). 5. Draw DF and EF (Asmt. 11).



Then △DEF is the required △.

Proof: 1. $\frac{DE}{CH} = \frac{AB}{x}$ (Const.). 2. Then DE x x = AB x CH (§386).

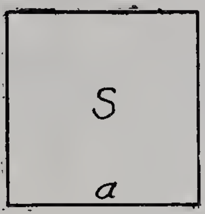
3. HF = x (Const.). 4. Then $\frac{1}{2}DE \times HF = \frac{1}{2}AB \times CH$ (Asmt. 4, 6). 5. But $\frac{1}{2}DE \times HF$ = area △DEF and $\frac{1}{2}AB \times CH$ = area △ABC (§273c). 6. ∴ △DEF = △ABC (Asmt. 7). 7. DF = FE (§378c). 8. ∴ △DEF is isos. (§56).

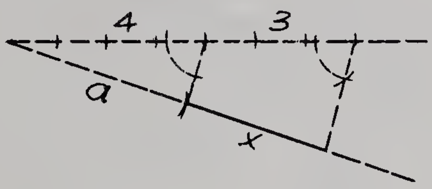
9. Given a square S with side a.

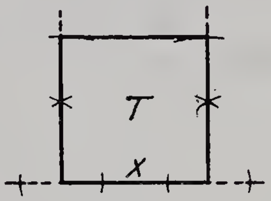
To construct a square $T = \frac{9}{16}S$.

Construction: 1. Construct a line segment x such that $\frac{4}{3} = \frac{a}{x}$ (§411a). 2. Construct a square T having x as a side (Ex. 1, page 224).

Then T is the required square.







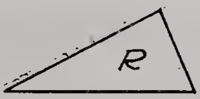
Proof: 1. T is a square (Const.). 2. $\frac{x}{a} = \frac{3}{4}$ (Const., §392). 3. ∴ $\frac{x^2}{a^2} = \frac{9}{16}$ (Asmt. 8). 4. But $S = a^2$ and $T = x^2$ (§273h). 5. ∴ $\frac{T}{S} = \frac{9}{16}$ (Asmt. 6). 6. ∴ $T = \frac{9}{16}S$ (Asmt. 3).

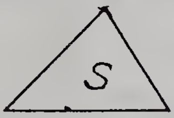
10. Given ⚠R and S.

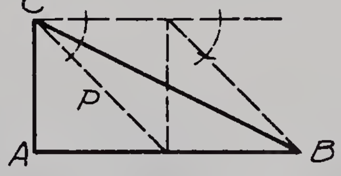
To construct a △ABC = △R + △S.

Construction: 1. *Construct square M = △R and square N = △S (Ex. 3, above). 2. *Construct square P = M + N (§262). 3. Construct △ABC = square P (§263).

Then ABC is the required △.







Proof: 1. △ABC = square P, square P = square M + square N, square M = △R, and square N = △S (Const.). 2. ∴ △ABC = △R + △S (Asmt. 6).

11. Given △ABC and line segment h.

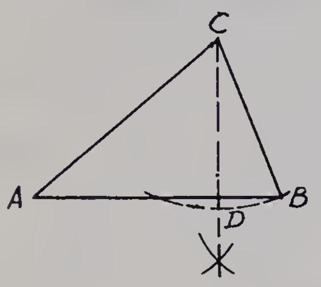
To construct a rt. △ with h the hypotenuse and = in area to △ABC.

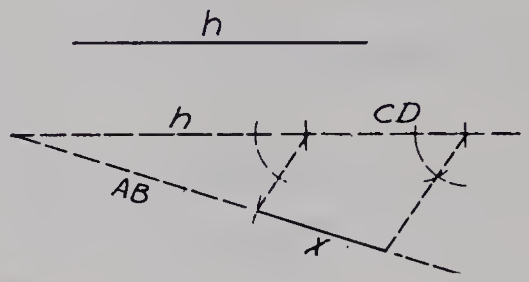
---

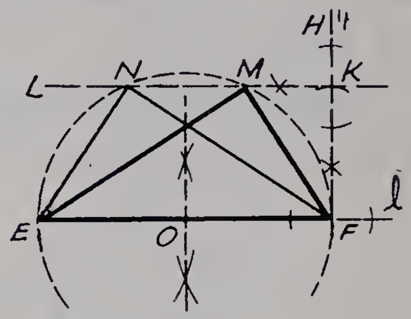
*Construction lines are not shown for the construction of squares M, N, and P.

Construction: 1. From C, construct the altitude CD of △ABC (§178a). 2. Construct x, the fourth proportional to h, CD, and AB (§411a). 3. On a line ℓ take EF = h (§9). 4. Bisect EF, calling the midpoint O (§177b). 5. With O as a center and OE as a radius draw a ⊙ (Asmt. 15). 6. At F, construct FH ⊥ EF (§178b). 7. On FH take FK = x (§9). 8. At K, construct KL ⊥ FK intersecting ⊙O in M and N (§178b). 9. Draw EM and FM (Asmt. 11).

Then △EFM is the required △.



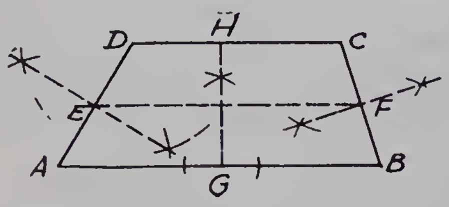


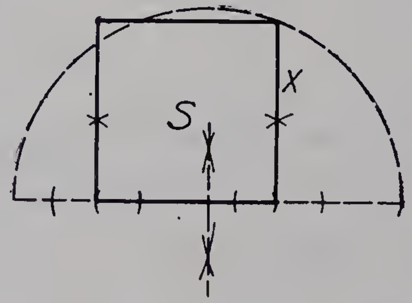


Proof: 1. ⊙O passes through F (§281b). 2. EF is a diameter and ∠EMF is an inscribed ∠ (Const.). 3. ∴ ∠EMF is a rt. ∠ (§319). 4. ∴ △EFM is a rt. △ (§57). 5. The altitude of △EFM upon EF = FK (§§160d, 193). 6. $\frac{h}{CD} = \frac{AB}{x}$ (Const.). 7. ∴ hx = CD x AB (§386). 8. ∴ $\frac{1}{2}$hx = $\frac{1}{2}$CD x AB (Asmt. 4). 9. △ABC = $\frac{1}{2}$CD x AB and △EFM = $\frac{1}{2}$EF x alt. on EF (§273c). 10. △EFM = $\frac{1}{2}$hx (Asmt. 6 and Const.). 11. ∴ △EFM = △ABC (Asmt. 7).

Discussion: 1. In the construction shown above, $\frac{1}{2}h > x$ and hence 2 solutions are possible. In addition to △EMF, it is also possible to draw △ENF as shown. This △ fulfills the required conditions by a proof similar to the one above. 2. If $\frac{1}{2}h < x$, it will be seen that there is no solution possible, since KL will not intersect the ⊙. 3. If $\frac{1}{2}h = x$, there will be only one solution, since KL will be tangent to the ⊙.

12. Given the trapezoid ABCD.
To construct a square = ABCD.





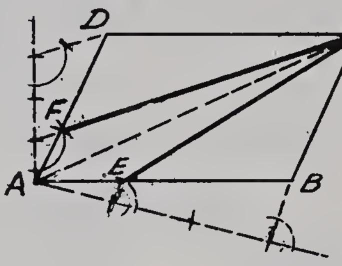
Construction: 1. Construct EF, the median of ABCD (§177b, Asmt. 11). 2. Construct GH the altitude of ABCD (§178b). 3. Construct x, the mean proportional between EF and GH (§429). 4. Construct a square S having x as a side (Ex. 1, page 224).

Then S is the required square.

Proof: 1. S is a square with side x (Const.). 2. HG is the altitude and EF the median of trapezoid ABCD (Const.). 3. $x^2 = GH \times EF$ (Const.). 4. $x^2 = S$ (§273h). 5. $ABCD = \frac{1}{2}GH(AB + DC)$ (§273e). 6. But $\frac{1}{2}(AB + DC) = EF$ (§207). 7. $\therefore ABCD = GH \times EF$ (Asmt. 6). 8. $\therefore S = ABCD$ (Asmt. 7).

13. Given ▱ABCD.

To divide ▱ABCD into 3 = parts by lines through C.



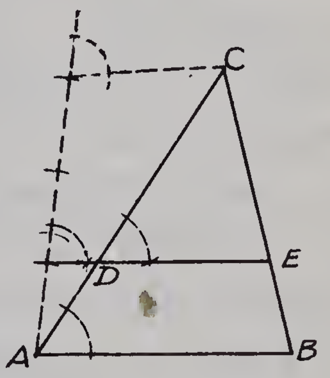
Construction: 1. Draw AC (Asmt. 11). 2. Divide AB into 2 parts so that $\frac{AE}{EB} = \frac{1}{2}$ (§401). 3. Draw CE (Asmt. 11). 4. In the same manner, divide AD and draw CF (Steps 2-3).

Then CE and CF are the required lines.

Proof: 1. $\frac{AE}{EB} = \frac{1}{2}$ (Const.). 2. $\frac{EB}{AE} = \frac{2}{1}$ (§392). 3. $\frac{AB}{AE} = \frac{3}{1}$ (§393). 4. $\frac{AE}{AB} = \frac{1}{3}$ (§392). 5. △ AEC and ABC have the same altitude (§90). 6. $\frac{\triangle AEC}{\triangle ABC} = \frac{AE}{AB}$ (§272b). 7. $\therefore \frac{\triangle AEC}{\triangle ABC} = \frac{1}{3}$ (Asmt. 7). 8. $\therefore \triangle AEC = \frac{1}{3}\triangle ABC$ (Asmt. 3). 9. $\triangle ABC = \frac{1}{2}$▱ABCD (§187). 10. $\therefore \triangle AEC = \frac{1}{6}$▱ABCD (Asmt. 6). 11. In like manner, $\triangle AFC = \frac{1}{6}$▱ABCD (Statements 3-10). 12. $\therefore$ polygon $AECF = \frac{1}{3}$▱ABCD (Asmt. 1). 13. From 9 and 10, $\triangle EBC = \frac{1}{3}$▱ABCD (Asmt. 2). 14. In like manner, $\triangle FCD = \frac{1}{3}$▱ABCD (Statements 9, 10, 13). 15. $\therefore \triangle EBC =$ polygon $AECF = \triangle FCD$ (Asmt. 7).

14. Given △ABC with base AB.

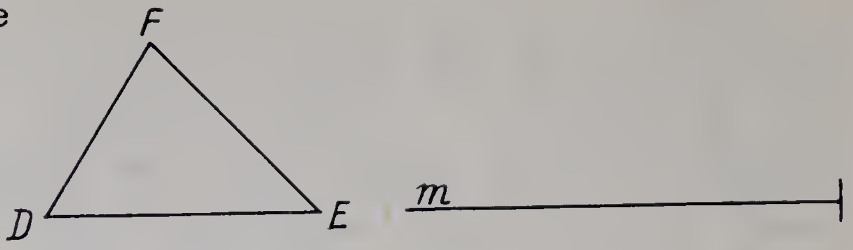
To construct a line ∥ AB so that the △ABC is divided into 2 parts having the ratio 4 : 5.



Construction: 1. Divide AC into 2 parts, CD and DA, so that $\frac{CD}{DA} = \frac{2}{1}$ (§401). 2. Through D, construct a line ∥ AB and intersecting BC in E (§179).

Then DE is the required line.

Proof: 1. DE ∥ AB (Const.). 2. $\frac{CD}{DA} = \frac{2}{1}$ (Const.). 3. $\frac{DA}{CD} = \frac{1}{2}$ (§392). 4. $\frac{CA}{CD} = \frac{3}{2}$ (§393). 5. $\angle CDE = \angle CAB$ (§158a). 6. $\triangle ABC \sim \triangle DEC$ (§415). 7. $\therefore \frac{\triangle ABC}{\triangle DEC} = \frac{\overline{CA}^2}{\overline{CD}^2}$ (§438). 8. But $\frac{\overline{CA}^2}{\overline{CD}^2} = \frac{9}{4}$ (Asmt. 8). 9. $\therefore \frac{\triangle ABC}{\triangle DEC} = \frac{9}{4}$ (Asmt. 7). 10. ∴ $\frac{ABED}{\triangle DEC} = \frac{5}{4}$ (§393). 11. $\therefore \frac{\triangle DEC}{ABED} = \frac{4}{5}$ (§392).



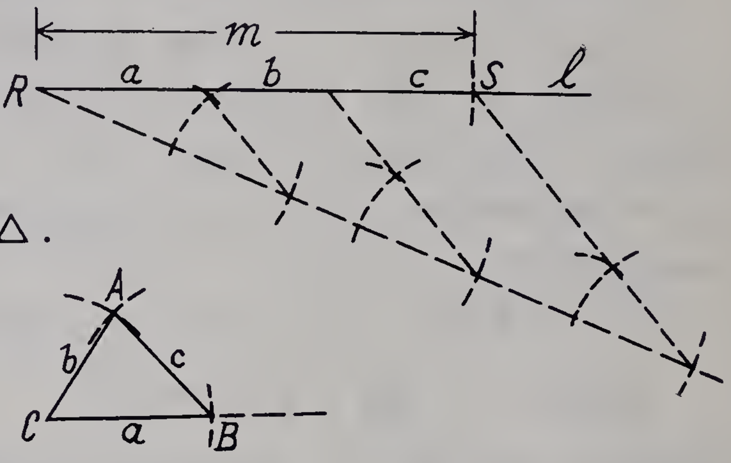
15. Given △DEF and line segment m.

To construct △ABC which is ~ △DEF and has a perimeter = m.

Construction: 1. On a line ℓ lay off a segment RS = m (§9). 2. Divide RS into three segments a, b, and c, proportional to DE, DF, and FE (§401). 3. Construct △ABC, having sides = a, b, and c respectively (§181a).

Then △ABC is the required △.

Proof: 1. AB = c, BC = a, and AC = b (Const.). 2. $\frac{a}{DE} = \frac{b}{DF} = \frac{c}{EF}$ (Const.). 3. $\therefore \frac{BC}{DE} = \frac{AC}{DF} = \frac{AB}{EF}$ (Asmt. 6). 4. ∴ △ABC ~ △DEF (§423). 5. AB + BC + AC = a + b + c (Asmt. 1). 6. a + b + c = m (Asmt. 9, Const.).



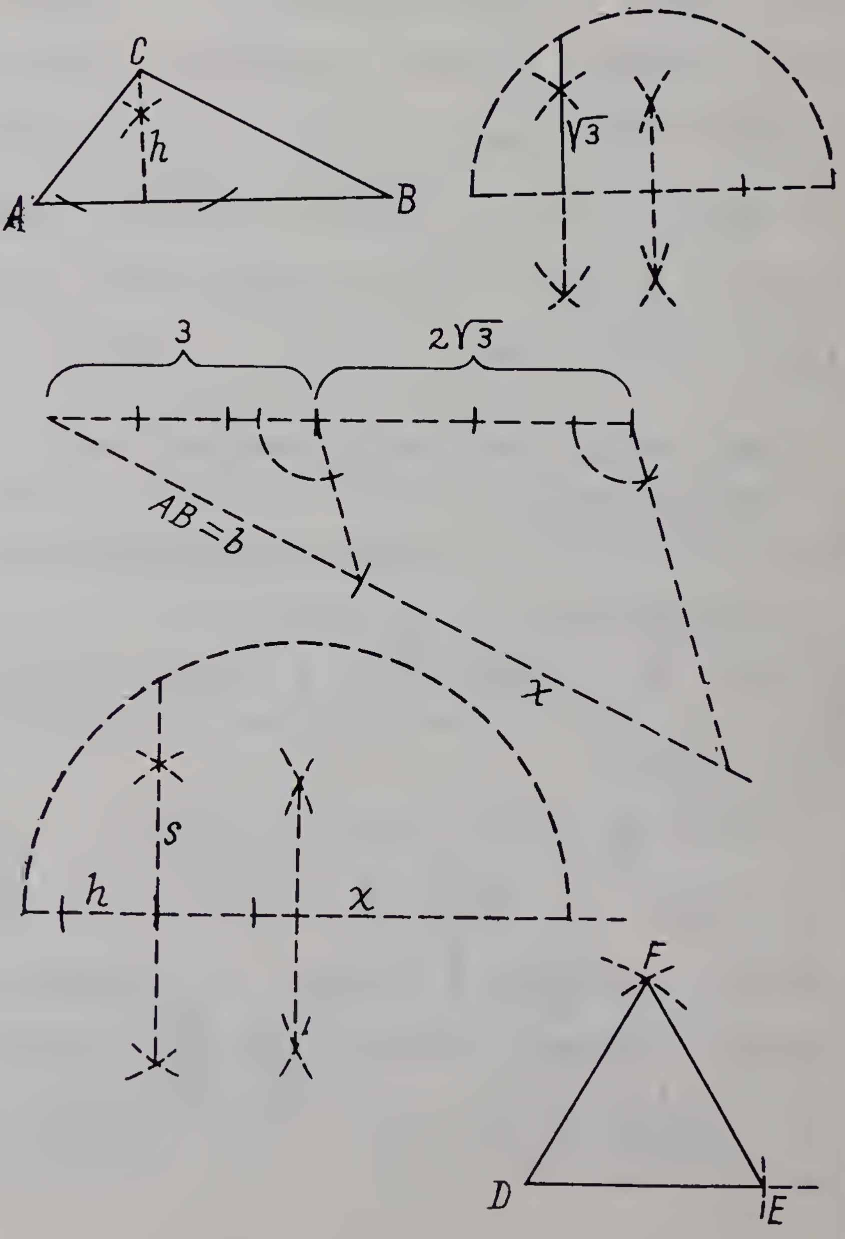
16. Given △ABC.

To construct an equilateral △ = △ABC.

Construction: 1. Construct the altitude h of △ABC (§178a). 2. Construct $\sqrt{3}$ (Similar to Ex. 2, page 416). 3. Construct x, the fourth proportional to 3, AB (or b), and $2\sqrt{3}$ (§400). 4. Construct s, the mean proportional between x and h (§429). 5. Construct equilateral △DEF having a side = s (§181a).

Then △DEF is the required △.

Proof: 1. △DEF is equilateral (Const.). 2. $\frac{3}{b} = \frac{2\sqrt{3}}{x}$ (Const.).



3. $3x = 2b\sqrt{3}$ (§386). 4. $x = \frac{2b\sqrt{3}}{3}$ (Asmt. 4). 5. $\frac{h}{s} = \frac{s}{x}$ (Const.). 6. Then $s^2 = hx$ (§386). 7. Then $s^2 = \frac{2b\sqrt{3h}}{3}$ (Asmt. 6). 8. $\frac{s^2}{4} = \frac{b\sqrt{3h}}{6}$ (Asmt. 4). 9. $\frac{s^2\sqrt{3}}{4} = \frac{1}{2}bh$ (Asmt. 3). 10. $\triangle DEF = \frac{s^2\sqrt{3}}{4}$ (§273f). 11. $\triangle ABC = \frac{1}{2}bh$ (§273c). 12. $\therefore \triangle DEF = \triangle ABC$ (Asmt. 7).

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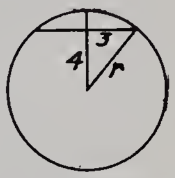
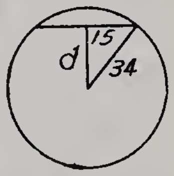
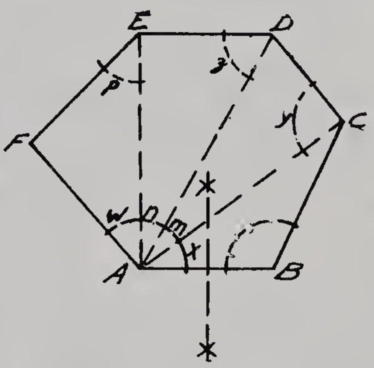
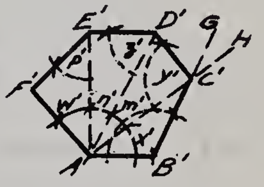


1. Let r = the length of the radius. Then $r^2 = 4^2 + 3^2$ (Asmt. 21, §§309a, 430); whence $r^2 = 25$, and $r = 5$. 5 in. Ans.



2. Let d = the distance from the chord to the center of the ⊙ (Asmt. 21). Then $d^2 + 15^2 = 34^2$ (§§309a, 430); whence $d^2 = 931$, and $d = 30.512+$.
30.51 in. Ans.





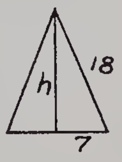
3. Given the hexagon ABCDEF.

To construct a polygon ~ ABCDEF and having its sides $\frac{1}{2}$ as long.

Construction: (This construction is based on §437 with $A'B' = \frac{1}{2}AB$.)

Proof: 1. In $\triangle$s A'B'C' and ABC, $\angle x' = \angle x$ and $\angle B' = \angle B$ (Const.). 2. $\therefore \triangle A'B'C' \sim \triangle ABC$ (§415). 3. $\therefore \frac{B'C'}{BC} = \frac{A'B'}{AB} = \frac{A'C'}{AC}$ (§412). 4. But $\frac{A'B'}{AB} = \frac{1}{2}$ (Const.). 5. $\therefore \frac{B'C'}{BC} = \frac{1}{2}$ (Asmt. 6). 6. Similarly, $\triangle A'C'D' \sim \triangle ACD$, $\frac{D'C'}{DC} = \frac{1}{2}$, $\triangle A'D'E' \sim \triangle ADE$, $\frac{D'E'}{DE} = \frac{1}{2}$, $\triangle E'F'A' \sim \triangle EFA$, and $\frac{E'F'}{EF} = \frac{F'A'}{FA} = \frac{1}{2}$ (Statements 1-5). 7. $\therefore$ A'B'C'D'E'F' ~ ABCDEF (§436).

4. $d = s\sqrt{2}$ (§253). Then $7\sqrt{2} = s\sqrt{2}$ (Asmt. 6); whence $s = 7$ (Asmt. 4).

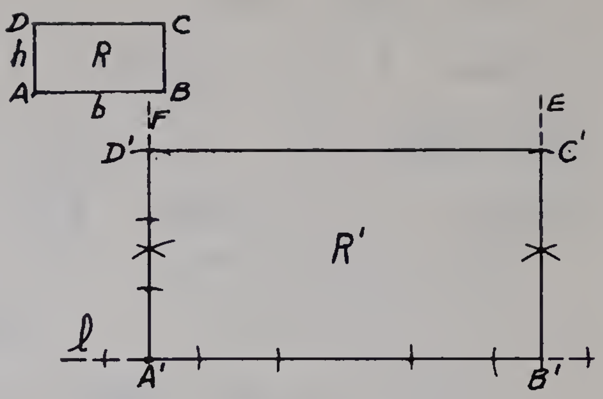


5. One of the equal sides = $\frac{1}{2}(50 - 14) = 18$ (in.). Let h = the altitude. Then $h^2 + 49 = 324$ (§430); whence $h^2 = 275$, and $h = 16.583$. 16.58 in. Ans.

6. (1) Given the rectangle R with base b and altitude h.

To construct a rectangle R' ~ R and having a base = 3b.

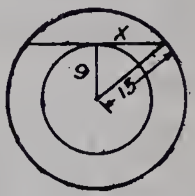
Construction: 1. On any line ℓ construct A'B' = 3AB (§9). 2. At A' and B' construct A'F and B'E respectively ⊥ A'B' (§178b). 3. On A'F construct A'D' = 3AD and on B'E construct B'C' = A'D' (§9). 4. Draw D'C' (Asmt. 11).



Then rectangle R' is the required rectangle.

Proof: 1. $A'D' \perp A'B'$ and $B'C' \perp A'B'$ (Const.). 2. $\therefore A'D' \parallel B'C'$ (§160d). 3. $A'D' = B'C'$ (Const.). 4. $\therefore A'B'C'D'$ is a ▱ (§223c). 5. $\angle A'$ is a rt. $\angle$ (Const.). 6. $\therefore$ ▱$A'B'C'D'$ is a rect. (§185). 7. $A'B' = 3AB$ and $A'D' = 3AD$ (Const.). 8. $B'C' = A'D'$ and $D'C' = A'B'$ (§217a). 9. ABCD is a rect. (Given). 10. $\therefore AD = BC$ and $DC = AB$ (§217a). 11. $\therefore B'C' = 3BC$ and $D'C' = 3DC$ (Asmt. 7). 12. $\frac{A'D'}{AD} = \frac{3}{1} = \frac{B'C'}{BC} = \frac{3}{1} = \frac{D'C'}{DC} = \frac{3}{1} = \frac{A'B'}{AB} = \frac{3}{1}$ (Asmt. 4, 7). 13. The corresponding $\angle$s of R and R' are = (§§185, 50a). 14. $\therefore R \sim R'$ (§412).

(2) $\frac{R'}{R} = \frac{9}{1}$ (§440).



7. Let 2x = the length of the chord. Then $x^2 + 9^2 = 15^2$ (§§309a, 311b, 430). Solving, $x^2 = 144$; whence x = 12. Then 2x = 24. 24 in. Ans.

8. Let x = the number of square inches in the area of the second polygon. Then $\frac{x}{48} = \frac{9^2}{6^2}$ (§440). Solving, x = 108 (sq.in.).

9. No, because the corresponding sides are not proportional. Since the altitude of a rect. = a side (§217b), we have $\frac{56}{18} = \frac{27}{24}$, or 1344 = 486, which is obviously untrue.

10. (1) $\overline{CD}^2 = AD \times DB$ (§428). AD = 26 − 8 = 18. Then $\overline{CD}^2 = 8 \times 18 = 144$; whence CD = 12. 12 in. Ans.

(2) $\overline{AC}^2 = AD \times AB$ (§427). Then $\overline{AC}^2 = 18 \times 26 = 468$; whence AC = 21.633+. 21.63 in. Ans.

(3) $\overline{BC}^2 = DB \times AB$ (§427). Then $\overline{BC}^2 = 8 \times 26 = 208$; whence BC = 14.422. 14.42 in. Ans.

11. Let x and x' represent two corresponding sides of the two similar triangles and let p and p' represent their perimeters. Then $\frac{16}{25} = \frac{x^2}{x'^2}$ (§440). Then $\frac{4}{5} = \frac{x}{x'}$ (Asmt. 8). But $\frac{p}{p'} = \frac{x}{x'}$ (§434). Then $\frac{p}{p'} = \frac{4}{5}$ (Asmt. 7).

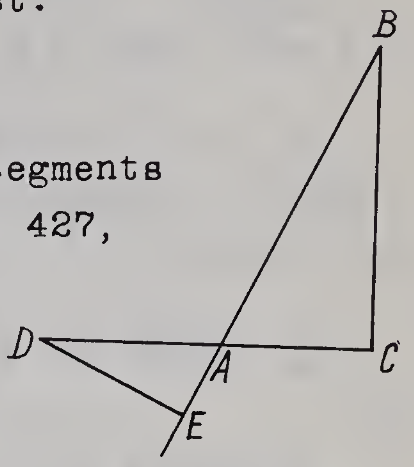
12. Let p and p' represent the perimeters and A and A' represent the areas of the two similar polygons.

(1) Then $\frac{p}{p'} = \frac{15}{18}$, or $\frac{5}{6}$ (§434). (2) $\frac{A}{A'} = \frac{5^2}{6^2}$, or $\frac{25}{36}$ (§440).

13. Given △ABC, ∠C a rt. ∠, BAE and DAC st. lines, and DE ⊥ EB.

To prove that DA : AB = DE : CB.

Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 417, 420, 425, 427, 428, 433. 2. We shall use §412.

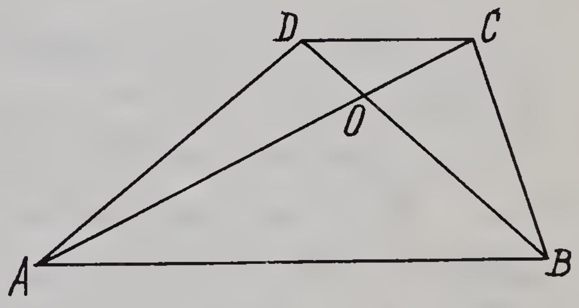


Proof: 1. ∠DAE = ∠BAC (§50d). 2. DE ⊥ BE (Given). 3. △DAE is a rt.△ (§57). 4. △ACB is a rt. △ (Given). 5. Rt. △ACB ~ rt. △DAE (§416). 6. DA : AB = DE : CB (§412).

14. The areas of the given squares are 9, 16, and 144 (§273h). The area of the required square = 9 + 16 + 144 = 169. Let x = a side of the required square. Then $x^2 = 169$ (§273h); x = 13 (Asmt. 8).

15. (1) △AOB ~ △DOC (§50d, Ex, 13, page 409, and §422). Then $\frac{\triangle AOB}{\triangle COD} = \frac{3^2}{1^2}$, or $\frac{9}{1}$ (§438).

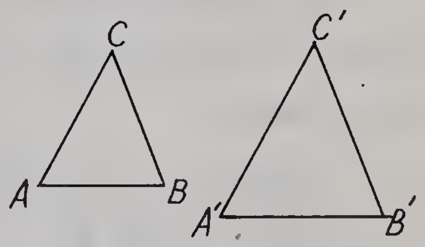
(2) △AOD = △BOC (Ex. 9, page 246). ∴ △AOD : △BOC = 1 : 1.



16. Given △ABC and △A'B'C' with AB ∥ A'B', BC ∥ B'C', and AC ∥ A'C'.

To prove that △ABC ~ △A'B'C'.

Planning the Proof: 1. We can prove △s ~ by §§412, 415, 416, 418, 422, 423, 424. 2. We shall use §415.



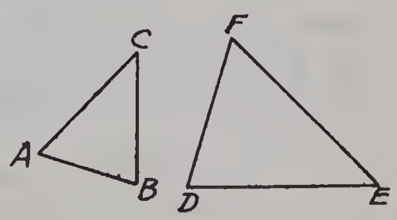
Proof: 1. AB ∥ A'B', BC ∥ B'C', and AC ∥ A'C' (Given). 2. ∴ ∠A = ∠A' and ∠B = ∠B' (§158b). 3. ∴ △ABC ~ △A'B'C' (§415).

17. Given △ABC and △DEF with AB ⊥ DF, BC ⊥ DE, and AC ⊥ EF.

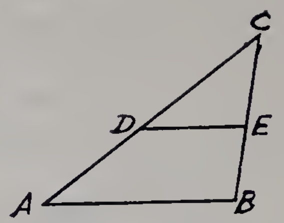
To prove that △ABC ~ △DEF.

Planning the Proof: (Same as in Ex. 16).

Proof: 1. AB ⊥ DF, BC ⊥ DE, and AC ⊥ EF (Given). 2. ∴ ∠B = ∠D and ∠A = ∠F (§158b). 3. ∴ △ABC ~ △DEF (§415).



18. Given the △ABC with DE ∥ AB and DE intersecting AC in D and BC in E.



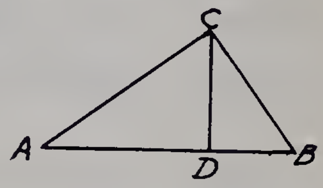
To prove that △DEC ∼ △ABC.

Planning the Proof: (Same as in Ex. 16).

Proof: 1. DE ∥ AB (Given). 2. ∴ ∠CDE = ∠A and ∠DEC = ∠B (§158a). 3. ∴ △DEC ∼ △ABC (§415).

19. Let x = the hypotenuse. Then $x^2 = (2ab)^2 + (a^2 - b^2)^2$ (§430). Solving, $x^2 = 4a^2b^2 + a^4 - 2a^2b^2 + b^4$, or $x^2 = a^4 + 2a^2b^2 + b^4$; whence $x = a^2 + b^2$.

20. Given the rt. △ABC with CD the altitude on the hypotenuse AB.

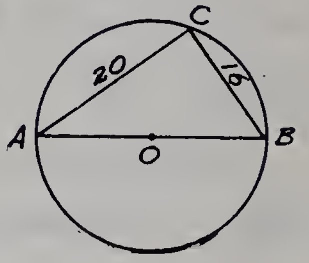


To prove that AC × BC = AB × CD.

Planning the Proof: 1. We can find the value of products of line segments by §§421, 425, 427, 428, 431, 433. 2. We shall use §421.

Proof: 1. CD is the altitude on the hypotenuse AB (Given). 2. Then △ABC ∼ △ACD (§424). 3. ∴ $\frac{AB}{AC} = \frac{BC}{CD}$, or AC × BC = AB × CD (§421).

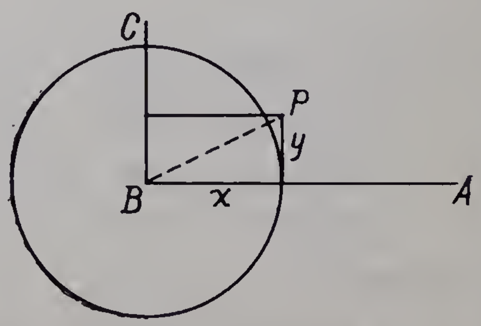
21. Let x = the length of the radius of the circumcircle. AB, the hypotenuse of the rt. △, is the diameter of the circumcircle (§§364, 205). Then, AB = 2x, and $4x^2 = 16^2 + 20^2$ (§430). Solving, $4x^2 = 656$; whence $x^2 = 164$, and $x = 2\sqrt{41} = 2 \times 6.403 = 12.806$. 12.81 ft. Ans.



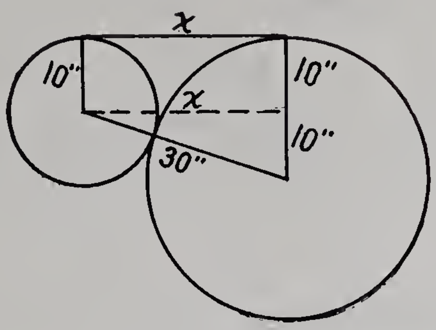
22. Planning the Proof: 1. We can prove a line segment the mean proportional by §§420c, 425, 427, 428, 433. 2. We shall use §420c.

Proof: 1. Draw DB (Asmt. 11). 2. ∠A = ∠A (Iden.). 3. ∠ADB is a rt. ∠ (§319). 4. BC is tangent to ⊙O at B (Given). 5. BC ⊥ AB (§295). 6. △ABC and △ABD are rt. △s (§57). 7. △ABD ∼ △ABC (§416). 8. AD : AB = AB : AC (§420c). 9. $\overline{AB}^2$ = AD × AC (§386). 10. AB = $\sqrt{AD \times AC}$ (Asmt. 8).

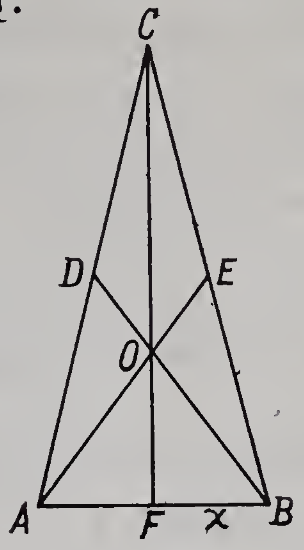
23. $\overline{BP}^2 = x^2 + y^2 = 25$. Then BP = 5. Then the locus is a circle whose center is B and radius is 5 (Asmt. 47).



24. $x^2 + 10^2 = 30^2$; $x^2 + 100 = 900$; $x^2 = 800$; whence $x = 20\sqrt{2}$, or 28.28. $20\sqrt{2}$ in., or 28.28 in. Ans.



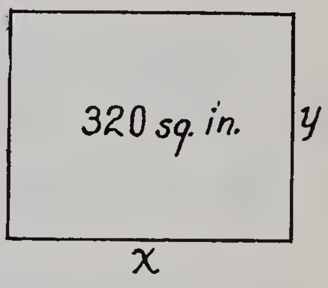
(Ex. 24)



(Ex. 25)

25. CF = 48, BD = 30, and AE = 30. BO = 20 and FO = 16 (§379d). Then $x^2 + 16^2 = 20^2$; $x^2 + 256 = 400$; $x^2 = 144$; $x = 12$. AB = 24. Area of $\triangle$ABC = $\frac{1}{2} \times 24 \times 48 = 576$. 576 sq.in. Ans.

26. Let x and y = the length and width of the rectangle, respectively, in inches. Then $2x + 2y = 72$, and $xy = 320$. Solving the first equation for x, $x + y = 36$, and $x = 36 - y$. Substituting this value for x in the second equation, $y(36 - y) = 320$. Solving, $36y - y^2 = 320$; $-y^2 + 36y - 320 = 0$; $y^2 - 36y + 320 = 0$; $(y - 20)(y - 16) = 0$; $y = 20$ or 16. Then $x = 16$ or 20. 16 in. and 20 in. Ans.



27. Let w = the width of the rectangle. Then the area of the rectangle = ℓw (§273a). $w^2 + \ell^2 = d^2$ (§430); $w^2 = d^2 - \ell^2$; and $w = \sqrt{d^2 - \ell^2}$. Substituting this value of w in the first equation, the area of the rectangle = $\ell\sqrt{d^2 - \ell^2}$.

28. a. 1. $\triangle$s ADC and DCE are rt. $\triangle$s (§57). 2. DE $\perp$ AC (Given). 3. $\triangle$AOD $\sim$ $\triangle$DOC and $\triangle$DOC $\sim$ $\triangle$COE (§424). 4. $\therefore$ $\triangle$AOD $\sim$ $\triangle$COE (§418).

b. $\triangle$AOD $\sim$ $\triangle$ADC (Statements 1, 2 of Part a, and §424).

c. 1. $\triangle$ADC $\sim$ $\triangle$DOC and $\triangle$DCE $\sim$ $\triangle$DOC (As in Part b). 2. $\therefore$ $\triangle$ADC $\sim$ $\triangle$DCE (§418).

d. $\frac{AD}{DC} = \frac{DC}{CE}$ (Statement 2 of Part c, and §412).

e. 1. AD = BC, DC = FE = AB, and CE = DF (§217a). 2. $\frac{DC}{CE} = \frac{AB}{DF}$ (Asmt. 4). 3. $\frac{AD}{DC} = \frac{DC}{CE}$ (Part d). 4. $\frac{BC}{FE} = \frac{AB}{DF}$ (Asmt. 6).

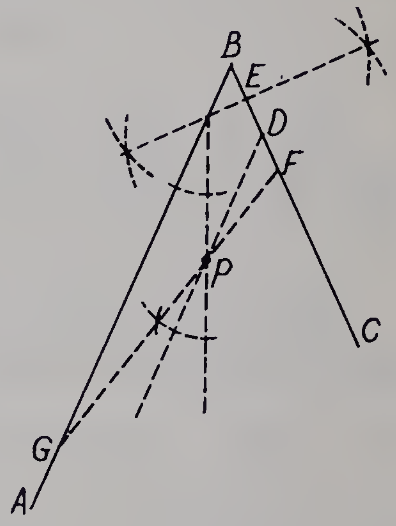
5. $\therefore \frac{AD}{DC} = \frac{DC}{CE} = \frac{AB}{DF} = \frac{BC}{FE}$ (Asmt. 7). 6. The corresponding ∡ of the rectangles are = (§§185, 50a). 7. ∴ Rect. ABCD ~ rect. FECD (§412).

29. Let h = the altitude of the triangle on side c. Then $h^2 + \frac{c^2}{4} = b^2$ (§430); $4h^2 + c^2 = 4b^2$ (Asmt. 3); $4h^2 = 4b^2 - c^2$ (Asmt. 2); $h^2 = \frac{4b^2 - c^2}{4}$ (Asmt. 4); $h = \frac{1}{2}\sqrt{4b^2 - c^2}$ (Asmt. 8). The area of the $\triangle = \frac{1}{2}c \times \frac{1}{2}\sqrt{4b^2 - c^2} = \frac{c}{4}\sqrt{4b^2 - c^2}$ (§273c).

30. Given point P within ∠ABC.

To construct a line through P whose end points are in AB and BC and which is divided by P into segments having the ratio 1 : 2.

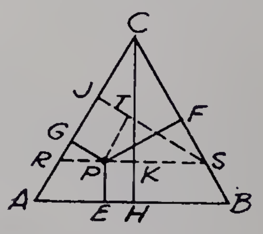
Construction: 1. Through P construct PD ∥ AB meeting BC in D (§179). 2. Bisect BD in E (§177b). 3. Take DF on BC = $\frac{1}{2}$BD (§9). 4. Draw FP meeting AB in G (Asmt. 11). 5. Then $\frac{FP}{PG} = \frac{1}{2}$.



Proof: 1. PD ∥ AB (Const.). 2. $\frac{DF}{BD} = \frac{FP}{PG}$ (§409a). 3. DF = $\frac{1}{2}$BD (Const.). 4. $\frac{DF}{BD} = \frac{1}{2}$ (Asmt. 4). 5. $\frac{FP}{PG} = \frac{1}{2}$ (Asmt. 6).

31. Given the equilateral △ABC, P any point within △ABC, PE ⊥ AB, PG ⊥ AC, and PF ⊥ BC.

To prove that PE + PG + PF = a constant.

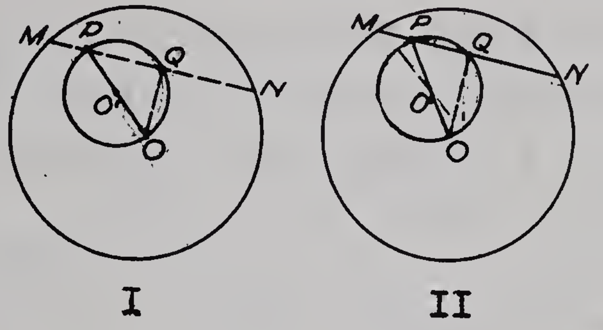


Planning the Proof: 1. We can find the value of sums of line segments by Asmt. 2. We shall use Asmt.

Proof: 1. Draw the altitude CH (§178a). 2. Through P draw a line ∥ AB intersecting AC in R, CH in K, and BC in S (§179). 3. Draw SJ ⊥ AC and PT ⊥ SJ (§178a). 4. AC = AB = BC (Given). 5. ∴ ∠A = ∠B = ∠C (§98a). 6. ∠CRS = ∠A and ∠CSR = ∠B (§158a). 7. ∴ ∠CRS = ∠CSR = ∠C (Asmt. 7). 8. PG ⊥ AC (Given). 9. ∴ From 3 and 8, PG ∥ TJ and PT ∥ GJ (§160d). 10. ∴ GPJT is a ▱ (§185). 11. ∴ GP = TJ (§217a). 12. ∠SRJ = ∠SPT (§158a). 13. ∴ From 7 and 12, ∠SPT = ∠PSF (Asmt. 7). 14. PS = PS (Iden.). 15. PF ⊥ BC (Given). 16. ∴ From 3 and 15, ◬PTS and PFS are rt. ◬ (§57). 17. ∴ △PTS ≅ △PFS (§156a). 18. ∴ PF = TS (§97a). 19. From 11 and 18, GP + PF = TJ + TS, or GP + PF = JS (Asmt. 1, 6, 9). 20. From 7, RS = CS (§157a). 21. From 1-3, ◬RJS and

CKS are rt. $\angle$s (Const. and §159a). 22. ∴ △s RJS and CKS are rt. △s (§57). 23. From 7 and 20, △RJS ≅ △CKS (§156a). 24. ∴ JS = CK (§97a). 25. PE ⊥ AB (Given). 26. From 1 and 25, PE ∥ KH (§160d). 27. PK ∥ EH (Const.). 28. ∴ EHKP is a ▱ (§185). 29. ∴ PE = KH (§217a). 30. From 19, 24, and 29, PG + PF + PE = CK + KH, or PG + PF + PE = CH (Asmt. 6, 1). 31. ∴ PG + PF + PE = a constant.

32. The locus of the midpoints of the chords of a given circle which pass through a given point within the circle is a circle having as a diameter the line joining the center of the given circle with the given point.



PART I.

Any point on the circle which is the locus is the midpoint of a chord of the given circle through the given point.

Given the ⊙O with P any point within it, ⊙O' on OP as a diameter, and Q any point on ⊙O'.

To prove that Q is the midpoint of MN, a chord of ⊙O through P.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309, 378. 2. We shall use §309a.

Proof: 1. Draw OQ (Asmt. 11). 2. OQ ⊥ MN (§335). 3. ∴ MQ = QN (§309a). 4. If Q coincides with P, MN is tangent to ⊙O' (§277), and O'P ⊥ MN (§311b). 5. ∴ MP = PN (§309a). 6. If Q coincides with O, MO = ON (§308a). 7. ∴ Any point on ⊙O' is the midpoint of a chord of ⊙O through P.

PART II.

The midpoint of any chord of the given circle through the given point is on the circle which is the locus.

Given the ⊙O with P any point within it, ⊙O' on OP as a diameter, and Q the midpoint of MN any chord of ⊙O through P.

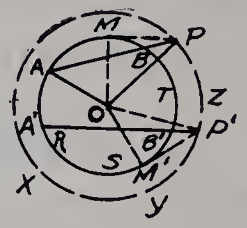
To prove that Q is on the ⊙O'.

Planning the Proof: 1. We can prove that a point is on a ⊙ by Asmt. 42, Asmt. 47, §378d. 2. We shall use §378d.

Proof: 1. Draw OQ (Asmt. 11). 2. OQ ⊥ MN (§311a). 3. △OQP is a rt. △ (§57). 4. ∴ Q is on ⊙O' (§378d); that is, the midpoint of any chord of ⊙O through P is on ⊙O'.

∴ ⊙O' is the locus of the midpoints of chords of ⊙O which pass through P (§§339, 342).

33. The required locus is a circle having the same center as the given circle and a radius equal to the distance from this center to the point P.



PART I.

For any point P on the circle which is the locus, PA x PB is constant.

Given concentric Ⓢ RST and XYZ with center O and radii OA and OP respectively, and PBA a secant to ⊙RST.

To prove that if a secant P'B'A' is drawn from any point P' of ⊙XYZ, P'A' x P'B' = PA x PB.

Planning the Proof: 1. We can find products of line segments = by §§421, 425, 427, 428, 431, 433. 2. We shall use §433, Asmt. 7.

Proof: 1. Let PM and P'M' be tangents to ⊙RST from P and P' respectively and draw OP', OM, and OM' (§330, Asmt. 11). 2. OM ⊥ PM and OM' ⊥ P'M' (§311b). 3. ∴ △OMP and △OM'P' are rt. △s (§§14, 57). 4. OM = OM' and OP = OP' (§308a). 5. ∴ △OMP ≅ △OM'P' (§156b). 6. ∴ PM = P'M' (§97a). 7. ∴ $\overline{PM}^2 = \overline{P'M'}^2$ (Asmt. 8). 8. PA · PB = $\overline{PM}^2$ and P'A' · P'B' = $\overline{P'M'}^2$ (§433). 9. ∴ PA · PB = P'A' · P'B' (Asmt. 7).

PART II.

For any point P placed such that PA x PB is constant, P is on the circle which is the locus.

Given concentric Ⓢ RST and XYZ with center O and radii OA and OP respectively, and PBA a secant to ⊙RST.

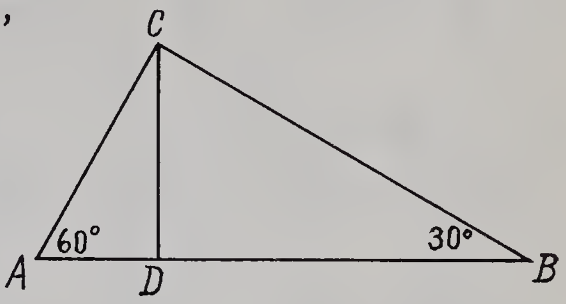
To prove that if a secant P'B'A' is so drawn from any point P' that P'A' x P'B' = PA x PB, then P' is on the ⊙XYZ.

Planning the Proof: 1. We can prove that a point is on a ⊙ by Asmt. 42, Asmt. 47, §378d. 2. We shall use Asmt. 42.

Proof: 1. Let PM and P'M' be tangents to ⊙RST from P and P' respectively and draw OP', OM, and OM' (§330, Asmt. 11). 2. PA · PB = $\overline{PM}^2$ and P'A' · P'B' = $\overline{P'M'}^2$ (§433). 3. PA · PB = P'A' · P'B' (Given). 4. ∴ $\overline{PM}^2 = \overline{P'M'}^2$ (Asmt. 7). 5. ∴ PM = P'M' (Asmt. 8). 6. OM = OM' (§308a). 7. OM ⊥ PM and OM' ⊥ P'M'

(§311b). 8. ∴ ∠OMP = ∠OM'P' (§14, Asmt. 19). 9. ∴ △OMP ≅ △OM'P' (§95a). 10. ∴ OP = OP' (§97a). 11. ∴ P' is on ⊙XYZ (Asmt. 42).

Then ⊙XYZ is the required locus (§§339, 342).

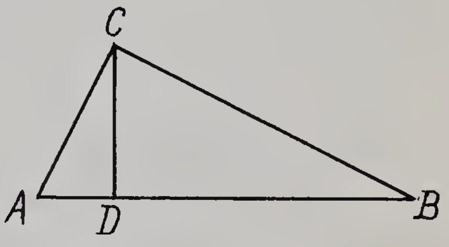


34. Given the rt. △ABC, ∠C a rt. ∠, ∠A = 60°, ∠B = 30°, and CD ⊥ AB.

To prove that AD : DB = 1 : 3.

Planning the Proof: 1. We can find ratios by §§395, 409, 412, 417, 420, 425, 427, 428, 433. 2. We shall use §427 and Asmt.

Proof: 1. CD ⊥ AB (Given). 2. ∠A = 60° and ∠B = 30° (Given). 3. $\frac{AD}{AC} = \frac{AC}{AB}$ (§427). 4. AC = $\frac{1}{2}$AB (§136). 5. $\frac{AD}{\frac{1}{2}AB} = \frac{\frac{1}{2}AB}{AB}$ (Asmt. 6). 6. AD = $\frac{1}{4}$AB (Asmt. 3). 7. ∴ DB = $\frac{3}{4}$AB (Asmt. 9, 2). 8. ∴ AD : DB = 1 : 3 (Asmt. 4).

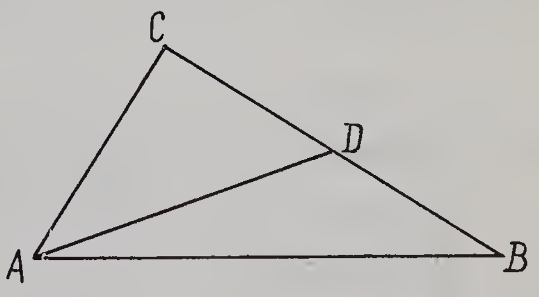


35. Given the rt. △ABC with ∠C = 90°, BC = 2AC, and altitude CD.

To prove that AD : DB = 1 : 4.

Planning the Proof: (Same as in Ex. 34).

Proof: 1. AC = $\frac{1}{2}$CB (Given). 2. $\frac{AD}{AC} = \frac{AC}{AB}$ (§427). 3. $\frac{AD}{\frac{1}{2}CB} = \frac{\frac{1}{2}CB}{AB}$ (Asmt. 6). 4. AD = $\frac{\frac{1}{4}\overline{CB}^2}{AB}$ (Asmt. 3). 5. $\frac{DB}{CB} = \frac{CB}{AB}$ (§427). 6. DB = $\frac{\overline{CB}^2}{AB}$ (Asmt. 3). 7. ∴ $\frac{AD}{DB} = \frac{1}{4}$ (Asmt. 4).



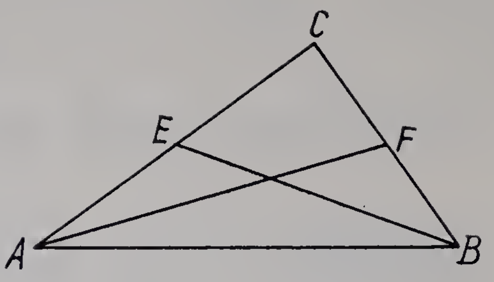
36. Given the rt. △ABC, ∠C the rt. ∠, and D the midpoint of BC.

To prove that $\overline{AB}^2 - \overline{AD}^2 = 3\overline{CD}^2$.

Planning the Proof: 1. We can prove relations of squares of line segments by §430 and Asmt. 2. We shall use §430 and Asmt.

Proof: 1. ∠C is a rt. ∠ and AB is the hypotenuse of △ABC (Given). 2. $\overline{AB}^2 = \overline{AC}^2 + \overline{CB}^2$ (§430). 3. $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2$ (§430). 4. $\overline{AB}^2 - \overline{AD}^2 = \overline{CB}^2 - \overline{CD}^2$ (Asmt. 2). 5. CB = 2CD (Given). 6. ∴ $\overline{CB}^2 = 4\overline{CD}^2$ (Asmt. 8). 7. ∴ $\overline{AB}^2 - \overline{AD}^2 = 4\overline{CD}^2 - \overline{CD}^2 = 3\overline{CD}^2$ (Asmt. 6).

37. Given rt. △ABC, ∠C the rt. ∠, BE bisecting AC in E, and AF bisecting BC in F.



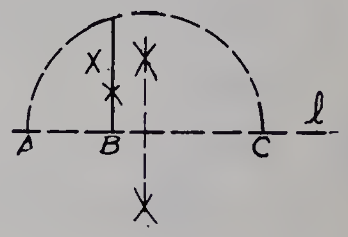
To prove that $4\overline{BE}^2 + 4\overline{AF}^2 = 5\overline{AB}^2$.

Planning the Proof: (Same as in Ex. 36).

Proof: 1. $\angle C$ is a rt. $\angle$ and AB is the hypotenuse of $\triangle ABC$ (Given). 2. $\overline{BE}^2 = \overline{BC}^2 + \overline{CE}^2$ and $\overline{AF}^2 = \overline{AC}^2 + \overline{CF}^2$ (§430). 3. $\overline{BE}^2 + \overline{AF}^2 = \overline{BC}^2 + \overline{AC}^2 + \overline{CE}^2 + \overline{CF}^2$ (Asmt. 1). 4. $4\overline{BE}^2 + 4\overline{AF}^2 = 4\overline{BC}^2 + 4\overline{AC}^2 + 4\overline{CE}^2 + 4\overline{CF}^2$ (Asmt. 3). 5. BC = 2CF and AC = 2CE (Given). 6. Then $\overline{BC}^2 = 4\overline{CF}^2$ and $\overline{AC}^2 = 4\overline{CE}^2$ (Asmt. 8). 7. $4\overline{BE}^2 + 4\overline{AF}^2 = 4\overline{BC}^2 + 4\overline{AC}^2 + \overline{AC}^2 + \overline{BC}^2$ (Asmt. 6). 8. $4\overline{BE}^2 + 4\overline{AF}^2 = 5\overline{BC}^2 + 5\overline{AC}^2$ (Asmt. 6). 9. $\overline{AB}^2 = \overline{BC}^2 + \overline{AC}^2$ (§430). 10. $5\overline{AB}^2 = 5\overline{BC}^2 + 5\overline{AC}^2$ (Asmt. 3). 11. $4\overline{BE}^2 + 4\overline{AF}^2 = 5\overline{AB}^2$ (Asmt. 6).

38. 1. $6x^2 = 6y^2$ (§431). 2. $x^2 = y^2$ (Asmt. 4). 3. $x = y$ (Asmt. 8). 4. $7y = 5x + 8$. 5. $2x = 8$. 6. $x = 4$. 7. $y = 4$. 8. $5x = 20$ and $7y = 28$. 9. The chords are 20 in. and 28 in.

39. Construction: 1. On any line ℓ construct AB = 2 units and BC = $3\frac{1}{2}$ units (§9). 2. Construct x, the mean proportional between AB and BC (§429).



Then $x = \sqrt{7}$.

Proof: 1. $x^2 = AB \times BC$, $AB = 2$, and $BC = 3\frac{1}{2}$ (Const.). 2. $\therefore x^2 = 2 \times 3\frac{1}{2}$, or $x^2 = 7$ (Asmt. 6). 3. $\therefore x = \sqrt{7}$ (Asmt. 8).

Discussion: It should be clearly understood that any 2 factors of 7 might be used for AB and BC.

40. Given the $\odot O$ with AB a tangent at B, AC a tangent at E, CD a tangent at D, EO a radius, and $AB \parallel CD$.

To prove that $\frac{AE}{EO} = \frac{EO}{EC}$.

Planning the Proof: 1. We can prove that a line segment is the mean proportional between the other line segments by §§420, 425, 427, 428, 433. 2. We shall use §420c and Asmt. 7.

Proof: 1. Draw AO and CO (Asmt. 11). 2. $AB \parallel CD$, CD and CE are tangents from C, and AE and AB are tangents from A (Given). 3. $\angle ECO = \frac{1}{2}\angle ECD$ and $\angle EAO = \frac{1}{2}\angle EAB$ (§300). 4. $\angle ECD + \angle EAB = 180°$ (§162). 5. $\therefore \angle ECO + \angle EAO = 90°$ (Asmt. 1, 6). 6. $\angle ECO + \angle EAO + \angle AOC = 180°$ (§120). 7. $\therefore \angle AOC = 90°$ (Asmt. 2). 8. $\therefore \triangle AOC$ is a rt. $\triangle$ (§57). 9. $EO \perp AC$ (§311b). 10. $\therefore \frac{AE}{EO} = \frac{EO}{EC}$ (§425).

41. Given the Ⓢ O and O' tangent internally at A, with AE a diameter of ⊙ O, AD a diameter of ⊙ O', and AB any chord of ⊙ O' drawn from A intersecting ⊙ O in C.

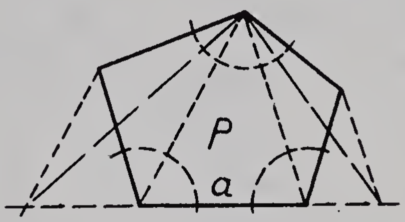
To prove that $\frac{AC}{CB}$ = a constant ratio.

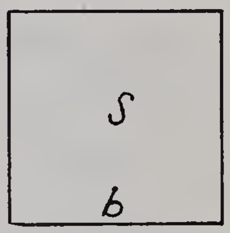
Planning the Proof: 1. We can find the values of ratios by §§395, 409, 412, 417, 420, 425, 427, 428, 433, 434. 2. We shall use §412 and Asmt. 6.

Proof: 1. Draw CE and BD (Asmt. 11). 2. Points A, O, and O' are in a st. line (§297). 3. AE and AD are diameters of Ⓢ O and O' respectively (Given). 4. ∴ ∠ ACE and ∠ ABD are rt. ∠s (§335). 5. ∠ A = ∠ A (Iden.). 6. ∴ △ ACE ∼ △ ABD (§416). 7. ∴ $\frac{AB}{AC} = \frac{AD}{AE}$ (§412). 8. But $\frac{AD}{AE}$ = a constant. ∴ as the position of B on the ⊙ changes, $\frac{AB}{AC}$ remains constant (Asmt. 6).

42. Given polygon P and square S.

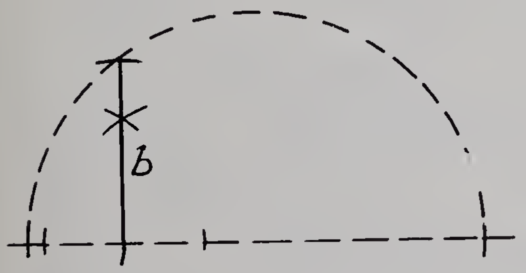
To construct a polygon similar to P and = S.

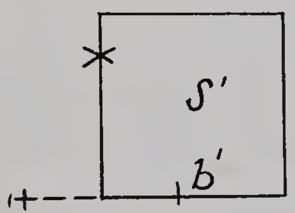


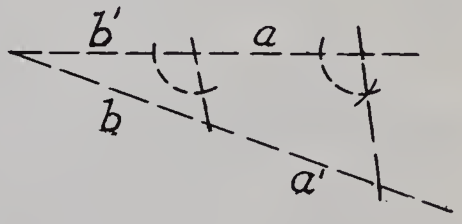


Construction: 1. Construct square S' = P (Ex. 6, page 431). 2. Construct a' the fourth proportional to b', b, and a (§411a). 3. Construct polygon P' ∼ P on side a' corresponding to side a of polygon P (§437).

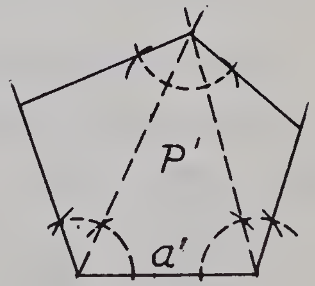
Then P' is the required polygon.







Proof: 1. $\frac{b'}{b} = \frac{a}{a'}$ (Const.). 2. $\frac{b'^2}{b^2} = \frac{a^2}{a'^2}$ (Asmt. 8). 3. $\frac{S'}{S} = \frac{b'^2}{b^2}$ (§440). 4. $\frac{P}{P'} = \frac{a^2}{a'^2}$ (§440). 5. $\frac{S'}{S} = \frac{P}{P'}$ (Asmt. 7). 6. But S' = P (Const.). 7. ∴ P' = S (§388). 8. P' ∼ P (Const.).



43. Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 417, 420, 425, 427, 428, 433, 434. 2. We shall use §412.

Proof: 1. Draw AD and BE (Asmt. 11). 2. ACB and DCE are st. lines (Given). 3. ∴ ∠ ACD = ∠ BCE (§50d). 4. AC and CB are diameters of ⊙s O and O' respectively (§277, Given). 5. ∴ ∠ADC and ∠CEB are rt. ∠s (§335). 6. ∴ △ACD ~ △CEB (§416). 7. ∴ $\frac{CE}{CD} = \frac{CB}{CA}$ (§412).

44. Given the ⊙O circumscribed about △ABC with h the altitude upon AB and CE a diameter of ⊙O.

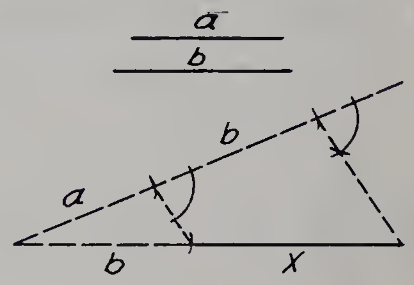
To prove that ab = hd.

Planning the Proof: 1. We can find the values of line segments by §§421, 425, 427, 428, 431, 433. 2. We shall use §421.

Proof: 1. Draw BE (Asmt. 11). 2. ∠A = ∠E (§334). 3. ∠ADC is a rt. ∠ and CE is a diameter of ⊙O (Given). 4. ∴ ∠EBC is a rt. ∠ (§335). 5. ∴ △ACD ~ △EBC (§§57, 416). 6. ∴ $\frac{b}{d} = \frac{h}{a}$, or ab = hd (§421).

45. Planning the Proof: 1. We can find the areas of polygons by §§273, 274. 2. We shall use §273c.

Proof: 1. CD ⊥ AB (Given). 2. ∠ADC is a rt. ∠ (§14). 3. ∠EBC is a rt. ∠ (§319). 4. △ADC and △EBC are rt. △s (§57). 5. ∠A = ∠E (§320). 6. △ADC ~ △EBC (§416). 7. $\frac{b}{h} = \frac{d}{a}$ (§412). 8. ab = dh (§386). 9. h = $\frac{ab}{d}$ (Asmt. 4). 10. d = 2R (Given). 11. A = $\frac{1}{2}$ch (§272c). 12. A = $\frac{1}{2}c(\frac{ab}{2R})$ (Asmt. 6). 13. A = $\frac{abc}{4R}$ (Asmt. 6).



46. Given the line segments a and b.

To construct a line segment x, so that ax = b^2.

Construction: 1. Construct x, the fourth proportional to a, b, and b (§411a). Then x is the required segment.

Proof: 1. $\frac{a}{b} = \frac{b}{x}$ (Const.). 2. Then ax = b^2 (§386).

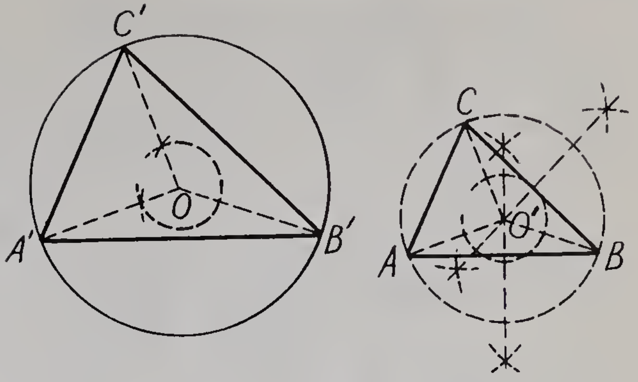
47. Planning the Proof: 1. We can prove that a line segment is the mean proportional between 2 other line segments by §§420, 421, 425, 427, 433. 2. We shall use §420c.

Proof: 1. ∠CAB $\overset{\circ}{=}$ $\frac{1}{2}\overset{\frown}{AB}$ of ⊙O' and ∠BAD $\overset{\circ}{=}$ $\frac{1}{2}\overset{\frown}{AB}$ of ⊙O (§333c). 2. ∠ADB $\overset{\circ}{=}$ $\frac{1}{2}\overset{\frown}{AB}$ of ⊙O' and ∠ACB $\overset{\circ}{=}$ $\frac{1}{2}\overset{\frown}{AB}$ of ⊙O (§333b). 3. ∴ ∠CAB = ∠ADB and ∠BAD = ∠ACB (Asmt. 7). 4. ∴ △ABC ~ △DBA (§415). 5. ∴ $\frac{BC}{AB} = \frac{AB}{BD}$ (§412).

48. Given the △ABC and the ⊙O.

To inscribe in ⊙O a △A'B'C' ~ △ABC.

Construction: 1. About △ABC circumscribe a ⊙O' (§364).



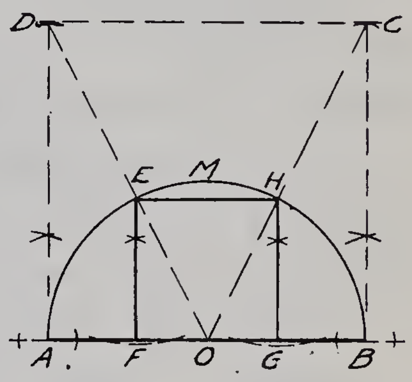
2. Draw radii O'A, O'B, and O'C (Asmt. 11). 3. Construct $\angle A'OB' = \angle AO'B$ and $\angle B'OC' = \angle BO'C$ (§180). 4. Draw A'B', B'C', and A'C' (Asmt. 11).

Then A'B'C' is the required $\triangle$.

Proof: 1. A', B', and C' are on $\odot O$ (Const.). 2. Then A'B'C' is inscribed in $\odot O$ (§288). 3. AO' = BO' and A'O = B'O (§308a). 4. $\therefore \frac{AO'}{A'O} = \frac{BO'}{B'O}$ (Asmt. 4). 5. $\angle A'OB' = \angle AO'B$ (Const.). 6. $\therefore \triangle A'OB' \sim \triangle AO'B$ (§422). 7. Similarly, $\triangle B'OC' \sim \triangle BO'C$ and $\triangle C'OA' \sim \triangle CO'A$ (Statements 3-6). 8. $\therefore \triangle A'B'C' \sim \triangle ABC$ (§436).

49. Given the semicircle AMB with AOB its diameter.

To inscribe a square in the semicircle.



Construction: 1. With AB as a side, construct a square ABCD (Ex. 1, page 224). 2. Draw OC and OD intersecting the semicircle in H and E respectively. 3. Draw EH (Asmt. 11). 4. From E draw EF $\perp$ AB and from H draw HG $\perp$ AB (§178a).

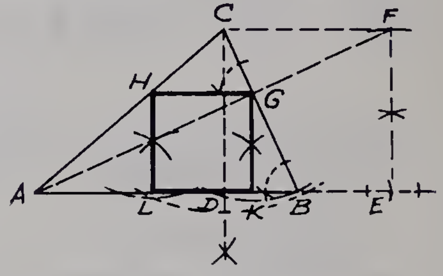
Then FGHE is the required square.

Proof: 1. ABCD is a square (Const.). 2. $\therefore$ AD $\perp$ AB, BC $\perp$ AB (§185). 3. EF $\perp$ AB and HG $\perp$ AB (Const.). 4. $\therefore$ AD $\parallel$ EF $\parallel$ HG $\parallel$ BC (§160d). 5. $\triangle$s OAD and OBC are rt. $\triangle$s (§57). 6. AD = BC (§194). 7. OA = OB (§308a). 8. $\therefore \triangle AOD \cong \triangle OBC$ (§96d). 9. $\therefore$ OD = OC (§97a). 10. In $\triangle EOH$, OE = OH (§308a). 11. $\therefore$ From 9 and 10, $\frac{OE}{OD} = \frac{OH}{OC}$ (Asmt. 4). 12. $\therefore$ EH $\parallel$ DC (§410b). 13. $\angle EOH = \angle DOC$ (Iden.). 14. $\therefore$ From 11 and 13, $\triangle OEH \sim \triangle ODC$ (§422). 15. $\frac{EH}{DC} = \frac{OH}{OC}$ (§412). 16. In $\triangle OGH$ and $\triangle OBC$, $\angle GOH = \angle BOC$ (Iden.), and $\angle OGH = \angle OBC$ = a rt. $\angle$ (Statements 1 and 4). 17. $\therefore \triangle OGH \sim \triangle OBC$ (§415). 18. $\therefore \frac{HG}{CB} = \frac{OH}{OC}$ (§412). 19. $\therefore$ From 15 and 18, $\frac{EH}{DC} = \frac{HG}{CB}$ (Asmt. 7). 20. But DC = CB (§194). 21. $\therefore$ EH = HG (§388). 22. Similarly, EH = EF (Statements 15-21). 23. $\therefore$ EF = HG (Asmt. 7). 24. $\therefore$ From 4 and 23, FGHE is a ▱ (§223c). 25. From 3, $\angle OFE$ = a rt. $\angle$ (§14). 26. $\therefore$ FGHE is a square (§185). 27. E and H are on the semicircle, and side FG lies on the diameter AB of the semicircle (Const.). 28. $\therefore$ FGHE is an inscribed square (§288).

50. This construction is based on §429 with AB and BC = 6 and 1, or any 2 factors of 6. (Cf. Ex. 39, above.)

51. Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 417, 420, 425, 427, 428, 433, 434. 2. We shall use §412, Asmt. 6.

Proof: 1. CD $\perp$ AB, and AE $\perp$ BC (Given). 2. $\therefore$ $\triangle$ADC, $\triangle$BDC, $\triangle$AEB, and $\triangle$ADF, are rt. ⨺ (§57). 3. $\angle$B = $\angle$B (Iden.). 4. $\therefore$ $\triangle$BDC $\sim$ $\triangle$AEB (§416). 5. $\angle$DAF = $\angle$BAE (Iden.). 6. $\therefore$ $\triangle$ADF $\sim$ $\triangle$AEB (§416). 7. $\therefore$ $\triangle$BDC $\sim$ $\triangle$ADF (§418). 8. $\therefore$ $\frac{BC}{AF} = \frac{CD}{AD}$ (§412). 9. AC = BC (Given). 10. CD = CD (Iden.). 11. $\therefore$ From 2, 9, and 10, $\triangle$ADC $\cong$ $\triangle$BDC (§156b). 12. $\therefore$ AD = DB (§97a). 13. $\therefore$ From 8, 9, and 12, $\frac{AC}{AF} = \frac{CD}{DB}$ (Asmt. 6).



52. Given the $\triangle$ABC with the base AB.

To inscribe a square in $\triangle$ABC so that one side of the square lies on AB.

Construction: 1. From C construct CD $\perp$ AB (§178a). 2. On CD as a side construct the square CDEF (Ex. 1, page 224). 3. Draw AF intersecting BC in G (Asmt. 11). 4. Through G construct a line $\parallel$ AB intersecting AC in H (§179). 5. From H construct HL $\perp$ AB and from G construct GK $\perp$ AB (§178a).

Then LKGH is the required square.

Proof: 1. HL $\perp$ AB and GK $\perp$ AB (Const.). 2. $\therefore$ HL $\parallel$ GK (§160d). 3. HG $\parallel$ LK and $\angle$LKG is a rt. $\angle$ (Const.). 4. $\therefore$ LKGH is a rect. (§185). 5. Quad. CDEF is a square (Const.). 6. $\therefore$ CF $\parallel$ AE (§185). 7. HG $\parallel$ AB (Const.). 8. $\therefore$ HG $\parallel$ CF (§160e). 9. $\therefore$ $\angle$AHG = $\angle$ACF (§158a). 10. $\angle$GAH = $\angle$FAC (Iden.). 11. $\therefore$ $\triangle$AGH $\sim$ $\triangle$AFC (§415). 12. $\therefore$ $\frac{HG}{CF} = \frac{AG}{AF}$ (§412). 13. GK $\perp$ AB and FE $\perp$ AB (Const.). 14. $\therefore$ $\triangle$AKG and $\triangle$AEF are rt. ⨺ (§57). 15. $\angle$KAG = $\angle$EAF (Iden.). 16. $\therefore$ $\triangle$AKG $\sim$ $\triangle$AEF (§416). 17. $\therefore$ $\frac{GK}{FE} = \frac{AG}{AF}$ (§412). 18. From 12 and 17, $\frac{HG}{CF} = \frac{GK}{FE}$ (Asmt. 6). 19. But from 5, CF = FE (§194). 20. $\therefore$ HG = GK (§388). 21. From 4 and 20, LKGH is a square (§185). 22. L and K are both in AB, G is in BC, and H is in AC (Const.). 23. $\therefore$ LKGH is an inscribed square (§288).

53. Given AB and CD 2 common chords of 3 intersecting ⓢO, O', and O"; C, D, A, B, and E known points of intersection of ⓢO, O', and O"; and P the intersection of chords AB and CD.

To prove that the common chord of ⓢO' and O" passes through P.

Planning the Proof: 1. We can prove three lines concurrent by §§355, 379. 2. We shall use §355.

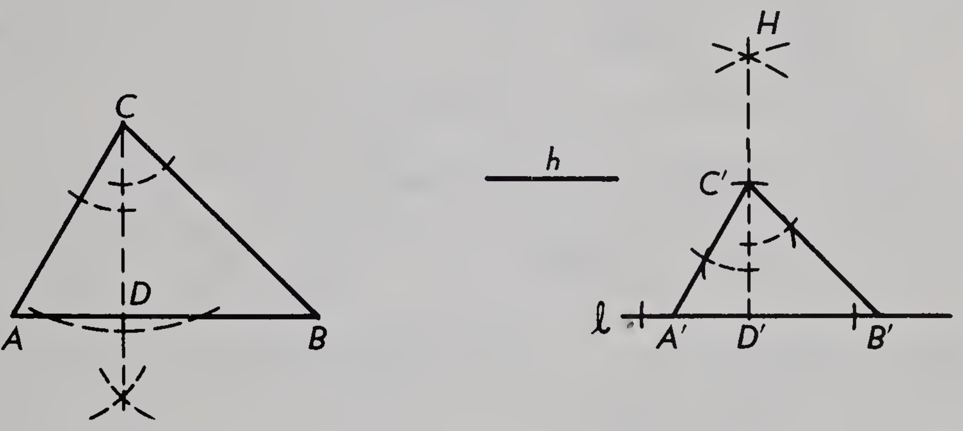
Proof: 1. Draw PE (Asmt. 11). 2. Produce EP to meet $\odot O'$ in F and $\odot O''$ in G (Asmt. 13). 3. EP $\times$ PF = CP $\times$ PD and EP $\times$ PG = AP $\times$ PB (§431). 4. But CP $\times$ PD = AP $\times$ PB (§431). 5. $\therefore$ EP $\times$ PF = EP $\times$ PG (Asmt. 7). 6. $\therefore$ PF = PG (Asmt. 4). 7. $\therefore$ F coincides with G and EPG is a common chord of ⓢ O' and O'' (Statements 1 and 2). 8. $\therefore$ The 3 common chords are concurrent (§355).

54. Given $\triangle$ABC and line segment h.

To construct a $\triangle \sim \triangle$ABC and having an altitude = h.

Construction: 1. From C construct CD the altitude on AB. (§178a). 2. On any line ℓ, draw D'H $\perp$ ℓ at any point as D' (§178b). 3. On D'H make D'C' = h (§9). 4. From C' draw a line intersecting ℓ in A' so that $\angle$A'C'D' = $\angle$ACD (§180). 5. From C' draw a line intersecting ℓ in B' so that $\angle$B'C'D' = $\angle$BCD (§180).

Then $\triangle$A'B'C' is the required $\triangle$.



Proof: 1. $\triangle$A'B'C' has the altitude h (Const., §90). 2. $\angle$A'C'D' = $\angle$ACD (Const.). 3. Rt. $\triangle$A'C'D' $\sim$ rt. $\triangle$ACD (§416). 4. In like manner, it can be proved that $\triangle$D'B'C' $\sim$ $\triangle$DBC (Steps 2-3). 5. $\therefore$ $\triangle$A'B'C' $\sim$ $\triangle$ABC (§436).

55. Given the point P in the arc of chord AB or $\odot$O.

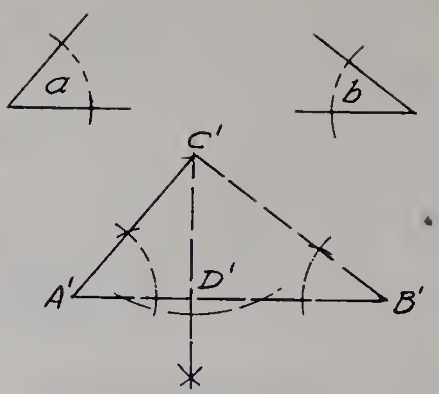
To construct a chord through P which shall be bisected by chord AB.

Construction: 1. Draw radius OP intersecting AB in C (Asmt. 11). 2. Extend PO to D making CD = CP (Asmt. 13, §9). 3. Through D construct DE $\parallel$ AB intersecting $\odot$O in E (§179). 4. Draw EP (Asmt. 11).

Then EP is the required chord.

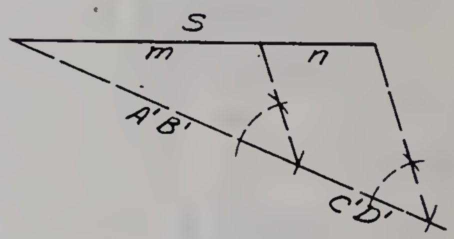
Proof: 1. In $\triangle$DEP, AC $\parallel$ ED and CD = CP (Const.). 2. $\therefore$ OE = OP, or chord EP is bisected by chord AB (§217e).

56. Given $\angle$a and $\angle$b two $\angle$s of a $\triangle$ and s the sum of the included side and the altitude on it.



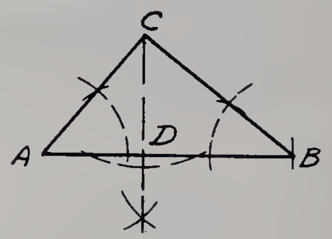
To construct the $\triangle$.

Construction: 1. Construct $\triangle A'B'C'$ with $\angle A' = \angle a$, $\angle B' = \angle b$, and side $A'B'$ = any convenient length (§181c). 2. From C' construct $C'D'$ the altitude on $A'B'$ (§178a). 3. Divide s into 2 parts, m and n, which are proportional to $A'B'$ and $C'D'$ (§411b). 4. Construct $\triangle ABC$ with side $AB = m$, $\angle A = \angle a$, and $\angle B = \angle b$ (§181c).



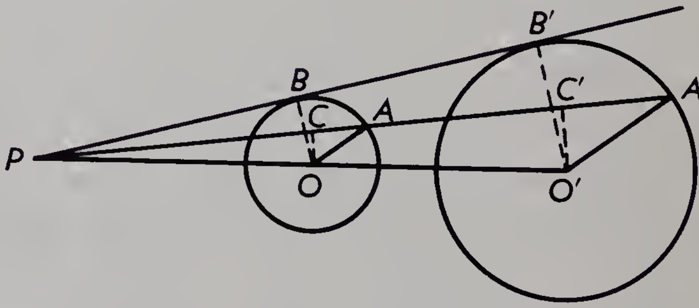
Then $\triangle ABC$ is the required $\triangle$.

Proof: 1. In $\triangle ABC$ and $\triangle A'B'C'$, $\angle A = \angle A' = \angle a$ and $\angle B = \angle B' = \angle b$ (Const.). 2. $\therefore \triangle ABC \sim \triangle A'B'C'$ (§415). 3. Construct CD the altitude on AB (§178a). 4. $\therefore \frac{CD}{C'D'} = \frac{AB}{A'B'}$ (§417).



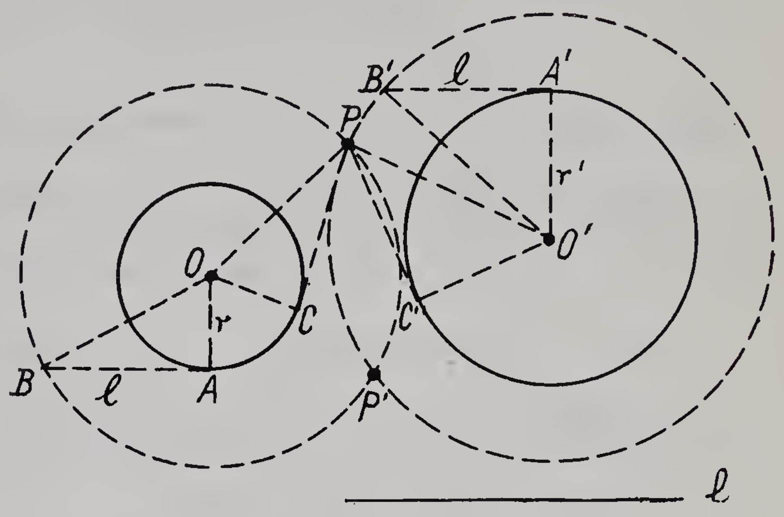
5. But $\frac{n}{C'D'} = \frac{m}{A'B'}$ and $AB = m$ (Const.). 6. $\therefore \frac{n}{C'D'} = \frac{AB}{A'B'}$, or $\frac{n}{C'D'} = \frac{CD}{C'D'}$ (Asmt. 6). 7. $\therefore n = CD$ (§388). 8. But $m + n = s$ (Const., Asmt. 9). 9. $\therefore AB + CD = s$ (Asmt. 6).

57. Planning the Proof: 1. We can prove lines $\parallel$ by §§160, 221, 410. 2. We shall use §160b.



Proof: 1. Draw OB and $O'B'$ (Asmt. 11). 2. $OB \perp PB'$ and $O'B' \perp PB'$ (§311b). 3. $\angle PBO$ and $\angle PB'O'$ are rt. $\angle$s (14). 4. $\angle PBO = \angle PB'O'$ (Asmt. 19). 5. $\angle OPB = \angle O'PB'$ (Iden.). 6. $\triangle OPB \sim \triangle O'PB'$ (§415). 7. $\frac{PO}{PO'} = \frac{OB}{O'B'}$ (§412). 8. $OB = OA$ and $O'B' = O'A'$ (Asmt. 26). 9. $\frac{PO}{PO'} = \frac{OA}{O'A'}$ (Asmt. 6). 10. Construct $OC \perp PA'$ and $O'C' \perp PA'$ (§178a). 11. $\angle PCO$ and $\angle PC'O'$ are rt. $\angle$s (§14). 12. $\angle PCO = \angle PC'O'$ (Asmt. 19). 13. $\angle OPC = \angle O'PC'$ (Iden.). 14. $\triangle POC \sim \triangle PO'C'$ (§415). 15. $\frac{PO}{PO'} = \frac{OC}{O'C'}$ (§412). 16. From statements 9 and 15, $\frac{OA}{O'A'} = \frac{OC}{O'C'}$ (Asmt. 7). 17. $\frac{OA}{OC} = \frac{O'A'}{O'C'}$ (§391). 18. $\frac{\overline{OA}^2}{\overline{OC}^2} = \frac{\overline{O'A'}^2}{\overline{O'C'}^2}$ (Asmt. 8). 19. $\frac{\overline{OA}^2 - \overline{OC}^2}{\overline{OC}^2} = \frac{\overline{O'A'}^2 - \overline{O'C'}^2}{\overline{O'C'}^2}$ (§393). 20. But $\overline{OA}^2 - \overline{OC}^2 = \overline{AC}^2$ and $\overline{O'A'}^2 - \overline{O'C'}^2 = \overline{A'C'}^2$ (§430, Asmt. 2). 21. $\therefore \frac{\overline{AC}^2}{\overline{OC}^2} = \frac{\overline{A'C'}^2}{\overline{O'C'}^2}$

(Asmt. 6). 22. $\frac{AC}{OC} = \frac{A'C'}{O'C'}$ (Asmt. 8). 23. $\frac{AC}{A'C'} = \frac{OC}{O'C'}$ (§391). 24. ∠OCA = ∠O'C'A' (Asmt. 19). 25. △COA ~ △C'O'A' (§422). 26. ∠CAO = ∠C'A'O' (§412). 27. OA ∥ O'A' (§160b).



58. Given ⓢ O and O'.

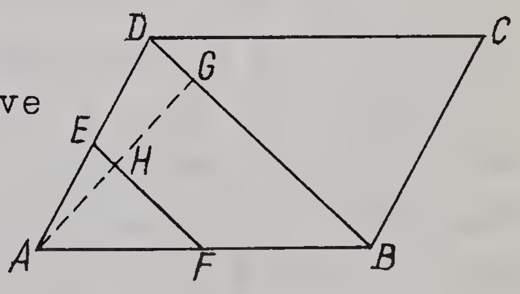
To find a point P such that the tangents from P to the two ⓢ are =.

Construction: 1. At any convenient points on ⓢ O and O', as A and A', draw tangents BA and B'A' = any convenient line segment ℓ not less than half OO' (§§336b, 9). 2. Draw OB, and O'B' (Asmt. 11). 3. With O and O' as centers and OB and O'B' as radii respectively, draw circles intersecting in points P and P' (Asmt. 15).

Then P or P' is the required point.

Proof: 1. Draw tangent PC to ⊙O and tangent PC' to ⊙O' (§336c). 2. Draw radii OC and O'C' to points of tangency (Asmt. 11). 3. PC ⊥ OC, PC' ⊥ O'C', OA ⊥ AB, and O'A' ⊥ A'B' (§311b). 4. OC = OA and O'C' = O'A' (Asmt. 26). 5. OB = OP and O'B' = O'P (Asmt. 26). 6. Rt. △OAB ≅ rt. △OPC and rt. △O'A'B' ≅ rt. △O'PC' (§156b). 7. ∴ PC = AB = ℓ and PC' = A'B' = ℓ (§97a, Const.). 8. Similarly, the tangents from P' may be proved = ℓ (Statements 1-7).

59. Given ▱ABCD with the diagonal BD, and EF joining the midpoints of AD and AB.



To prove that EFBD is a trapezoid and = $\frac{3}{8}$▱ABCD.

Planning the Proof: 1. We can (a) prove that a figure is a trapezoid by §206; (b) compare areas of polygons by §§272, 274, 438, 439, 440, and Asmt. 2. We shall use (a) §206; (b) Asmt.

Proof: (a) 1. EF ∥ DB (§221b). 2. ED and FB intersect in A (Given). 3. ∴ EFBD is a trapezoid (§206).

(b) 1. From A, draw a line ⊥ BD intersecting EF in H and DB in G (§178a). 2. AH ⊥ EF (§159a). 3. AH = HG (§217d). 4. Area of △ABD = $\frac{1}{2}$AG × BD = HG × BD (§273c). 5. Area of ▱ABCD =

$2\triangle ABD$ (§187). 6. Area of ▱ABCD = 2HG x BD (Asmt. 6). 7. Area of EFBD = $\frac{1}{2}$HG(EF + BD) (§273e). 8. EF = $\frac{1}{2}$BD (§218a). 9. ∴ Area of EFBD = $\frac{3}{4}$HG x BD (Asmt. 6). 10. ∴ $\frac{EFBD}{ABCD} = \frac{\frac{3}{4}HG \times BD}{2HG \times BD} = \frac{3}{8}$, or area of EFBD = $\frac{3}{8}$ area of ABCD (Asmt. 4, 6).

60. Proof: 1. Draw DF making $\angle FDC = \angle ADB$ (§180). 2. $\angle ABD = \angle FCD$ (§334). 3. $\triangle ABD \sim \triangle FDC$ (§415). 4. $\frac{AB}{FC} = \frac{BD}{DC}$ (§412). 5. FC x BD = AB x DC (§386). 6. FC = $\frac{AB \times DC}{BD}$ (Asmt. 4). 7. $\angle DAF = \angle DBC$ (§334). 8. $\angle BDF = \angle BDF$ (Iden.). 9. $\angle ADF = \angle BDC$ (Asmt. 1). 10. $\triangle ADF \sim \triangle BDC$ (§415). 11. $\frac{AF}{BC} = \frac{AD}{DB}$ (§412). 12. AF x DB = AD x BC (§386). 13. AF = $\frac{AD \times BC}{DB}$ (Asmt. 4). 14. AF + FC = $\frac{AB \times CD + AD \times BC}{DB}$ (Asmt. 1). 15. AC = same fraction (Asmt. 6, 9). 16. AC x DB = AB x CD + AD x BC (Asmt. 3).

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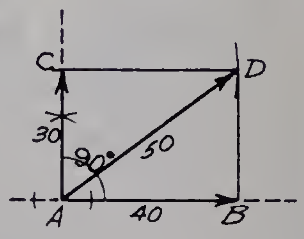
1. Room	Length	Width
Living Room	24'	22'
Dining Room	$14\frac{1}{2}'$	$11\frac{1}{2}'$
Kitchen	$14\frac{1}{2}'$	11'
Bedroom	14'	13'
Bedroom	13'	11'
Bedroom	13'	10'

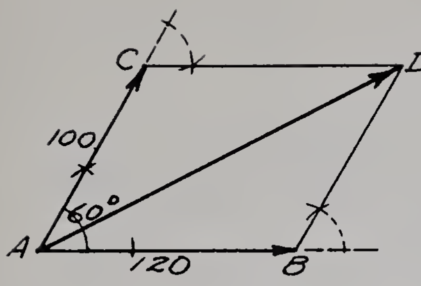
3. Let AB representing a force of 40 lb. = 1 in. Then AC representing a force of 30 lb. = $\frac{3}{4}$ in. Construct a ▱ABDC with AB = 1 in., AC = $\frac{3}{4}$ in., and $\angle BAC$ = 90° (Ex. 11, page 224). Draw AD. Then AD represents the resultant. By actual measurement, we find AD = $1\frac{1}{4}$ in. Then the resultant = $1\frac{1}{4}$ x 40 lb. = 50 lb.



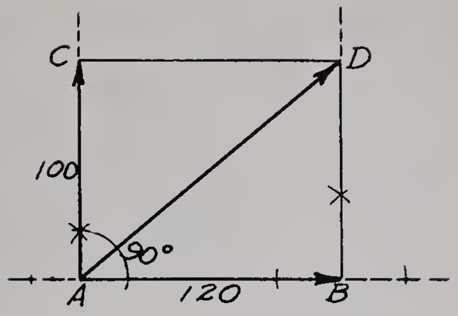
(The figure given here is reduced in scale.)

4\.

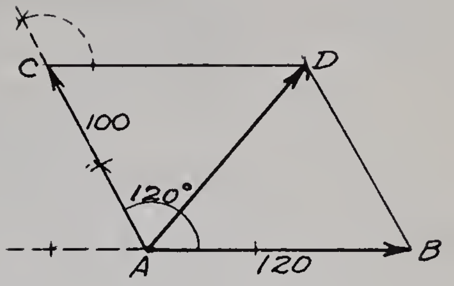
(1)



(2)



(3)



(4) The student should find by measurement that the resultant in (1) is the greatest.

PAGE 438

1. $a^2 = b^2 + c^2 - 2cb'$. $a^2 = 169 + 196 - 140$; $a^2 = 225$; $a = 15$.

2. $c^2 = a^2 + b^2 - 2ba'$.
$c^2 = 96^2 + 58^2 - 2 \times 36 \times 58$; $c^2 = 8404$; $c = 91.7$.

3. $a^2 = b^2 + c^2 - 2cb'$. $a^2 = 64 + 100 - 139$; $a^2 = 25$; $a = 5$.

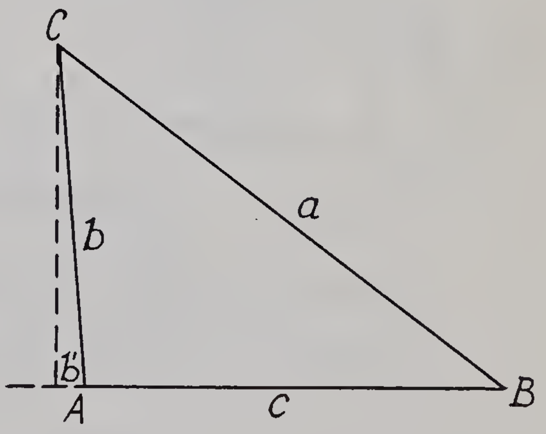
4. $a^2 = b^2 + c^2 - 2cb'$. $361 = 144 + 289 - 34b'$.
Solving, $34b' = 72$; $b' = 2\frac{2}{17}$.

5. Let d' = the projection of EF on DE. Then $144 = 100 + 225 - 30d'$. Solving, $d' = 6\frac{1}{30}$.

PAGE 439

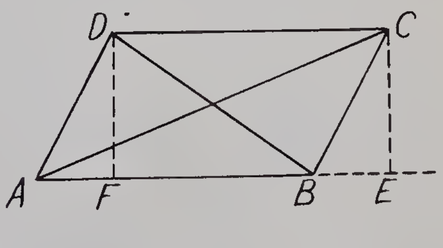
1. Acute (§446). 2. Right (§446).

3. $a^2 = b^2 + c^2 + 2b'c$.
$a^2 = 100 + 144 + 80$; $a^2 = 324$;
$a = 18$.



4. Given ▱ABCD with diagonals AC and BD.

To prove that $\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{AD}^2 = \overline{AC}^2 + \overline{BD}^2$.



Proof: 1. Draw DF $\perp$ AB and CE $\perp$ AB (§178a). 2. $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 + 2\overline{AB} \times \overline{BE}$ (§445). 3. $\overline{BD}^2 = \overline{AB}^2 + \overline{AD}^2 - 2\overline{AB} \times \overline{AF}$ (§444). 4. $\overline{AC}^2 + \overline{BD}^2 = \overline{AB}^2 + \overline{AB}^2 + \overline{AD}^2 + \overline{BC}^2 + 2\overline{AB}(BE - AF)$ (Asmt. 1). 5. AB = DC (§217a). 6. $\overline{AB}^2 = \overline{DC}^2$ (Asmt. 8). 7. AD = BC (§217a). 8. DF = CE (§217b). 9. Rt. $\triangle$AFD $\cong$ rt. $\triangle$BEC (§§57, 156b). 10. AF = BE (§97a). 11. From (4), (6), and (10), $\overline{AC}^2 + \overline{BD}^2 = \overline{AB}^2 + \overline{DC}^2 + \overline{AD}^2 + \overline{BC}^2$ (Asmt. 6).

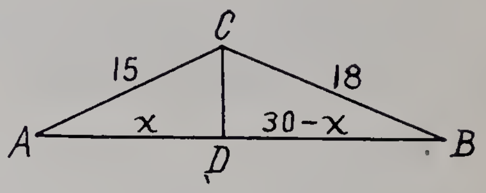
PAGE 440

1. $m_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$ (§447).

$m_c = \frac{1}{2}\sqrt{2(18^2 + 30^2) - 24^2} = 2\sqrt{117} = 21.6$.

2. The triangle is a right triangle because $8^2 + 15^2 = 17^2$ (§255). Then 8 is the altitude to the side 15.

3. $\frac{15}{x} = \frac{18}{30 - x}$ (§409d). Solving, $x = 13\frac{7}{11}$. Then $AD = 13\frac{7}{11}$ and $DB = 16\frac{4}{11}$. $\overline{CD}^2 = AC \times BC - AD \times DB$; $\overline{CD}^2 = 270 - \frac{27,000}{121}$; $CD^2 = \sqrt{\frac{5670}{121}}$; $CD = \frac{9}{11}\sqrt{70} = 6.8$.

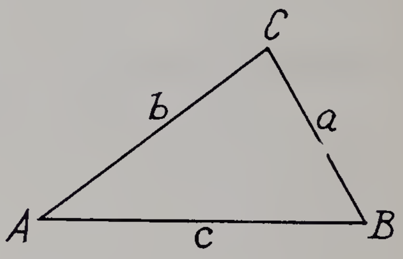


4. $m_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$ (§447).

$m_c = \frac{1}{2}\sqrt{2(225 + 64) - 289} = 8.5$

$m_a = \frac{1}{2}\sqrt{2(64 + 289) - 225} = \frac{21.9}{2} = 11.0$.

$m_b = \frac{1}{2}\sqrt{2(225 + 289) - 64} = 15.5$



5. From §447, $t_c = \frac{2}{a + b}\sqrt{abs(s - c)}$; $t_c = \frac{2}{36}\sqrt{20 \cdot 16 \cdot 27 \cdot 9} = 15.5$. $t_b = \frac{2}{a + c}\sqrt{acs(s - b)}$; $t_b = \frac{2}{38}\sqrt{20 \cdot 18 \cdot 27 \cdot 11} = 17.2$

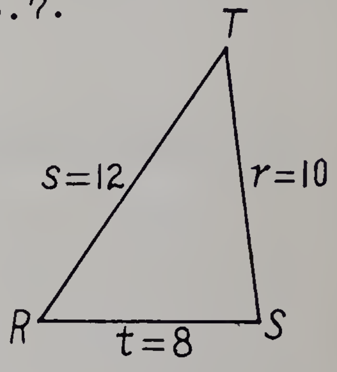
$t_a = \frac{2}{b + c}\sqrt{bcs(s - a)}$; $t_a = \frac{2}{34}\sqrt{16 \cdot 18 \cdot 27 \cdot 7} = 13.7$.

6. $h_c = \frac{2}{c}\sqrt{s(s - a)(s - b)(s - c)}$ (§258).

Then $h_t = \frac{2}{8}\sqrt{15(5)(3)(7)} = \frac{2 \times 15}{8}\sqrt{7} = 9.9$;

and $h_r = \frac{2}{10}\sqrt{15(5)(3)(7)} = \frac{2 \times 15}{10}\sqrt{7} = 7.9$;

and $h_s = \frac{2}{12}\sqrt{15(5)(3)(7)} = \frac{2 \times 15}{12}\sqrt{7} = 6.6$.



7. (1) $m_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$; $m_c = \frac{1}{2}\sqrt{576} = 12$.

$m_a = m_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2} = \frac{3}{2}\sqrt{41} = 9.6$.

(2) $h_c = m_c = 12$.

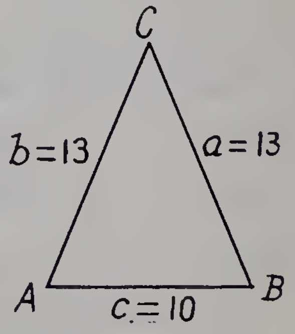
$h_a = h_b = \frac{2}{b}\sqrt{s(s - a)(s - b)(s - c)}$

$= \frac{2}{13}\sqrt{18 \cdot 5 \cdot 5 \cdot 8} = \frac{2}{13}(60) = 9.2$

(3) $t_c = \frac{2}{a + b}\sqrt{abs(s - c)}$

$= \frac{2}{26}\sqrt{13 \times 13 \times 18 \times 8} = 12$.

$t_a = t_b = \frac{2}{a + c}\sqrt{acs(s - b)} = \frac{2}{13}\sqrt{13 \times 10 \times 18 \times 5} = 9.4$.



PAGES 441-442 (Applied Problems)

1. $\triangle A'B'C' \sim \triangle ABC$ (§415). Then $\frac{AB}{15.5} = \frac{100}{8}$ (§412). Solving, 8AB = 1550; whence AB = 193.75. 193.75 ft. Ans.

2. (1) Given the △s AOC and DOB with $\frac{AO}{OB} = \frac{CO}{OD}$.

To prove that $\triangle AOC \sim \triangle DOB$.

Planning the Proof: 1. We can prove △s ~ by §§412, 415, 416, 418, 422, 423, 424, 435. 2. We shall use §422.

Proof: (1) 1. $\angle AOC = \angle DOB$ (§50d). 2. $\frac{AO}{OB} = \frac{CO}{OD}$ (Given). 3. $\therefore \triangle AOC \sim \triangle DOB$ (§422).

(2) $\frac{AC}{DB} = \frac{AO}{OB}$ (§412). If $\frac{AO}{OB} = \frac{3}{2}$, then $\frac{AC}{DB} = \frac{3}{2}$ (Asmt. 6).

(3) If $\frac{AO}{OB} = \frac{3}{2}$, adjust the dividers so that AC = the given segment. Then $\frac{DB}{AC} = \frac{OB}{AO}$, or $\frac{DB}{AC} = \frac{2}{3}$ (§392, Asmt. 6); whence $DB = \frac{2}{3}AC$.

Then DB = the required segment.

3. Given the pantograph AECB (Fig. 1) with AE ∥ BF, DB ∥ EC, point A fixed, points B and C movable, points A, B, and C in a st. line, and quad. I and quad. II.

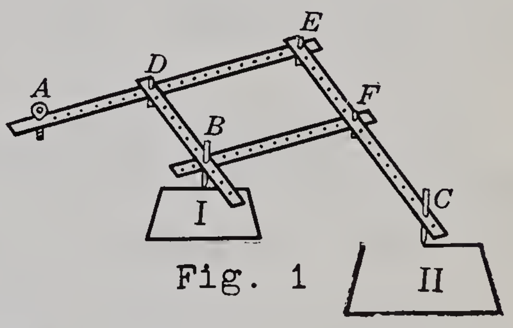


Fig. 1

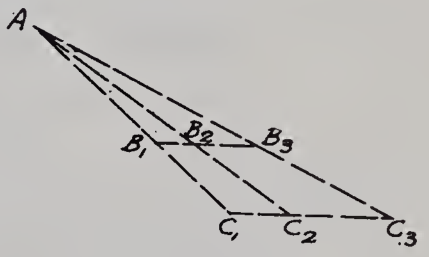


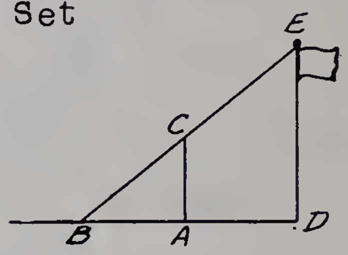
Fig. 2

To prove that (a) $\triangle ABD \sim \triangle ACE$, (b) $\frac{AD}{AE} = \frac{EF}{EC}$, (c) quad. I ~ quad. II.

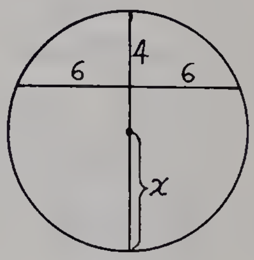
Planning the Proof: 1. We can prove (a) △s ~ (Same as Ex. 2); (b) lines proportional: §§396, 409, 412, 417, 420, 425, 427, 428, 433, 434; (c) polygons ~: §§412, 436. 2. We shall use: (a) §415; (b) §412; (c) §436.

Proof: 1. Draw line ABC (Asmt. 11). 2. $\angle DAB = \angle EAC$ (Iden.). 3. DB ∥ EC (Given). 4. $\therefore \angle ABD = \angle ACE$ (§158a). 5. $\therefore \triangle ABD \sim \triangle ACE$ (§415). 6. $\therefore \frac{AD}{AE} = \frac{DB}{EC}$ (§412). 7. DB ∥ EF and DE ∥ BF (Given). 8. ∴ BDEF is a ▱ (§185). 9. ∴ DB = EF (§217a). 10. $\therefore \frac{AD}{AE} = \frac{EF}{EC}$ (Asmt. 6). 11. From 5, $\frac{AB}{AC} = \frac{AD}{AE}$ (§412). 12. AD and AE are of fixed length (Given). 13. $\therefore \frac{AB}{AC}$ = a constant (Asmt. 6). 14. Let B_1, B_2, and B_3 be any 3 points in a st. line in quad. I and C_1, C_2, and C_3 be corresponding points in quad. II (Fig. 2). Then from 13, $\frac{AB_1}{AC_1} = \frac{AB_2}{AC_2} = \frac{AB_3}{AC_3}$ (Asmt. 6). 15. $\therefore B_1B_2 \parallel C_1C_2$ and $B_2B_3 \parallel C_2C_3$ (§410a). 16. ∴ From 14, $C_1C_2C_3$ is a st.

line and any st. line in quad. I is reproduced in quad. II as a st. line (§108). 17. Similarly, the converse of Statement 16 is true (Statements 13-16). 18. $\angle B_1AB_2 = \angle C_1AC_2$ (Iden.). 19. ∴ From 14, $\triangle AB_1B_2 \sim \triangle AC_1C_2$ (§422). 20. ∴ $\frac{B_1B_2}{C_1C_2} = \frac{AB_1}{AC_1}$ = a constant (§412, Asmt. 6). 21. Draw 2 corresponding diagonals of quad. I and quad. II (Asmt. 11). 22. From 16 and 20, $\frac{\text{each line in quad. I}}{\text{each corresponding line in quad. II}} = \frac{AB_1}{AC_1}$ (Asmt. 6). 23. ∴ Each $\triangle$ of quad. I $\sim$ the corresponding $\triangle$ of quad. II (§423). 24. ∴ Quad. I $\sim$ quad. II (§436).



4. In the figure, ED represents a flagpole. Set a stake AC at A any point on the ground at a convenient distance from ED. Find a point B on the ground in line with C and E. Measure BA, BD, and AC. Find the value of ED from the proportion $\frac{ED}{AC} = \frac{BD}{BA}$.



5. $4(2x - 4) = 36$ (§431); $8x - 16 = 36$; $8x = 52$; $x = 6\frac{1}{2}$. The radius is $6\frac{1}{2}$ ft.

6. Planning the Proof: 1. We can prove line segments proportional by §§396, 409, 412, 417, 420, 425, 427, 428, 433, 434, and Asmt. 2. We shall use Asmt.

Proof: 1. AB is a diameter of the circle (Given). 2. $\angle AEB$ is a rt. $\angle$ (§319). 3. $\triangle ABE$ is a rt. $\triangle$ (§57). 4. $CE \perp AB$ (Given). 5. $\overline{AE}^2 = AB \times AC$ and $\overline{BE}^2 = AB \times CB$ (§427). 6. AC = CD = DB (Given). 7. CB = CD + DB (Asmt. 9). 8. CB = 2AC (Asmt. 6). 9. $\overline{BE}^2 = 2AB \times AC$ (Asmt. 6). 10. $\frac{\overline{AE}^2}{\overline{BE}^2} = \frac{AB \times AC}{2AB \times AC}$ (Asmt. 4). 11. $\frac{AE}{BE} = \frac{1}{\sqrt{2}}$ (Asmt. 8).

7. Set stakes at A, C, and B. Find E the midpoint of AC and F the midpoint of BC. Measure EF and double the distance to find AB.

Another method is to decide on a point C as before, set a stake at any convenient point E on AC, and then locate F by using the proportion $\frac{FC}{EC} = \frac{BC}{AC}$. With AC, EC, and EF known, AB can be found by using the proportion $\frac{AB}{EF} = \frac{AC}{EC}$.

8. Let x = the number of feet in the height of the tree. Then $\frac{x}{42} = \frac{4\frac{1}{4}}{6}$. Solving, $x = 29\frac{3}{4}$ (ft.).

9. Let x = the number of inches the object is from the camera. $\frac{x}{10} = \frac{8}{1}$. Solving, $x = 80$ (in.).

PAGE 443 (Space Geometry)

1. $\frac{AE}{EB} = \frac{CF}{FD}$ (§448). $\frac{24}{15} = \frac{36}{FD}$. Solving, $FD = 22\frac{1}{2}$.

2. $\frac{AE}{EB} = \frac{CF}{FD}$ (§448). $\frac{12}{18} = \frac{CF}{40 - CF}$. Solving, $CF = 16$.

3. $\frac{EG}{AC} = \frac{EB}{AB}$. $\frac{EG}{8} = \frac{6}{16}$. Solving, $EG = 3$.

4. $\frac{BD}{GF} = \frac{CD}{CF}$. $\frac{BD}{6} = \frac{20}{8}$. Solving, $BD = 15$.

5. If BG = GC, AE = EB. Then AB = 2AE, or AB = 8. Similarly, CD = 2CF = 12.

6. The locus is a plane parallel to and lying between the two planes and whose distances from the planes have the ratio 1 : 2.

7. The locus is a right conical surface making a 45° angle with the given line as its axis.

PAGE 444 (Space Geometry)

1. 1. $\frac{A'B'C'D'}{ABCD} = \frac{\overline{PE'}^2}{\overline{PE}^2}$ (Given). 2. $\frac{12}{ABCD} = \frac{2^2}{4^2} = \frac{1}{4}$ (Asmt. 6). 3. $\therefore ABCD = 48$ (Asmt. 3). 48 sq.in. Ans.

2. Let x and x' = the distances of the light from ABCD and A'B'C'D' respectively. Then $\frac{\text{the intensity at ABCD}}{\text{the intensity at A'B'C'D'}} = \frac{x'^2}{x^2}$ (Given). $\frac{\text{Intensity at ABCD}}{\text{Intensity at A'B'C'D'}} = \frac{4}{36} = \frac{1}{9}$ (Asmt. 6). Then the intensity at A'B'C'D' is 9 times as great as it is at ABCD.

PAGE 445 (Review Questions)

1. When the lines are $\perp$ each other. 2. (1) §381. (2) §382. 3. Any 5 of the following: §§412, 415, 416, 418, 422, 423, 424, 435. 4. §413. 5. 3. 6. Any 2 of the following: §§425, 427, 428, 429, 433. 7. §§424, 425, 427. 8. To prove line segments =, $\angle$s =, and polygons ≅. 9. To prove line segments proportional, $\angle$s =, and polygons $\sim$. 10. §§417, 439. 11. Yes. [$d^2 = 2s^2$

(§430); whence $\frac{d}{s} = \frac{s}{\frac{1}{2}d}$ (§387, Asmt. 6)] 12. §415. 13. Equilateral △s. 14. Squares. 15. 20. 16. (1) No. (2) Yes. 17. §430. 18. §433. 19. $\sqrt{2}:1$. 20. §431. 21. Find the square root of the difference between the square of the hypotenuse and the square of the known leg. 22. Yes.

PAGE 447 (Test 32)

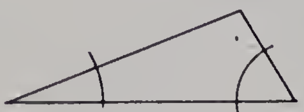
1. T. 2. T. 3. F. 4. F. 5. T. 6. T. 7. F. 8. F. 9. F.

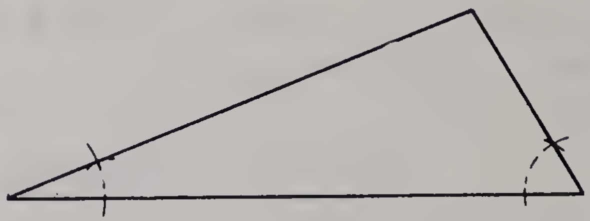
PAGE 448 (Test 33)

1. proportional, equal. 2. triangles, similar. 3. corresponding sides. 4 sides proportional. 5. segments, diameter. 6. corresponding sides, proportional. 7. corresponding angles, equal. 8. projection. 9. are similar. 10. secant, external segment.

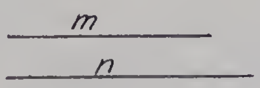
PAGE 448 (Test 34)

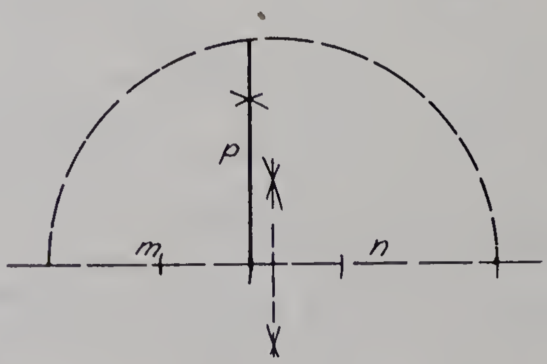
1. This construction depends on §§181a, 181b, or 181c.



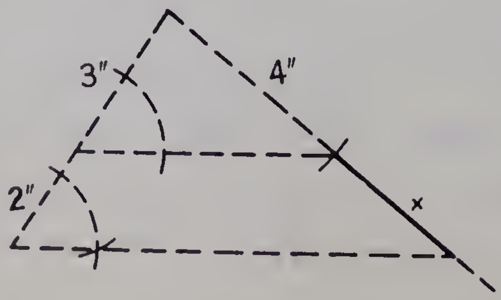


2. This construction depends on §429.





3. The construction is a direct application of §400. x is the fourth proportional to 3", 2", and 4".



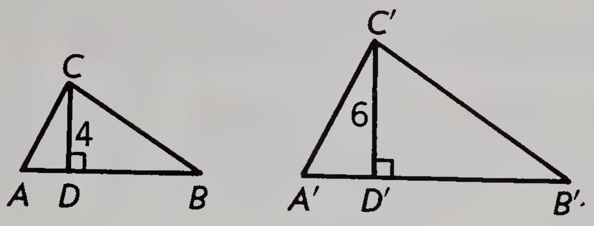
PAGE 449 (Test 35)

1. $\frac{x}{12} = \frac{20}{8}$ (§412); whence $8x = 240$, and $x = 30$.

2. $AE \times EB = CE \times ED$ (§431). $3AE = 8 \cdot 6$ (Asmt. 6). Solving, $AE = 16$.

3. $\frac{p}{50} = \frac{12}{8}$ (§434); whence $8p = 600$, and $p = 75$. 75 in. Ans.

4. $\frac{B'C'}{7} = \frac{6}{4}$ (§417); whence $4B'C' = 42$, and $B'C' = 10\frac{1}{2}$.



5. From §253, $s = \frac{d\sqrt{2}}{2} = \frac{\sqrt{8} \times \sqrt{2}}{2} = 2$.

6. (1) $\triangle FHG \sim \triangle KLM$.
   (2) $\overline{FG}^2 = 25 + 25 = 50$ (§430); whence $FG = 5\sqrt{2}$.
   (3) $\overline{RS}^2 = 25 - 16 = 9$ (§430); whence $RS = 3$.

7. $\overline{CH}^2 = AH \times HB$ (§425); whence $HB = \frac{36}{4}$, or $HB = 9$.

8. $\overline{CH}^2 = 5 \times 45$ (§425); whence $\overline{CH}^2 = 225$, and $CH = 15$.

9. $\overline{AC}^2 = 4 \times 16$ (§427); whence $\overline{AC}^2 = 64$, and $AC = 8$.

10. $\overline{CD}^2 = AD \times DB$ (§428); or $\overline{CD}^2 = 9 \times 16$; whence $\overline{CD}^2 = 144$, and $CD = 12$. CD = 12 in. Ans.

11. (1) $\overline{AB}^2 = AD \times AC$ (§433), or $36 = AD \times 4$; whence $AD = 9$.
    (2) $\overline{AB}^2 = 27 \times 3$ (§433); whence $\overline{AB}^2 = 81$, and $AB = 9$.

12. $\frac{p}{43} = \frac{3}{14}$ (§434); whence $14p = 129$, and $p = 9\frac{3}{14}$.

13. $x = \frac{1}{2}(10)$ (§136) $= 5$. 5 in. Ans.

14. $\frac{5}{4}$ (§414).

Chapter 14. Regular Polygons and the Circle

PAGE 451

1. (1) A regular polygon has all of its sides = and all of its ∠s =. (2) Two polygons are similar when their corresponding ∠s are = and their corresponding sides are proportional.

2. (1) Equilateral △. (2) Square. (3) Regular pentagon. (4) Regular hexagon.

3. (1) A polygon is inscribed in a ⊙ when its vertices lie on the ⊙. (2) A polygon is circumscribed about a ⊙ when its sides are tangent to the ⊙. (3) In the first case the ⊙ is circumscribed about the polygon and in the second, the ⊙ is inscribed in the polygon.

4. See §451c.

5. See §455b.

6. (1) In a ⊙ = chords have = arcs and are equally distant from the center. (2) In a ⊙ = arcs have = central ∠s and = chords.

7. (1) The arc which is intercepted by the sides of the ∠. (2) An arc which is $\frac{1}{2}$ the arc intercepted by the sides of the ∠. (3) An arc which is $\frac{1}{2}$ the arc intercepted by the sides of the ∠.

8. A rhombus that is not a square. 9. See §208.

10. See §210.

11. polygon; polygon. 12. Yes; Yes. 13. No; No.

14. Yes; No.

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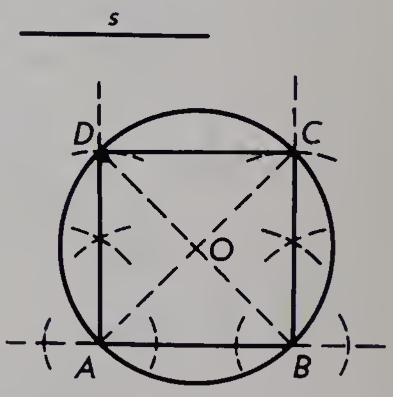


1. (a) Given a side s of a square.

To construct the square.

Construction: Construct square ABCD having a side s (Ex. 1, p. 224).

(b) Given square ABCD.

To circumscribe a circle about ABCD.

Construction: 1. Draw AC and BD intersecting in a point O (Asmt. 11). 2. With A as a center and a radius = AO draw a circle O (Asmt. 15).

⊙O is the required circle.

Proof: 1. ABCD is a square (by construction). 2. ABCD is a rectangle and a ▱ (§185). 3. AC and BD bisect each other (§195). 4. AC = BD (Ex. 8, p. 206). 5. AO + OC = BO + OD (Asmt. 9, 6).

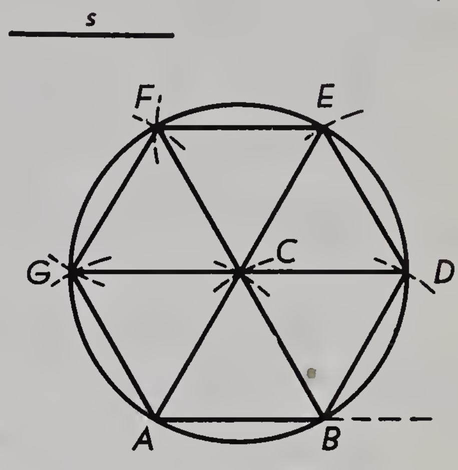
6. $2AO = 2BO$ (Asmt. 6). 7. $AO = BO$ (Asmt. 4). 8. $AO = BO = CO = DO$ (Asmt. 6). 9. $\odot O$ passes through A, B, C, D (Asmt. 42). 10. $\odot O$ is circumscribed about ABCD (§288).

2. (a) Given a side s of a regular hexagon.

To construct the hexagon.

Construction: 1. Construct equilateral $\triangle$ ABC, BCD, DCE, ECF, and FCG (Ex. 1, page 186). 2. Draw AG (Asmt. 11).

Then ABDEFG is the required regular hexagon.



Proof: 1. $AB = s$ (by construction). 2. $\triangle ABC$, $\triangle BCD$, $\triangle DCE$, $\triangle ECF$, and $\triangle FCG$ are equilateral (Const.). 3. These $\triangle$ are equiangular (§84). 4. Each angle of these $\triangle = 60^\circ$ (§120, Asmt. 9, 6, 4). 5. $\angle ACB + \angle BCD + \angle DCE + \angle ECF + \angle FCG = 300^\circ$ (Asmt. 1). 6. $\angle ACB + \angle BCD + \angle DCE + \angle ECF + \angle FCG + \angle ACG = 360^\circ$ (Asmt. 22). 7. $\angle ACG = 60^\circ$ (Asmt. 2). 8. $\angle CAG + \angle CGA + \angle ACG = 180^\circ$ (§120). 9. $\angle CAG + \angle CGA = 120^\circ$ (Asmt. 2). 10. $\angle CAG = \angle CGA$ (§98a). 11. $2\angle CAG = 120^\circ$ (Asmt. 6). 12. $\angle CAG = 60^\circ$ (Asmt. 4). 13. $\angle CGA = 60^\circ$ (Asmt. 6). 14. $AG = AC$ (§157a). 15. $AG = AB = BD = DE = EF = FG$ (Asmt. 6). 16. $\angle ABD = \angle BDE = \angle DEF = \angle EFG = \angle FGA = \angle GAB$ (Asmt. 1, 9, 6). 17. ABDEFG is a regular hexagon (§184).

(b) To circumscribe a $\odot$ about ABDEFG.

Construction: 1. With C as a center and a radius = AC, draw a circle (Asmt. 15).

This is the required circle.

Proof: 1. $CA = CB = CD = CE = CF = CG$ (Const.). 2. The circle will pass through A, B, D, E, F, and G (Asmt. 42). 3. Then the circle is circumscribed about ABDEFG (§288).

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1. a. $c = 120^\circ$.

$i = \dfrac{180^\circ}{3} = 60^\circ$.

b. $c = 72^\circ$.

$i = \dfrac{180(5-2)^\circ}{5} = 108^\circ$.

c. $c = 90^\circ$.

$i = \dfrac{180(4-2)^\circ}{4} = 90^\circ$.

d. $c = 60^\circ$.

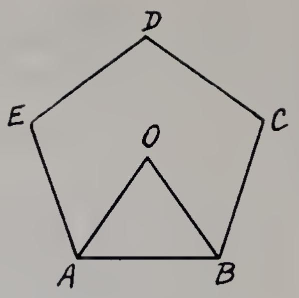
$i = \dfrac{180(6-2)^\circ}{6} = 120^\circ$.

e. $c = 45^\circ$.

$i = \dfrac{180(8-2)^\circ}{8} = 135^\circ$.

f. $c = 51\frac{3}{7}^\circ$.

$i = \dfrac{180(7-2)^\circ}{7} = 128\frac{4}{7}^\circ$.

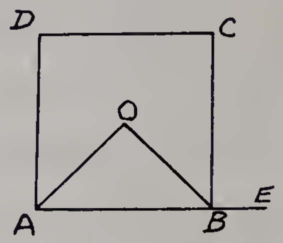


2. Given the regular polygon ABCDE with the central $\angle AOB$ and with n sides.

To prove that $\angle AOB$ is supp. to $\angle ABC$.

Planning the Proof: 1. We can prove $\angle$s supp. by §§53, 162, 222, 321. 2. We shall use §53a.

Proof: 1. $\angle AOB = \frac{360°}{n}$ (§462). 2. $\angle ABC = \frac{180(n-2)°}{n}$ (§211). 3. $\angle AOB + \angle ABC = \frac{360°}{n} + \frac{180(n-2)°}{n} = 180°$ (Asmt. 1).
4. $\therefore \angle AOB$ and $\angle ABC$ are supp. (§53a).

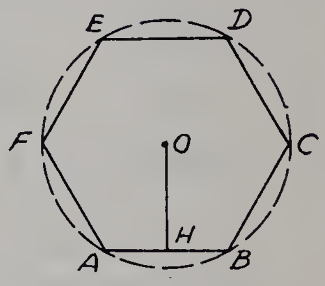


3. Given the regular polygon ABCD with a central $\angle AOB$ and exterior $\angle EBC$.

To prove that $\angle AOB = \angle EBC$.

Planning the Proof: 1. We can prove $\angle$s = by §§50, 98, 158, 219, 310, 334, 413. 2. We shall use Asmt. 7.

Proof: 1. $\angle EBC = \frac{360°}{n}$ (§209). 2. $\angle AOB = \frac{360°}{n}$ (§462).
3. $\therefore \angle EBC = \angle AOB$ (Asmt. 7).

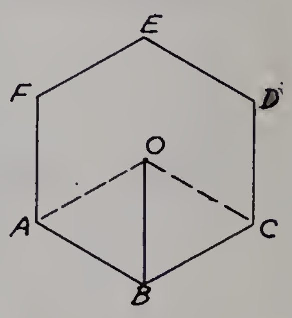


4. Given the regular polygon ABCDEF with apothem OH drawn to AB.

To prove that AH = HB.

Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309, 378. 2. We shall use §309a.

Proof: 1. Circumscribe a $\odot$ about the polygon (§458). 2. OH is the apothem of ABCDEF (Given). 3. $\therefore$ OH $\perp$ chord AB of $\odot O$ (§461). 4. $\therefore$ OH bisects AB (§309a).



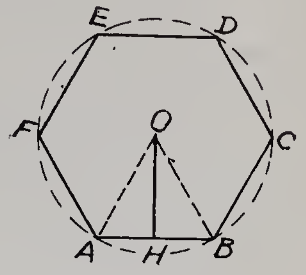
5. Given $\angle ABC$, any $\angle$ of a regular polygon ABCDEF and radius OB.

To prove that $\angle ABO = \angle OBC$.

Planning the Proof: 1. We can prove $\angle$s = by §§50, 98, 158, 219, 310, 334, 413. 2. We shall use §98b and §98a.

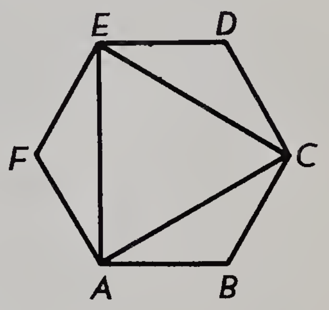
Proof: 1. Draw OA and OC (Asmt. 11). 2. O is the center of the circumscribed $\odot$ (§461).
3. $\therefore$ OA = OB = OC (Asmt. 26). 4. AB = BC (§184). 5. $\therefore \triangle ABO \cong \triangle OBC$ (§95c). 6. $\therefore \angle ABO = \angle BCO$ (§98b). 7. $\angle OBC = \angle BCO$ (§98a). 8. $\therefore \angle ABO = \angle OBC$ (Asmt. 7).

6. (1) Yes. (2) Yes. (3) No.



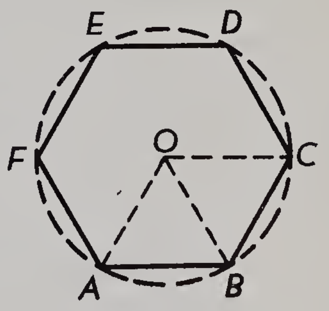
7. Proof: 1. Draw OA and OB (Asmt. 11). 2. $\angle AOB = \frac{360°}{6} = 60°$ (§462). 3. $\angle FAH = \frac{180(6-2)°}{6} = 120°$ (§211). 4. $\angle OAB = 60°$ (Ex. 5). 5. Likewise $\angle OBA = 60°$ (Statements 3, 4). 6. $\therefore OA = AB = OB$ (§129). 7. $OH \perp AB$ (§461). 8. Let $OA = AB = OB = R$. Then $OH = \frac{R}{2}\sqrt{3}$ (§254). 9. Area of $\triangle OAB = \frac{R^2}{4}\sqrt{3}$ (§273f). 10. In like manner, it can be shown that the area of each of the other 5 $\triangle$s similarly formed $= \frac{R^2}{4}\sqrt{3}$ (Statements 1-9). 11. Then the area of $ABCDEF = 6 \times \frac{R^2}{4}\sqrt{3} = \frac{3}{2}R^2\sqrt{3}$ (Asmt. 1, 9, 6).

8. Area of a regular hexagon $= \frac{3}{2}s^2\sqrt{3}$ (Ex. 7, above) $= \frac{3}{2} \times 16\sqrt{3}$ sq.in. $= 24\sqrt{3}$ sq.in.



9. Planning the Proof: 1. We can prove a triangle equilateral by §§56, 84, 129, 184. 2. We shall use §56.

Proof: 1. ABCDEF is a regular polygon (Given). 2. $AB = BC = CD = DE = EF = AF$ and $\angle B = \angle BCD = \angle D = \angle DEF = \angle F = \angle BAF$ (§184). 3. $\triangle ABC \cong \triangle CDE \cong \triangle AFE$ (§95a). 4. $AC = CE = AE$ (§97a). 5. $\triangle AEC$ is equilateral (§56).



10. Proof: 1. Circumscribe ⊙O about reg. polygon ABCDEF (§458). 2. AB, BC, CD, etc. are chords of ⊙O (§277). 3. The $\perp$ bisectors of these chords pass through O (§292). 4. Draw OA, OB, and OC (Asmt. 11). 5. $OB = OB$ (Iden.). 6. $OA = OC$ (Asmt. 26). 7. $AB = BC$ (§184). 8. $\triangle OAB \cong \triangle OBC$ (§95c). 9. $\angle OBA = \angle OBC$ (§98b). 10. OB bisects $\angle ABC$ (§22). 11. The bisector of $\angle ABC$ passes through O (Asmt. 18). 12. Likewise, the bisectors of the other angles of ABCDEF pass through O, the point where the $\perp$ bisectors of the sides of ABCDEF are concurrent (Statements 3-10).

11. The area of the triangle with sides 8 in., 15 in., and 17 in. $= \sqrt{20(12)(5)(3)} = 60$ (sq.in.). Then $\frac{s^2}{4}\sqrt{3} = 60$, $s^2\sqrt{3} = 240$, $s^2 = 80\sqrt{3}$, $s = \sqrt{80\sqrt{3}} = 4\sqrt{5}\sqrt{\sqrt{3}}$. $4\sqrt{5}\sqrt[4]{3}$ sq.in. Ans.

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(1) A circle can be divided into four equal parts by drawing two of its diameters perpendicular to each other.

(2) A circle can be divided into six equal arcs by separating it into arcs each of which has a chord equal to the radius of the circle.

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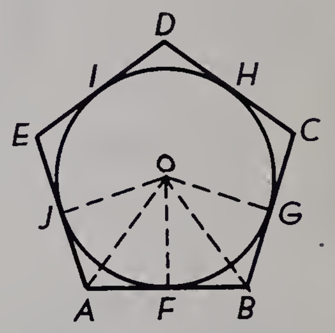
1. Given equilateral pentagon ABCDE circumscribed about ⊙O.

To prove that ABCDE is regular.

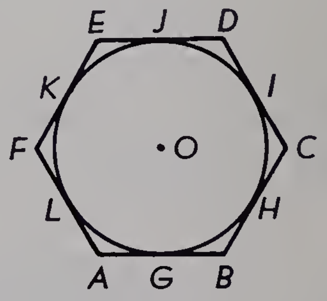


Planning the Proof: 1. We can prove a polygon regular by §§184, 463, 464, 465, 466. 2. We shall use §184.

Proof: 1. ABCDE is an equilateral pentagon circumscribed about ⊙O (Given). 2. AB = BC = CD = DE = AE (§183). 3. ID = DH, EI = EJ, AJ = AF, BF = BG, and CG = CH (§309b). 4. IE = HC (Asmt. 2). 5. EJ = CG (Asmt. 7). 6. AJ = BG (Asmt. 2). 7. AF = FB (Asmt. 7). 8. In like manner, we can prove BG = GC, CH = HD, DI = IE, and EJ = JA (Statements 2-7). 9. Draw OJ, OA, OF, OB, and OG (Asmt. 11). 10. EA, AB, and BC are tangents to ⊙O (§325). 11. OJ ⊥ AE, OF ⊥ AB, and OG ⊥ BC (§311b). 12. ⟁AOJ, OAF, OFB, and OBG are rt. ⟁ (§57). 13. OJ = OF = OG (Asmt. 26). 14. AJ = AF = FB = BG (Statements 2, 8, and Asmt. 2, 9, 6, 4). 15. △OAJ ≅ △OAF ≅ △OFB ≅ △OBG (§96d). 16. ∠OAJ = ∠OBF and ∠OAF = ∠OBG (§98b). 17. ∠JAF = ∠FBG (Asmt. 1, 9, 6). 18. In like manner we can prove ∠ABC = ∠GCD, etc. (Statements 9-17). 19. ∴ ABCDE is equiangular (§183a). 20. ∴ ABCDE is regular (§184).

2. Given equiangular hexagon ABCDEF circumscribed about ⊙O.

To prove that ABCDEF is regular.



Planning the Proof: 1. We can prove a polygon regular by §§184, 463, 464, 465, 466. 2. We shall use §466.

Proof: 1. ∠A = ∠B = ∠C = ∠D = ∠E = ∠F (Given, §183). 2. $\angle A° = \frac{1}{2}(\widehat{LK} + \widehat{KJ} + \widehat{JI} + \widehat{IH} + \widehat{GH} - \widehat{LG})$ and $\angle B° = \frac{1}{2}(\widehat{GL} + \widehat{LK} + \widehat{KJ} + \widehat{JI} + \widehat{IH} - \widehat{GH})$ (§333e, Asmt. 9, 6). 3. $\widehat{LK} + \widehat{KJ} + \widehat{JI} + \widehat{IH} + \widehat{GH} - \widehat{LG} = \widehat{GL} + \widehat{LK} + \widehat{KJ} + \widehat{JI} + \widehat{LH} - \widehat{GH}$ (Asmt. 3, 7). 4. $\widehat{GH} - \widehat{LG} = \widehat{GL} - \widehat{GH}$ (Asmt. 2). 5. $2\widehat{GH} = 2\widehat{LG}$ (Asmt. 1, 2). 6. $\widehat{GH} = \widehat{LQ}$ (Asmt. 4). 7. In like manner, $\widehat{GH} = \widehat{LG} = \widehat{HI} = \widehat{IJ} = \widehat{JK} = \widehat{KL}$ (Statements 2-6).

8. AB, BC, CD, DE, EF, and AF are tangents to ⊙O (§325). 9. Then ABCDEF is regular (§466).

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1. See §331.

2. See §332.

3. Inscribe a square in the ⊙ thus forming 4 = arcs. Bisect each of these arcs. Draw the chords of the 8 = arcs, forming a regular octagon (§465).

4. Inscribe a regular hexagon in the ⊙. Bisect each of the 6 = arcs. Draw the chords of the 12 = arcs (§465).

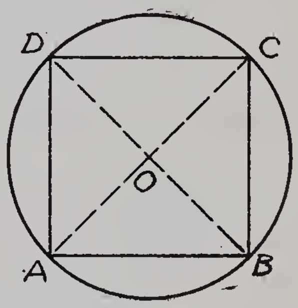
5. See Ex. 3, page 338.

6. Given the square ABCD.

To circumscribe a ⊙ about it.

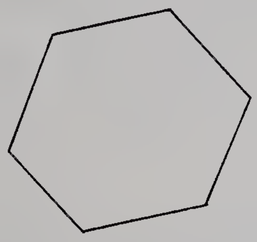
Construction: 1. Draw the diagonals AC and BD intersecting in O (Asmt. 11). 2. With O as a center and a radius = OA describe a ⊙ (Asmt. 15).

Then ⊙O is the required ⊙.



Proof: 1. Square ABCD is a ▱ and a rhombus (§185). 2. AC ⊥ BD (§220a). 3. AB = BC = AD (§194). 4. ∠DAB = ∠ABC (§191). 5. △DAB ≅ △ABC (§95a). 6. AC = BD (§97a). 7. AO = OC and DO = OB (§217c). 8. ∴ AO = OC = DO = OB (Asmt. 4, 6). 9. ∴ ⊙O passes through B, C, and D (§281b). 10. ∴ ⊙O is circumscribed about ABCD (§288).

7.



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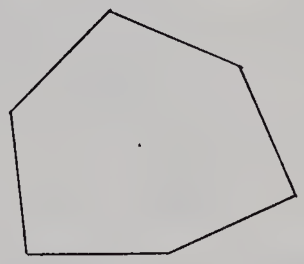
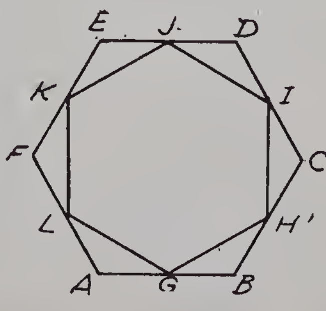


9. Given the regular polygon ABCDEF and polygon GHIJKL formed by joining the midpoints of the sides of ABCDEF in order.

To prove that GHIJKL is regular.



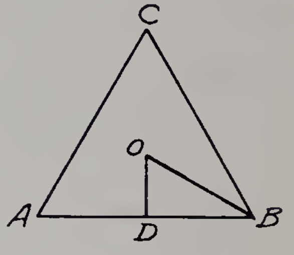
Planning the Proof: 1. We can prove polygons regular by §§184, 463, 464, 465, 466. 2. We shall use §184.

Proof: 1. AB = BC = CD, etc. (§184). 2. AG = GB, BH = HC, etc. (Given). 3. ∴ GB = BH = HC

= CI (Asmt. 4). 4. $\angle B = \angle C = \angle D$, etc. (§184). 5. $\therefore \triangle GBH \cong \triangle HCI \cong \triangle IDJ$, etc. (§95a). 6. $\therefore GH = HI = IJ$, etc. (§97a). 7. $\angle AGL = \angle BGH = \angle BHG = \angle CHI$, etc. (§§98a, 98b, Asmt. 7). 8. $\therefore \angle LGH = \angle GHI$, etc. (Asmt. 1, §50c). 9. $\therefore$ GHIJKL is a regular polygon (§184).

10. Given equilateral $\triangle ABC$ with apothem OD and radius OB.

To prove that $OD = \frac{1}{2}OB$.

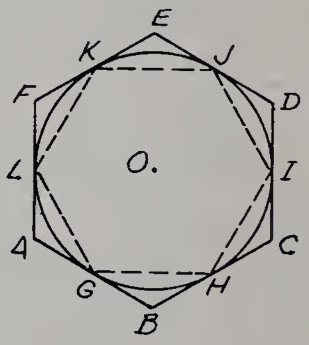


Planning the Proof: 1. We can prove a line segment half another by §136. 2. We shall use §136.

Proof: 1. $\triangle ABC$ is equilateral (Given). 2. $\therefore \triangle ABC$ is equiangular (§84). 3. Then $\triangle ABC$ is regular (§184). 4. $\angle ABC = 60°$ (§211). 5. $\therefore \angle OBD = 30°$ (Ex. 5, page 455). 6. $\triangle ODB$ is a rt. $\triangle$ (§§461, 57). 7. $\therefore OD = \frac{1}{2}OB$ (§136).

11. Given equiangular polygon ABCDEF circumscribed about $\odot$O.

To prove that ABCDEF is regular.

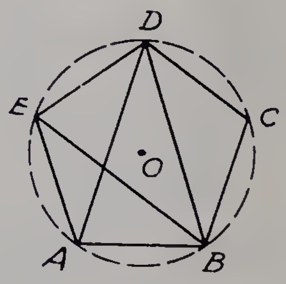


Planning the Proof: 1. We can prove a polygon regular by §§184, 463, 464, 465, 466. 2. We shall use §466.

Proof: 1. Draw GH, HI, IJ, etc. connecting the points of tangency (Asmt. 11). 2. $\angle B = \angle C$ (Given). 3. $BG = BH$, $CH = CI$ (§309b). 4. $\therefore \angle BGH = \angle BHG$, $\angle CHI = \angle CIH$ (§98a). 5. $\angle B + \angle BGH + \angle BHG = \angle C + \angle CHI + \angle CIH$ (§120, Asmt. 6). 6. $\therefore \angle BGH + \angle BHG = \angle CHI + \angle CIH$ (Asmt. 2). 7. $\therefore 2\angle BHG = 2\angle CHI$ (Asmt. 6). 8. $\therefore \angle BHG = \angle CHI$ (Asmt. 4). 9. $\angle BHG \overset{\circ}{=} \frac{1}{2}\overset{\frown}{GH}$ and $\angle CHI \overset{\circ}{=} \frac{1}{2}\overset{\frown}{HI}$ (§333c). 10. $\therefore \frac{1}{2}\overset{\frown}{GH} = \frac{1}{2}\overset{\frown}{HI}$ (Asmt. 7). 11. $\therefore \overset{\frown}{GH} = \overset{\frown}{HI}$ (Asmt. 3). 12. In like manner, $\overset{\frown}{HI} = \overset{\frown}{IJ}$, etc. (Statements 2-11). 13. $\therefore$ ABCDEF is regular (§466).

12. Given the regular pentagon ABCDE, with three of its diagonals AD, BE, and BD.

To prove that $AD = BE = BD$.



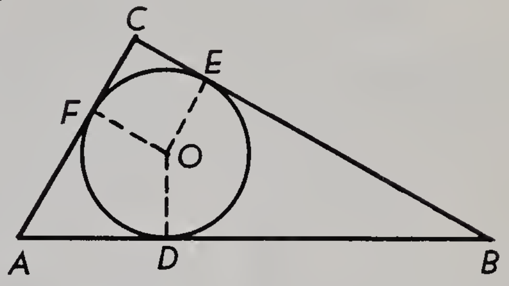
Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309, 378. 2. We shall use §308b.

Proof: 1. Circumscribe a $\odot$ about ABCDE (§458). 2. The pentagon has 5 equal sides AB, BC, etc. (§§183b, 184). 3. From each vertex, as D, there are 2 diagonals (§182). 4. $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD}$, etc. (§312b). 5. $\therefore \overset{\frown}{BC} + \overset{\frown}{CD} = \overset{\frown}{AE} + \overset{\frown}{ED} = \overset{\frown}{BA} + \overset{\frown}{AE}$ (Asmt. 1).

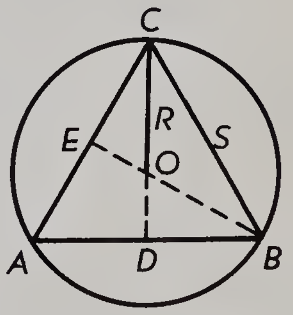
6. ∴ $\widehat{BD} = \widehat{AD} = \widehat{BE}$ (Asmt. 9, 6). 7. ∴ chord BD = chord AD = chord BE (§308b).

13. The perimeter = 6 x 4 in. = 24 in.

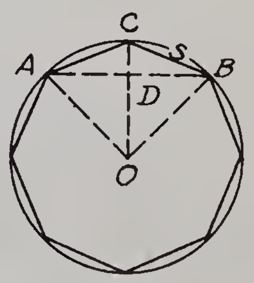
14. Given ⊙O inscribed in △ABC having ∠C a rt. ∠.
To prove that AC + BC − AB = diam. ⊙O.
Proof: 1. ⊙O is inscribed in △ABC and ∠C is a rt. ∠ (Given). 2. Draw radii OD, OE, and OF (D, E, F the points of tangency (Asmt. 11). 3. AC = AF + FC, BC = BE + EC, and AB = AD + DB (Asmt. 9). 4. AC + BC = AF + FC + BE + EC (Asmt. 1). 5. AF = AD and BE = BD (§309b). 6. AC + BC = AD + FC + BD + EC (Asmt. 6). 7. AC + BC − AB = FC + CE (Asmt. 2). 8. OF ⊥ FC and OE ⊥ EC (§295). 9. FC ⊥ CE (§14). 10. OF ∥ CE and OE ∥ FC (§160d). 11. FC = OE and CE = FO (§217b). 12. CE + CF = OF + OE (Asmt. 1). 13. OF + OE = a diameter (§277). 14. CE + CF = a diameter (Asmt. 6). 15. AC + BC − AB = a diameter of ⊙O (Asmt. 6).



15. Proof: 1. Extend radius CO(R) intersecting AB in D (Asmt. 13). 2. Draw line BO intersecting AC in E (Asmt. 11). 3. AC = BC (§56). 4. O is equidistant from A and B (Asmt. 26). 5. Then CD is the ⊥ bisector of AB (§140). 6. Likewise, BE is the ⊥ bisector of AC (Statements 3-5). 7. CD and BE are medians of △ABC (§89). 8. Then R = $\frac{2}{3}$CD (§362). 9. CD is an altitude of △ABC (§90). 10. CD = $\frac{s}{2}\sqrt{3}$ (§254). 11. R = $\frac{2}{3} \times \frac{s}{2}\sqrt{3}$ (Asmt. 6). 12. Solving for s, s = $R\sqrt{3}$. (Asmt. 6, 3, 4).



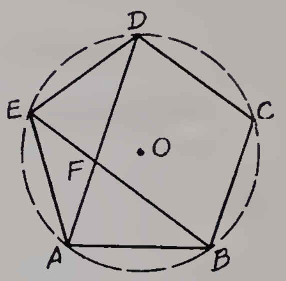
16. Proof: 1. Draw BO, AO, CO, and AB (Asmt. 11). 2. $\widehat{CB} = \widehat{AC} = \frac{1}{8}$ of 360° = 45° (§312b, Asmt. 6, 4). 3. $\widehat{ACB}$ = 90° (Asmt. 1). 4. AC = CB (Given). 5. AO = BO (Asmt. 26). 6. ∴ CO is the ⊥ bisector of AB (§140). 7. ∠AOB = 90° (§333a). 8. ∴ $\overline{AB}^2 = R^2 + R^2 = 2R^2$ (§430). 9. ∴ AB = $R\sqrt{2}$ (Asmt. 8). 10. DB = $\frac{R}{2}\sqrt{2}$ (Asmt. 6, 4). 11. $\overline{OD}^2 = R^2 - (\frac{R\sqrt{2}}{2})^2 = R^2 - \frac{R^2}{2}$ (§430). 12. OD = $\sqrt{R^2 - \frac{R^2}{2}}$ (Asmt. 8). 13. OD = $\frac{R}{2}\sqrt{2}$ (Asmt. 6). 14. CD = $R - \frac{R}{2}\sqrt{2}$ (Asmt. 9, 6). 15. $s^2 = \overline{CD}^2 + \overline{DB}^2$ (§430). 16. $s^2 = (R - \frac{R}{2}\sqrt{2})^2 + (\frac{R}{2}\sqrt{2})^2 = R^2 - R^2\sqrt{2} + \frac{R^2}{2} + \frac{R^2}{2} =$



$2R^2 - R^2\sqrt{2}$ (Asmt. 6). 17. $s = \sqrt{2R^2 - R^2\sqrt{2}} = R\sqrt{2 - \sqrt{2}}$ (Asmt. 8, 6).

17. Given the regular pentagon ABCDE with two diagonals AD and BE intersecting in F.

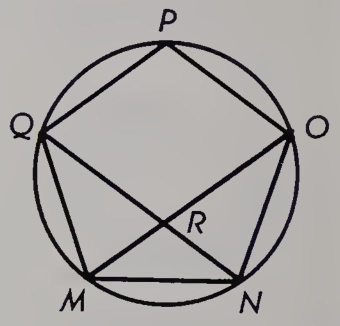
To prove that DF = ED.



Planning the Proof: 1. We can prove line segments = by §§97, 157, 217, 308, 309, 378. 2. We shall use §157a.

Proof: 1. Circumscribe a $\odot$ about ABCDE (§458). 2. AB = BC = CD, etc. (§184). 3. $\therefore \overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD}$, etc. (§312b). 4. $\therefore \overset{\frown}{AB} = \overset{\frown}{BC}$, etc. $= 72°$ (§314, Asmt. 9, 6, 4). 5. $\angle FED \stackrel{\circ}{=} \frac{1}{2}(\overset{\frown}{BC} + \overset{\frown}{CD}) = 72°$ (§333b, Asmt. 6). 6. $\angle EFD \stackrel{\circ}{=} \frac{1}{2}(\overset{\frown}{ED} + \overset{\frown}{AB}) = 72°$ (§333d, Asmt. 6). 7. $\therefore \angle EFD = \angle FED$ (Asmt. 7). 8. $\therefore$ DF = ED (§157a).

18. Planning the Proof: 1. We can prove line segments proportional by §§409, 451, 453. 2. We shall use §451a, Asmt.



Proof: 1. MNOPQ is a regular pentagon inscribed in $\odot$O and having diagonals MO and NQ intersecting in point R (Given). 2. MN = NO = OP = PQ = MQ (§184). 3. $\overset{\frown}{MN} = \overset{\frown}{NO} = \overset{\frown}{OP} = \overset{\frown}{PQ} = \overset{\frown}{MQ}$ (§312b). 4. $\frac{1}{2}\overset{\frown}{MN} = \frac{1}{2}\overset{\frown}{MQ}$ (Asmt. 4). 5. $\angle MON \stackrel{\circ}{=} \frac{1}{2}\overset{\frown}{MN}$ and $\angle MNQ \stackrel{\circ}{=} \frac{1}{2}\overset{\frown}{MQ}$ (§333b). 6. $\angle MON = \angle MNQ$ (Asmt. 7). 7. $\angle NMO = \angle NMR$ (Iden.). 8. $\triangle MNR \sim \triangle MNO$ (§450a). 9. $\frac{MR}{MN} = \frac{MN}{MO}$ (§451a). 10. $\angle RNO \stackrel{\circ}{=} \frac{1}{2}(\overset{\frown}{QPO})$ (§333b). 11. $\angle RNO \stackrel{\circ}{=} \frac{1}{2}(\overset{\frown}{QP} + \overset{\frown}{PO})$ (Asmt. 9, 6). 12. $\angle RNO \stackrel{\circ}{=} \frac{1}{2}(\overset{\frown}{QM} + \overset{\frown}{NO})$ (Asmt. 6). 13. $\angle NRO \stackrel{\circ}{=} \frac{1}{2}(\overset{\frown}{QM} + \overset{\frown}{NO})$ (§333d). 14. $\angle RNO = \angle NRO$ (Asmt. 7). 15. RO = NO (§157a). 16. MN = RO (Asmt. 6). 17. $\frac{MR}{RO} = \frac{RO}{MO}$ (Asmt. 6).

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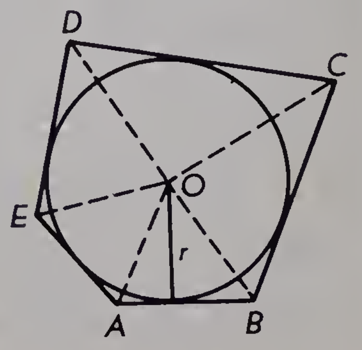
1. Given polygon ABCDE circumscribed about $\odot$O with radius r.

To prove that the area of ABCDE = $\frac{1}{2}r(AB + BC + CD + DE + EA)$.

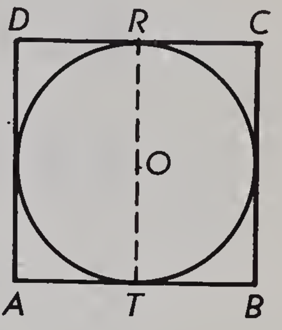


Planning the Proof: 1. We can find areas by §§273, 467. 2. We shall use §273c.

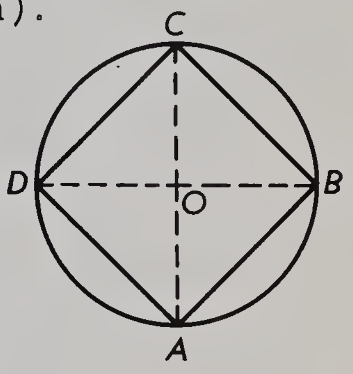
Proof: 1. Draw OA, OB, OC, OD, and OE (Asmt. 11). 2. AB, BC, etc. are tangent to $\odot$O (§325). 3. AB, BC, etc. $\perp$ to the radius drawn to the point of tangency (§311b).

4. $\triangle AOB = \frac{1}{2}r \times AB$, $\triangle BOC = \frac{1}{2}r \times BC$, etc. (§273c). 5. $\triangle AOB + \triangle BOC + \triangle COD + \cdots = \frac{1}{2}r \times AB + \frac{1}{2}r \times BC + \frac{1}{2}r \times CD + \cdots$ (Asmt. 1). 6. Polygon ABCDE = $\frac{1}{2}r(AB + BC + CD + DE + EA)$ (Asmt. 9, 6).

2. Each side of the hexagon = $\frac{1}{6}$ of 12 in. = 2 in. The apothem = $\frac{s}{2}\sqrt{3} = \frac{2}{2}\sqrt{3} = \sqrt{3}$. $\sqrt{3}$ in. Ans.



3. Proof: 1. ABCD is circumscribed about ⊙O (Given). 2. Draw OR and OT, radii to points of tangency (Asmt. 11). 3. Then OR ⊥ DC and OT ⊥ AB (§§325, 295). 4. AB ∥ DC (§185). 5. OT ⊥ DC (§114). 6. OT extended through O coincides with OR and ROT is a st. line (Asmt. 20). 7. RT ∥ CB (§107). 8. RT = CB (§217b). 9. Area ABCD = $\overline{BC}^2 = (2\overline{RO})^2 = 4r^2$ (§273h, Asmt. 6).



4. 1. ABCD is a square inscribed in ⊙O (Given). 2. AB = BC = CD = AD (§§185, 194). 3. Each ∠ of ABCD is a rt. ∠ (§§185, 191). 4. If AC is not a diameter, ∠ABC is not a rt. ∠ (§318). Then since ∠ABC is a rt. ∠, AC is a diameter. 5. Likewise BD is a diameter. 6. ABCD is a rhombus (§185). 7. Area ABCD = $\frac{1}{2}AC \times DB$ (§273g). 8. Area ABCD = $\frac{1}{2} \times 2r \times 2r$ (Asmt. 6). 9. Area ABCD = $2r^2$ (Asmt. 6).

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Ex. The area of EFGH = 2 x 64 sq.in. = 128 sq.in.

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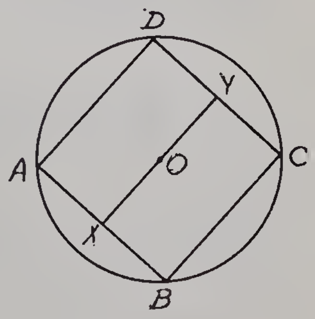
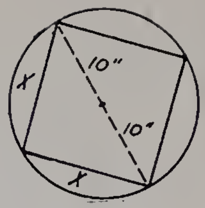
1. XY = BC = 10 in. OX = $\frac{1}{2}XY$ = 5 in.



2. (1) $\frac{p}{p_1} = \frac{2}{6} = \frac{1}{3}$ (§451c).
   (2) $\frac{S}{S_1} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$ (§469).

3. (1) - (3) $\frac{p}{p'} = \frac{s}{s'} = \frac{r}{r'} = \frac{a}{a'} = \frac{3}{5}$ (§§451c, 470, Asmt. 7). (4) $\frac{S}{S'} = \frac{3^2}{5^2} = \frac{9}{25}$ (§469).



4. Diameter = 20 in. (Given). If x = the number of inches in a side, $x^2 + x^2 = 20^2$ (§430). Solving, $2x^2 = 400$; $x^2 = 200$; whence $x = 14.142$.

14.14 in. Ans.

5. $S = \frac{3}{2}R^2\sqrt{3}$ (Ex. 7, page 455).

$S = \frac{3}{2} \times 16^2\sqrt{3} = 384\sqrt{3} = 665.09$ sq.in.

Also $S = 6 \times \frac{s^2}{4}\sqrt{3} = 384\sqrt{3} = 665.09$ sq.in.

6. $\frac{p}{p'} = \frac{r}{r'}$ (§470); $\frac{36}{63} = \frac{6}{r'}$; $r' = 10.5$ (in.).

7. $\frac{S}{S'} = \frac{r^2}{r'^2} = \frac{a^2}{a'^2}$ (§471).

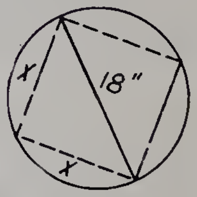
$\frac{r^2}{r'^2} = \frac{a^2}{a'^2} = \frac{112}{28} = \frac{4}{1}$; $\frac{r}{r'} = \frac{a}{a'} = \frac{2}{1}$.

8. $\frac{S}{S'} = \frac{s^2}{s'^2} = \frac{p^2}{p'^2} = \frac{25}{50}$ (§§469, 451c, Asmt. 8).

$\frac{p}{p'} = \sqrt{\frac{25}{50}} = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} = \frac{1}{2} \times 1.414 = 0.707$. 0.71. Ans.

9. (1) $\frac{h}{h'} = \frac{s}{s'} = \frac{p}{p'} = \frac{3}{8}$ (§451b, c). (2) $\frac{S}{S'} = \frac{h^2}{h'^2} = \frac{3^2}{8^2} = \frac{9}{64}$ (§455a).

10. $\frac{28}{S_1} = \frac{3^2}{(4\frac{1}{2})^2}$; $S_1 = 63$ (sq.in.).



11. $x^2 + x^2 = 18^2$ (§430). Solving, $2x^2 = 324$; $x^2 = 162$ (sq.in.), area of square.

12. $s = R\sqrt{3}$ (Ex. 15, page 458); $R = \frac{12}{\sqrt{3}} = 4\sqrt{3}$. $a = \frac{1}{2}R$ (Ex. 10, p. 458) $= \frac{1}{2} \times 4\sqrt{3} = 2\sqrt{3} = 2 \times 1.732 = 3.464$.

3.46 ft. Ans.

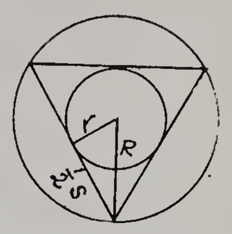
13. $s = R\sqrt{3}$ (Ex. 15, page 458); $R = \frac{16}{\sqrt{3}} = \frac{16}{3} \times 1.732 = 9.237+$.

9.24 ft. Ans.

14. $\frac{S}{S'} = \frac{r^2}{r'^2} = \frac{p^2}{p'^2} = \frac{2\frac{1}{4}}{1} = \frac{9}{4}$ (§§471, 470); $\frac{p}{p'} = \frac{3}{2}$.

15. $\frac{S}{S'} = \frac{h^2}{h'^2} = \frac{9}{1}$ (§455a); $\frac{h}{h'} = \frac{3}{1}$.

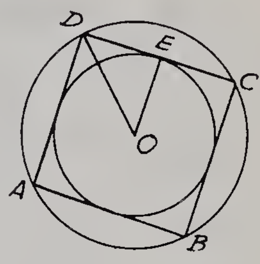
16. From Ex. 15, page 458, $R = \frac{s}{\sqrt{3}}$.



$r = \sqrt{(\frac{s}{\sqrt{3}})^2 - (\frac{1}{2}s)^2} = \sqrt{\frac{s^2}{3} - \frac{s^2}{4}} = \sqrt{\frac{s^2}{12}} = \frac{s}{\sqrt{12}}$.

$R : r = \frac{s}{\sqrt{3}} : \frac{s}{\sqrt{12}} = \sqrt{12} : \sqrt{3} = 2\sqrt{3} : \sqrt{3} = 2 : 1$.

17. OE = DE. $\overline{DO}^2 = \overline{DE}^2 + \overline{OE}^2$. $\overline{DO}^2 = 2\,\overline{OE}^2$. $DO = OE\sqrt{2}$. Then the radius of the circumcircle is $\sqrt{2}$ times the radius of the incircle.



18. Using the figure of Ex. 17, AB = 6 in. and BC = 6 in. Then AC, the diameter of the circumscribed ⊙, = $6\sqrt{2}$ in., or 8.48 in.

19. If the radius = 8, the diagonal = 16. The area = $\frac{1}{2} \times 16^2 = 128$ (§§273g, 185).

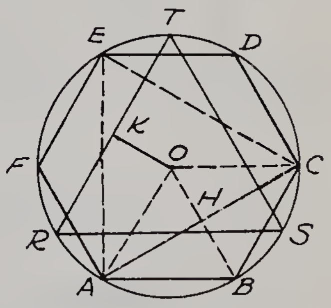
20. (1) Let x = the number of inches in a side of the inscribed square. The diameter of the ⊙ = 16 in. Then $x^2 + x^2 = 16^2$. Solving, $x = 8\sqrt{2}$. The perimeter of the inscribed square = $32\sqrt{2}$ in. and the area = $(8\sqrt{2})^2$ sq.in. = 128 sq.in.

(2) Each side of the circumscribed square = the diameter = 16 in. The perimeter = 64 in. and the area = 16^2 sq.in. = 256 sq.in.

21. No. An example is an inscribed rectangle.

22. Given ⊙O with ABCDEF, an inscribed regular hexagon and RST, an inscribed equilateral △ with OK an apothem.

To prove that AB = 2 OK.



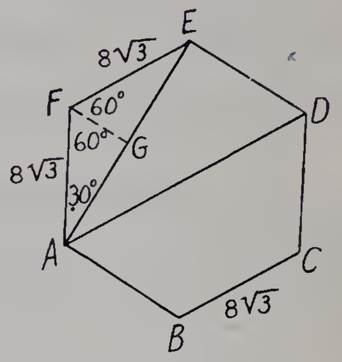
Planning the Proof: 1. We can prove a line segment twice another: §§136, 218. 2. We shall use Asmt. 6.

Proof: 1. Draw CO, AO, BO, AC, EC, and AE (Asmt. 11). 2. AB = BC = CD, etc. (§184). 3. ∴ $\widehat{AB} = \widehat{BC} = \widehat{CD}$, etc. (§312b). 4. ∴ $\widehat{AB} + \widehat{BC} = \widehat{CD} + \widehat{DE} = \widehat{EF} + \widehat{FA}$ (Asmt. 1). 5. ∴ $\widehat{AC} = \widehat{CE} = \widehat{EA}$ (Asmt. 9, 6). 6. ∴ △AEC is a regular polygon (§463). 7. OA = OC (Asmt. 26). 8. AB = BC (§184). 9. ∴ OB ⊥ AC (§159b). 10. OH is the apothem of △ACE (§461). 11. OH = OK (§§312a, 308b, d). 12. $\widehat{AB} = 60°$ (§314, Asmt. 9, 4). 13. ∠AOB = 60° (§333a). 14. ∠OAB + ∠OBA + ∠AOB = 180° (§120). 15. ∴ ∠OAB + ∠OBA = 120° (Asmt. 2). 16. OA = OB (Asmt. 26). 17. ∠OAB = ∠OBA (§98a). 18. 2∠OBA =

120° (Asmt. 6). 19. $\angle OBA = 60°$ (Asmt. 4). 20. OB = AB (§97a). 21. AO = 2 OH (Ex. 16, page 463). 22. ∴ AB = 2 KO (Asmt. 6).

23. The rhombus consists of two = equilateral △ having 18-inch sides. The area of each △ = $\frac{s^2}{4}\sqrt{3} = \frac{18^2\sqrt{3}}{4} = 81\sqrt{3}$ sq.in. The area of rhombus = $162\sqrt{3}$ sq.in., or 280.58 sq.in.

24. ABCDEF = $6(\frac{s^2}{4}\sqrt{3}) = \frac{3}{2}s^2\sqrt{3}$; ABCDEF = $\frac{3}{2}(8\sqrt{3})^2\sqrt{3} = 288\sqrt{3}$; ADEF = $\frac{1}{2}(288\sqrt{3})^2 = 144\sqrt{3}$. Draw FG ⊥ AE; FG = $4\sqrt{3}$ (§136). $\overline{AG}^2 = (8\sqrt{3})^2 - (4\sqrt{3})^2$; $\overline{AG}^2 = 144$; AG = 12. $\triangle AFE = \frac{1}{2}(24)4\sqrt{3} = 48\sqrt{3}$. $\triangle AED = 144\sqrt{3} - 48\sqrt{3} = 96\sqrt{3}$ sq.in., or 166.27 sq.in.



PAGES 464-465

1. (1) The polygon of 32 sides has the greater apothem.

(2) The smaller chord is at a greater distance from the center.

(3) The polygon of 32 sides has the greater perimeter.

(4) The sum of two sides of a △ is greater than the third side; and if unequals are added to unequals in the same order, the sums are unequal in the same order.

2. (1) The radius of the ⊙ is the limit of the apothem of the polygon.

(2) The ⊙ circumscribed about the polygon is the limit of the perimeter of the polygon.

3. $\frac{2}{3}$. 4. $\frac{7}{9}$. 5. $\frac{8}{9}$. 6. a. The ⊙. b. The area of the ⊙. c. No. 7. No.

PAGE 466 (First Group)

1. §476. 2. (1) m = n. (2) §477. 3. pr ⟶ pc.
4. (1) 180°. (2) 0°.

PAGE 466 (Second Group)

1. (1) The circumscribed square. (2) The circumscribed square.
2. The polygon of 16 sides.

PAGES 473-475

1. a. $c = 2\pi r = 25.133$ b. $c = 2\pi r = 37.699.$
c. $c = 2\pi r = 75.398$ d. $c = 2\pi r = 785.40$

2. a. $S = \pi r^2 = 28.274$ b. $S = \pi r^2 = 706.86$
c. $S = \pi r^2 = 5541.8$ d. $S = \pi r^2 = 64\pi^3$, or 1984.4

3. a. $d = \frac{c}{\pi} = 4.0000$ b. $d = \frac{c}{\pi} = 25.000$
c. $d = \frac{c}{\pi} = 11.000$ d. $d = \frac{c}{\pi} = 31.8309.$
31.831. Ans.

4. a. $\pi r^2 = 13.0946$; $r^2 = 4.1681$; $r = 2.041+$. 2.04. Ans.
b. $\pi r^2 = 18$; $r^2 = 5.7296$; $r = 2.3936.$
c. $\pi r^2 = 324\pi$; $r^2 = 324$; $r = 18.$

5. a. $2\pi r = 30$; $r = 4.7746$ b. $2\pi r = 45$; $r = 7.1620$
c. $2\pi r = 68$; $r = 10.823$

6. Area of ⊙ $= \pi r^2 = \pi 6^2 = 113.0976$. Area of semicircle = $\frac{1}{2}$ of 113.0976 = 56.549.

7. (1) $\frac{c}{c_1} = \frac{r}{r_1} = \frac{5}{10} = \frac{1}{2}$ (§479). (2) $\frac{S}{S_1} = \frac{r^2}{{r_1}^2} = \frac{5^2}{10^2} = \frac{1}{4}$ (§489).

8. Area of square = 16. Area of ⊙ $= \pi r^2 = 16$. Solving, $r = 2.2568.$

9. Area of ring $= \pi R^2 - \pi r^2 = \pi 8^2 - \pi 6^2 = 87.965$ (sq.in.).

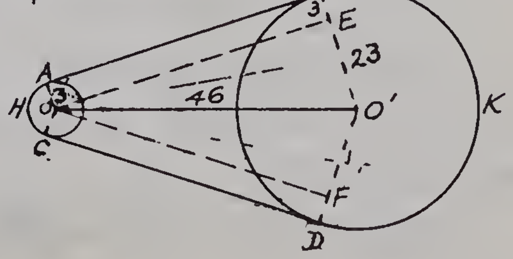
10. $c = \pi d$. $c = 3.1416\ (34.15) = 107.29$ (in.), or 107.3 in.

11. The number of inches in the belt $= \frac{1}{2} \times 8\pi + 20 + \frac{1}{2} \times 8\pi + 20 = 65.133.$

12. $\frac{S}{36} = \frac{(3r)^2}{r^2}$ (§489). Solving, $S = 324$. 324 sq.in. Ans.

13. 2 A. = 320 sq.rd. = 9680 sq.yd. Let r = the number of yards in the radius of the pond. Then $\pi r^2 = 9680$. Solving, $r = 55.5$. The radius of the ⊙ which is the other boundary of the walk = 57.5 yd. The number of square yards in the walk = $\pi 57.5^2 - 9680 = 706.92$. $706.92 \times \$.09 = \$63.62$.

14. Draw OA, O'B, OC, and O'D, radii to points of tangency of AB and CD. Draw OE and OF ∥ AB and CD respectively. AB = CD = OE = OF (§§217a, 160d, 311b)



$= \sqrt{46^2 - 23^2} = 39.837$ in. O'E $= \frac{1}{2}$OO'. ∴ ∠O'OE = 30° and ∠OO'E = 60°. Likewise ∠O'OF = 30° and ∠OO'F = 60°. $\widehat{AHC} = 360° - 90° - 30° - 30° = 120°$ (§§185, 191, 136). $\widehat{AHC} = \frac{120}{360} \times 2\pi 3$ in. = 6.2832 in. $\widehat{BKD} = 360° - 60° - 60° = 240°$. $\widehat{BKD} = \frac{240}{360} \times 2\pi 26$ in. = 108.91 in. Length of belt = 39.837 in. + 39.837 in. + 6.2832 in. + 108.91 in. = 194.87 in.

15. Area of ⊙ in square inches $= \pi r^2 = 78.540$. Area of square in square inches $= \frac{1}{2} \times 10^2 = 50$ (§273g).

78.54 sq.in. − 50 sq.in. = 28.540 sq.in.

16. 1. AO = OB (Given). 2. Area of largest semicircle $= \frac{\pi}{2}\overline{AO}^2$ (§488). 3. Area of each smaller semicircle $= \frac{1}{2}\pi\left(\frac{AO}{2}\right)^2 = \frac{\pi}{8}\overline{AO}^2$ (§488, Asmt. 6). 4. Area of both smaller ⓢ $= \frac{\pi}{4}\overline{AO}^2$ (§488). 5. Area of shaded portion $= \frac{\pi}{2}\overline{AO}^2 - \frac{\pi}{4}\overline{AO}^2 = \frac{\pi}{4}\overline{AO}^2$ (Asmt. 2). 6. ∴ Area of shaded portion = area of both smaller semicircles (Asmt. 6).

17. Let d = the diameter. Then πd = the circumference and $\frac{\pi d^2}{4}$ = the area. $\pi d = \pi\frac{d^2}{4}$. Solving, d = 4.

18. Area of arch = area of sector ABC + sector BAC − area of △ABC $= \frac{1}{6}\pi 6^2 + \frac{1}{6}\pi 6^2 - \frac{6^2}{4}\sqrt{3} = 12\pi - 9\sqrt{3} = 22.111$.

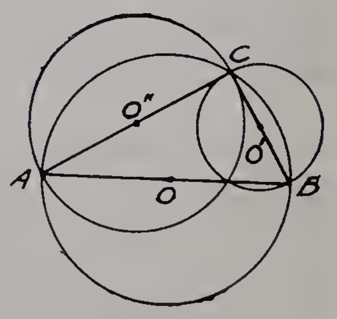
22.111 sq.ft. Ans.

19. $\frac{S}{S_1} = \frac{(\frac{1}{2})^2}{(\frac{1}{8})^2} = \frac{16}{1}$ (§489). Then $S = 16S_1$.

20. 1. AO = OB (Given). 2. Area of unshaded surface $= \frac{\pi}{2}\left(\frac{AO}{2}\right)^2 + \frac{\pi}{2}\overline{AO}^2 - \frac{\pi}{2}\left(\frac{BO}{2}\right)^2 = \frac{\pi}{2}\overline{AO}^2$ (§488, Asmt. 9, 6). 3. Area of shaded surface $= \frac{\pi}{2}\left(\frac{BO}{2}\right)^2 + \frac{\pi}{2}\overline{BO}^2 - \frac{\pi}{2}\left(\frac{AO}{2}\right)^2 = \frac{\pi}{2}\overline{BO}^2$ (Same as 2). 4. ∴ from 1, area of shaded portion = area of unshaded portion (Asmt. 7).

21. 1 mi. = 5280 ft. = 63,360 in. Circumference of wheel $= 60\pi = 188.50$ in. 60 mi. an hour is 1 mi. a minute. 63,360 ÷ 188.50 = 336.13, the number of revolutions.

22. Given rt. △ABC with hypotenuse AB, ⊙O with diameter AB, ⊙O' with diameter BC, and ⊙O" with diameter AC.



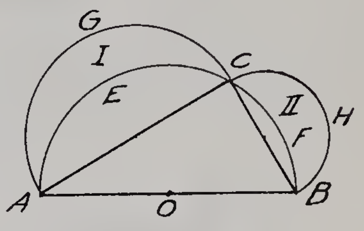
To prove that ⊙O = ⊙O' + ⊙O".

Planning the Proof: 1. We can prove the sum of 2 ⓢ = another ⊙ by Asmt. 2. We shall use Asmt. 7.

Proof: 1. ⊙O $= \frac{\pi}{4}\overline{AB}^2$, ⊙O' $= \frac{\pi}{4}\overline{BC}^2$, and

$\odot O'' = \frac{\pi}{4}\overline{AC}^2$ (§488, Asmt. 4). 2. $\odot O' + \odot O'' = \frac{\pi}{4}(\overline{BC}^2 + \overline{AC}^2)$ (Asmt. 1). 3. $\overline{BC}^2 + \overline{AC}^2 = \overline{AB}^2$ (§430). 4. ∴ $\odot O' + \odot O'' = \frac{\pi}{4}\overline{AB}^2$ (Asmt. 6). 5. ∴ $\odot O = \odot O' + \odot O''$ (Asmt. 7).

23. Given rt. △ABC with hypotenuse AB, semicircle ACB with diameter AB, semicircle AGC with diameter AC, semicircle BHC with diameter BC, and crescents I and II formed by the semicircles.

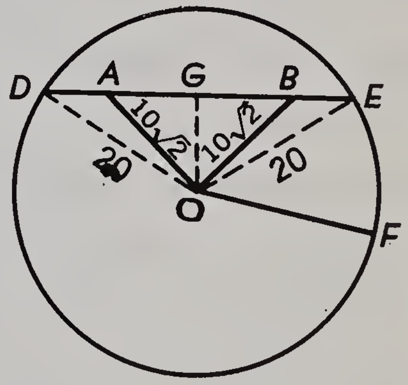


To prove that △ABC = I + II.

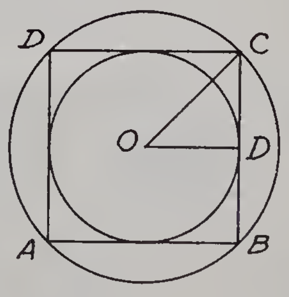
Planning the Proof: 1. We can prove a △ = parts of ⓢ by Asmt. 2. We shall use Asmt. 7.

Proof: 1. Semicircle AGC = $\frac{\pi}{8}\overline{AC}^2$, semicircle BHC = $\frac{\pi}{8}\overline{BC}^2$, and semicircle AEB = $\frac{\pi}{8}\overline{AB}^2$ (§488, Asmt. 4). 2. △ABC = $\frac{\pi}{8}\overline{AB}^2$ - segment AEC - segment BFC (Asmt. 9, 2). 3. Crescent I = $\frac{\pi}{8}\overline{AC}^2$ - segment ACE and crescent II = $\frac{\pi}{8}\overline{BC}^2$ - segment BFC (Asmt. 9, 2). 4. I + II = $\frac{\pi}{8}(\overline{AC}^2 + \overline{BC}^2)$ - segment ACE - segment BFC (Asmt. 1). 5. $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$ (§430). 6. ∴ I + II = $\frac{\pi}{8}\overline{AB}^2$ - segment ACE - segment BFC (Asmt. 6). 7. ∴ △ABC = crescent I + crescent II (Asmt. 7).

24. 1. Draw OG ⊥ AE. 2. Draw OE and DO. 3. ∠GOB = 45° and ∠GBO = 45°. 4. OG = GB. 5. $\overline{OG}^2 + \overline{GB}^2 = (10\sqrt{2})^2$. 6. $2\overline{OG}^2 = 200$. 7. $\overline{OG}^2 = 100$. 8. OG = 10. 9. ∠GOE = 60°. 10. ∠DOE = 120°. 11. The area of sector DHE = $\frac{2}{3}$ area of ⊙O. 12. The area of the shaded portion of the figure = the area of sector DHE - area △AOB = $\frac{2}{3}\pi\overline{20}^2 - \frac{1}{2}(10\sqrt{2})^2 = 737.76$. 737.76. Ans.



25. Given square ABCD with its incircle and circumcircle with center O, OC the radius of the circumcircle and OD the radius of the incircle.



To prove that the area of the incircle is half the area of the circumcircle.

Planning the Proof: 1. We can compare areas of circles by §§281a, 489. 2. We shall use Asmt.

Proof: 1. Area of incircle = $\pi\overline{OD}^2$ and area of circumcircle = $\pi\overline{OC}^2$ (§488). 2. OD ⊥ BC (§461). 3. ∠OCD = 45° (§185, Ex. 5, page 455). 4. ∠COD = 45° (§161). 5. ∴ CD = OD (§157).

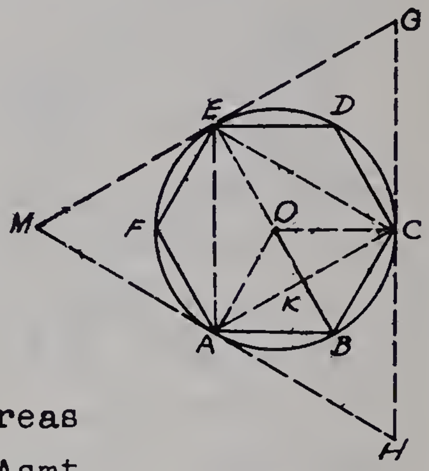
6. $\overline{OC}^2 = \overline{OD}^2 + \overline{CD}^2$ (§430). 7. $\overline{OC}^2 = 2\,\overline{OD}^2$ (Asmt. 6). 8. $\frac{\text{Area incircle}}{\text{Area circumcircle}} = \frac{\overline{OD}^2}{\overline{OC}^2} = \frac{\overline{OD}^2}{2\,\overline{OD}^2} = \frac{1}{2}$ (Asmt. 4, 6). 9. ∴ Area of incircle = $\frac{1}{2}$ area of circumcircle (Asmt. 3).

26. Side of △ = 6". Radius of circumcircle = $\frac{2}{3}$ altitude of △ = $2\sqrt{3}$. Sum of areas of 3 segments = $\pi(2\sqrt{3})^2$ - area of △ = $12\pi - 9\sqrt{3}$. Sum of areas of 3 semicircles = $\frac{3}{2}\pi 3^2 = \frac{27}{2}\pi$. Sum of areas of crescents = $\frac{27\pi}{2} - (12\pi - 9\sqrt{3}) = \frac{3}{2}\pi + 9\sqrt{3} = 20.30$.

20.30 sq.in. Ans.

27. Given ⊙O, an inscribed regular hexagon ABCDEF with radius OB, an inscribed equilateral △, and a circumscribed equilateral △.

(Note: The two ⚠ are not shown in the drawing.)



To prove that $\frac{\text{an inscribed equilateral }\triangle}{\text{ABCDEF}} = \frac{\text{ABCDEF}}{\text{a circumscribed equilateral }\triangle}$.

Planning the Proof: 1. We can compare areas of polygons by §§272, 274. 2. We shall use Asmt.

Proof: 1. ABCDEF is a regular hexagon (Given). 2. AB = BC = CD, etc. (§184). 3. $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD}$, etc. (§312b). 4. $\overset{\frown}{AC} = \overset{\frown}{CE} = \overset{\frown}{AE}$ (Asmt. 1). 5. Draw AC, CE, and AE (Asmt. 11). 6. △ACE is equilateral (§308b). 7. Draw OA, OE, and OC (Asmt. 11). 8. OB is the ⊥ bisector of AC (§140, Asmt. 25). 9. Then OK is the apothem of △ACE (§461). 10. OK = KB (Ex. 10, page 458). 11. ABCO is a ▱ (§223d). 12. ∴ △OAC = $\frac{1}{2}$OABC (§187). 13. In like manner, △COE = $\frac{1}{2}$OCDE and △AOE = $\frac{1}{2}$OAFE (Statements 9-13). 14. ∴ △ACE = $\frac{1}{2}$ABCDEF (Asmt. 1). 15. Draw tangents to ⊙O at A, C, and E forming equilateral △HMG (§§329, 466). 16. EG = GC = EC (§§333c, e, 157) and AC = AE = EC (Statement 6). 17. AE = EG and AC = CG (Asmt. 7). 18. ∴ △ACE ≅ △ECG (§95c). 19. In same manner, △ACE ≅ ⚠ AEM and ACH (Statements 17, 18). 20. △AEC = $\frac{1}{4}$△HMG (Statements 18, 19). 21. From 14, $\frac{\triangle\text{ ACE}}{\text{ABCDEF}} = \frac{1}{2}$ (Asmt. 4). 22. $\frac{\text{ABCDEF}}{\triangle\text{ HMG}} = \frac{2\triangle\text{AEC}}{4\triangle\text{AEC}} = \frac{1}{2}$ (Asmt. 6, 4). 23. $\frac{\triangle\text{ ACE}}{\text{ABCDEF}} = \frac{\text{ABCDEF}}{\triangle\text{ HMG}}$ (Asmt. 7, 6). 24. Any equilateral △ inscribed in ⊙O ≅ △ACE (§95c). 25. In same manner as above, any circumscribed equilateral △ is = 4 times an inscribed equilateral △ and hence ≅ △HMG (Asmt. 7). 26. ∴ $\frac{\text{an inscribed equilateral }\triangle}{\text{ABCDEF}} = \frac{\text{ABCDEF}}{\text{a circumscribed equilateral }\triangle}$ (Asmt. 6).

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2. Let x = the number of inches in the longer segment. Then $5 - x$ = the number of inches in the shorter segment. Then $x^2 = 5(5 - x)$. Solving, $x = 3.09$ and $5 - x = 1.91$.

3.09 in., 1.91 in. Ans.

3. Direct application of §493, with AB = 5 in.

4. Let x = the length of the longer segment. Then $a - x$ = the length of the shorter segment. Then $x^2 = a(a - x)$. $x^2 = a^2 - ax$. $x^2 + ax = a^2$. $x^2 + ax + \frac{a^2}{4} = \frac{5}{4}a^2$. $x + \frac{a}{2} = \frac{a}{2}\sqrt{5}$. $x = \frac{a}{2}\sqrt{5} - \frac{a}{2} = \frac{a}{2}(\sqrt{5} - 1) = .618a$. Then $a - x = .382a$.

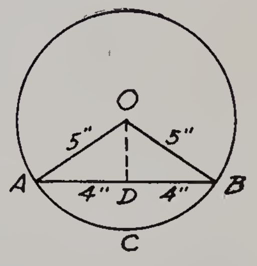
5. $x^2 = 180(180 - x)$. $x^2 = 32{,}400 - 180x$. $x^2 + 180x = 32{,}400$. Solving, $x = 111.25°$ and $180° - x° = 68.75°$.

6. (1) The construction is a direct application of §493, with the line segment 6 in. long.

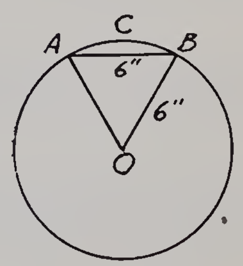
(2) Let x = the number of inches in the longer segment. Then $6 - x$ = the number of inches in the shorter segment. Then $x^2 = 6(6 - x)$. Solving, $x = 3.708$ and $6 - x = 2.292$.

3.708 in. and 2.292 in. Ans.

7. $\angle O \overset{\circ}{=} \overset{\frown}{ACB} = 106°16'$. Area of sector OACB $= \frac{106\frac{4}{15}}{360} \times \pi \cdot 5^2 = \frac{1594}{15 \times 360} \times 3.1416 \times 25 = 23.184$. $OD = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$. Area of $\triangle AOB = \frac{1}{2} \times 8 \times 3 = 12$. Area of segment ACB $= 23.184 - 12 = 11.184$ (sq.in.).



8. Area of sector AOB $= \frac{1}{6}\pi\overline{BO}^2 = \frac{\pi}{6}36 = 6\pi$. Area of $\triangle AOB = \frac{\overline{BO}^2}{4}\sqrt{3} = 9\sqrt{3}$. Segment ABC = Sector AOB − $\triangle AOB = 6\pi - 9\sqrt{3} = 3.262$, the area in square inches.



9. Area of square = 64 sq.in. Area of each semicircle $= \frac{1}{2}\pi 4^2 = 8\pi$ sq.in. Area of 2 figures bounded by side of square and arcs $= 64 - 2 \times 8\pi = 64 - 16\pi$. Area of 4 of these figures $= (128 - 32\pi)$ sq.in. Area of shaded portion = 64 sq.in. − $(128 - 32\pi)$ sq.in. $= (32\pi - 64)$ sq.in. = 36.53 sq.in.

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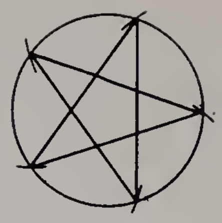
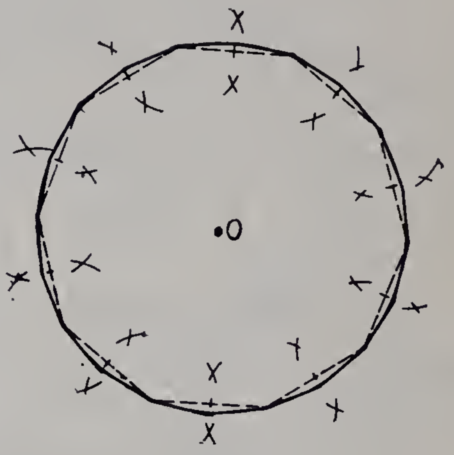


1. Draw a $\odot$ and divide it into 5 = arcs by §495. Join every other point of division as shown in the figure.

2. Given $\odot$O.

To construct a regular inscribed polygon of 20 sides.

Construction: 1. Inscribe a regular decagon in $\odot$O (§494). 2. Bisect each of the 10 = arcs (§328). 3. Draw the chords of these arcs (Asmt. 11).

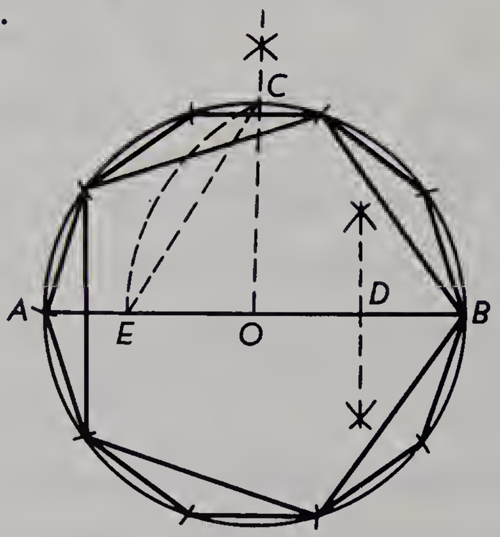


Then the resulting polygon is a regular inscribed polygon of 20 sides.

Proof: The chords joining the end points of the 20 = arcs form a regular inscribed polygon of 20 sides (§465).

3. Divide the given $\odot$ into 15 = arcs by means of §496. Bisect the = arcs by §328 and draw the chords of these arcs. The proof is the same as for Ex. 2.

5.



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1. 1. $\frac{AB}{AE} = \frac{AE}{EB}$ (§493). 2. $\overline{AE}^2 = AB \times EB$ (§386). 3. Area of AF = $\overline{AE}^2$ (§273h). 4. Area of a rect. with base AB and altitude EB = AB × EB (§273a). 5. ∴ Square AF = a rect. with base AB and altitude EB (Asmt. 7).

2. 1. $\frac{AB}{AE} = \frac{AE}{EB}$ (§493). 2. AE = BC (Const.). 3. $\frac{AB}{BC} = \frac{BC}{EB}$

(Asmt. 6). 4. $\therefore \frac{AB}{BC} = \frac{BC}{CF} = \frac{CD}{FE} = \frac{DA}{EB}$ (Asmt. 6). 5. The corresponding $\angle$s of the two rectangles are = (§191, Asmt. 19). 6. $\therefore$ ABCD $\sim$ BCFE (§413).

3. 1. $\frac{BC}{AB} = \frac{EB}{BC}$ (Ex. 2, §412). 2. $\frac{BC + AB}{AB} = \frac{EB + BC}{BC}$ (§393). 3. $\frac{BC + AB}{AB} = \frac{AB}{BC}$ (Asmt. 6).

4. 1. ABCD $\sim$ BCFE (Ex. 2). 2. $\triangle$ABC $\sim$ $\triangle$BCF (§435). 3. $\angle$OAB = $\angle$OBC (§412). 4. $\angle$ABO + $\angle$OBC = a rt. $\angle$ (Asmt. 9, §191). 5. $\therefore$ $\angle$ABO + $\angle$OAB = a rt. $\angle$ (Asmt. 6). 6. $\angle$ABO + $\angle$AOB + $\angle$OAB = 2 rt. $\angle$s (§120). 7. $\angle$AOB = a rt. $\angle$ (Asmt. 2). 8. BF $\perp$ AC (§14). 9. $\therefore \frac{AO}{OB} = \frac{OB}{OC} = \frac{OC}{OF}$ (§453a, Asmt. 7).

5. 1. Area of ABCD = AB x AD and area of EBCF = EB x AD (§273a, Asmt. 6). 2. ABCD x EBCF = AB x EB x $\overline{AD}^2$ (Asmt. 3). 3. AD = 1 (Given). 4. AB x EB = $\overline{AD}^2$ (Ex. 1, Asmt. 6). 5. $\therefore$ ABCD x EBCF = 1 (Asmt. 6). 6. EBCF = $\frac{1}{ABCD}$ (Asmt. 4).

PAGE 485 (Space Geometry)

1. 1. Let A = the area of one base. 2. $A = \pi r^2$ (§488). 3. $2A = 2\pi r^2$ (Asmt. 3). 4. $S = 2\pi rh$ (§500). 5. $T = 2A + S$ (Asmt. 9). 6. $T = 2\pi r^2 + 2\pi rh$ (Asmt. 6). 7. $T = 2\pi r(r + h)$ (Asmt. 6).

2. a. $S = 2\pi rh$ (§500).
$S = 2 \times 3.1416 \times 4 \times 6 = 150.7968$. 150.80 sq.in. Ans.

b. $S = 2\pi rh$ (§500).
$S = 2 \times 3.1416 \times 2 \times 10 = 125.664$. 125.66 sq.ft. Ans.

c. $S = 2\pi rh$ (§500).
$S = 2 \times 3.1416 \times \frac{7}{2} \times 4 = 87.9648$. 87.965 sq.ft. Ans.

d. $S = 2\pi rh$ (§500). $c = 2\pi r$.
$S = ch$. $S = 32 \times 14$. $S = 448$ (sq.in.).

3. a. $V = \pi r^2 h$ (§500). $V = 3.1416 \times 25 \times 12 = 942.48$ (cu.in.).

b. $V = \pi r^2 h$ (§500). $V = 3.1416 \times 11^2 \times 20 = 7602.672$.
7602.7 sq.ft. Ans.

c. $V = \pi r^2 h$ (§500). $V = 3.1416 \times 5^2 \times 16 = 1256.6$ (sq.in.).

d. $V = \pi r^2 h$ (§500). $c = 2\pi r$. $r = \frac{c}{2\pi}$.

$V = \pi \cdot \frac{c^2}{4\pi^2} \cdot h = \frac{c^2 h}{4\pi}$. $V = \frac{80^2 \times 72}{12.5664} = 36669$ (cu.in.),
or 21.220 (cu.ft.).

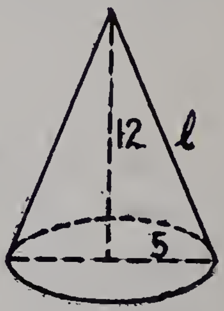
4. (1) $S = 2\pi rh$ (§500). $S = 2 \times 3.1416 \times 5 \times 20 = 628.32$.
$2A = 2\pi r^2$. $2A = 2 \times 3.1416 \times 25 = 157.08$.
Total area = 628.32 sq.ft. + 157.08 sq.ft.
= 785.40 sq.ft.
(2) $V = \pi r^2 h$ (§500). $V = 3.1416 \times 25 \times 20 = 1570.8$ (cu.ft.).

5. $S = ch$ (§500). $S = 28 \times 15 = 420$ (sq.in.).

6. $V = \pi r^2 h$ (§500). $231 = 3.1416 \times 4^2 \times h$.
Solving, $h = 4.595$. 4.60 in. Ans.

PAGES 485-486

1. 1. $S = \pi r \ell$ (§501). 2. $A = \pi r^2$ (§488). 3. $T = S + A = \pi r \ell + \pi r^2$ (Asmt. 9, 6). 4. $T = \pi r(\ell + r)$ (Asmt. 6).



2. (1) $\ell^2 = 12^2 + 5^2$ (§430). $\ell^2 = 169$ (Asmt. 6).
$\ell = 13$ (Asmt. 8). 13 in. Ans.
(2) $S = \pi r \ell$ (§501). $S = 3.1416 \times 5 \times 13$.
$S = 204.204$. 204.20 sq.in. Ans.

3. (1) $V = \frac{1}{3}\pi r^2 h$ (§501). $V = \frac{1}{3} \times 3.1416 \times 6^2 \times 8$.
$V = 301.5936$. 301.59 cu.in. Ans.
(2) $\ell^2 = 8^2 + 6^2$. Solving, $\ell = 10$.
$S = \pi r \ell$ (§501). $S = 3.1416 \times 6 \times 10 = 188.496$.
188.50 sq.in. Ans.

4. (1) $S = \pi r \ell$ (§501).
$S = 3.1416 \times 8 \times 204 = 5127.0912$.
5127.1 sq.in., or 35.605 sq.ft. Ans.
(2) $h^2 = 204^2 - 8^2$. Solving, $h = 203.84$.
$V = \frac{1}{3}\pi r^2 h$ (§501).
$V = \frac{1}{3} \times 3.1416 \times 64 \times 203.84 = 13{,}661.519872$.
13,662 cu.in., or 7.9063 cu.ft. Ans.

5. $T = \pi r(r + \ell)$ (Ex. 1). $\ell^2 = 24^2 + 7^2$. $\ell = 25$.
$T = 3.1416 \times 7(7 + 25)$. $T = 703.7184$. 703.72 sq.in. Ans.

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1. a. $S = 4\pi r^2$ (§502). $S = 4 \times 3.1416 \times 6^2 = 452.3904$.
452.39 sq.in. Ans.
$V = \frac{4}{3}\pi r^3$ (§502). $V = \frac{4}{3} \times 3.1416 \times 6^3 = 904.7808$.
904.78 cu.in. Ans.

b. $S = 4\pi r^2$ (§502). $S = 4 \times 3.1416 \times 4^2 = 201.0624$.
201.06 sq.in. Ans.
$V = \frac{4}{3}\pi r^3$ (§502). $V = \frac{4}{3} \times 3.1416 \times 4^3 = 268.0832$.
268.08 cu.in. Ans.

c. $S = 4\pi r^2$ (§502). $S = 4 \times 3.1416 \times 3^2 = 113.0976$.
113.10 sq.ft. Ans.
$V = \frac{4}{3}\pi r^3$ (§502). $V = \frac{4}{3} \times 3.1416 \times 3^3 = 113.0976$.
113.10 cu.ft. Ans.

d. $S = 4\pi r^2$ (§502). $S = 4 \times 3.1416 \times 5^2 = 314.16$.
314.16 sq.ft. Ans.
$V = \frac{4}{3}\pi r^3$ (§502). $V = \frac{4}{3} \times 3.1416 \times 5^3 = 523.60$.
523.60 cu.ft. Ans.

2. (1) $S = 4\pi r^2$ (§502). $S = 4 \times 3.1416 \times 36 = 452.3904$.
452.39 sq.in. Ans.

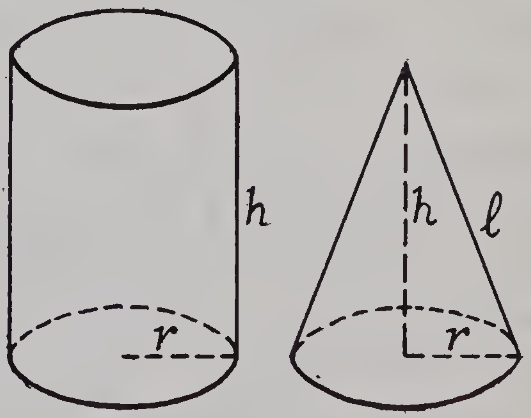
(2) $S = 2\pi rh$ (§500). $S = 2 \times 3.1416 \times 6 \times 12 = 452.3904$.
452.39 sq.in. Ans.

(3) $V = \frac{4}{3}\pi r^3$ (§502). $V = \frac{4}{3} \times 3.1416 \times 6^3 = 904.7808$.
904.78 cu.in. Ans.

(4) $V = \pi r^2 h$ (§500). $V = 3.1416 \times 6^2 \times 12 = 1357.1712$.
1357.2 cu.in. Ans.

3. $S = 4\pi r^2$. $4\pi r^2 = S$. $r^2 = \frac{S}{4\pi}$. $r = \sqrt{\frac{S}{4\pi}}$. $r = \frac{1}{2\pi}\sqrt{\pi S}$.

4. (1) Let T = the area of the cylinder. Let T' = the area of the cone. $T = 2\pi r(r + h)$ and $T' = \pi r(r + \ell)$. Then $\frac{T}{T'} = \frac{2(r + h)}{r + \ell}$.



(2) Let V = the volume of the cylinder. Let V' = the volume of the cone. $V = \pi r^2 h$ and $V' = \frac{1}{3}\pi r^2 h$. Then $\frac{V}{V'} = \frac{3}{1}$.

5. $S = \frac{1}{2} \times 4\pi r^2$. $S = \frac{1}{2} \times 4 \times 3.1416 \times (\frac{25}{2})^2 = 981.75$ (sq.ft.).
981.75 × 3 = 2945.25. $2945.25. Ans.

6. (1) $S = 4\pi r^2$ (§502). $S = 4 \times 3.1416 \times 7^2 = 615.7536$.
615.75 sq.in. Ans.

(2) $S = 2\pi rh$ (§500). $S = 2 \times 3.1416 \times 7 \times 14 = 615.7536$.
615.75 sq.in. Ans.

(3) $V = \frac{4}{3}\pi r^3$ (§502). $V = \frac{4}{3} \times 3.1416 \times 7^3 = 1436.7584$.
1436.8 cu.in. Ans.

(4) $V = \pi r^2 h$ (§500). $V = 3.1416 \times 7^2 \times 14 = 2155.1376$.
2155.1 cu.in. Ans.

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1. These two sentences define a right angle in terms of two perpendicular lines. The first is a simple declarative sentence and the second is a complex sentence. This is not reasoning in a circle.

2. The grocer reasoned in a circle.

3. If Celia had proved Theorem 21 without using Theorem 20, she could have proved Theorem 20 by using Theorem 21. If she had not proved Theorem 21 or had proved it by Theorem 20, her proof was not correct. Proving Theorem 21 by Theorem 20 and proving Theorem 20 by Theorem 21 is reasoning in a circle.

PAGE 488 (Review Questions)

1. Incenter, circumcenter, orthocenter, and centroid. 2. a. Yes. b. Not in general. c. Not in general. d. Yes. e. Yes. f. Yes. 3. The polygon must be equilateral and equiangular. 4. Triangle, rhombus. 5. Two regular polygons are similar if they have the same number of sides. 6. a. The radius. b. Zero. c. The circumference. d. The area of the circle. e. 180°. f. 0°. 7. Eleven by methods given in the text. (Sides are 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24.) 8. Because it is the radius of the circumscribed ⊙. 9. Regular hexagon. 10. a. No. b. Yes. 11. (1) A constant is a quantity which has a fixed value throughout a given discussion. (2) A variable is a quantity which may have different successive values during a discussion. (3) No. 12. (1) 2 : 5 (§479). (2) 4 : 25 (§489). 13. No (§481). 14. d. 15. a. (Ex. 4, p. 459). $S = 2(25) = 50$ sq.in. $9\pi - 4\pi = 5\pi$ sq.in. (§488).

PAGES 490-491 (Test 36)

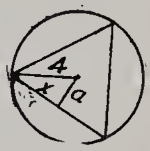
1. F. 2. T. 3. T. 4. T. 5. F. 6. F (§§467, 485 and Ex. 4, page 480). 7. T. 8. F. 9. T. 10. F. 11. T. 12. T. 13. F. 14. F. 15. T. 16. F. 17. F. 18. F.

PAGE 491 (Test 37)

1. $\frac{S}{S'} = \frac{d^2}{d'^2} = \frac{16}{25}$ (§506d).

2. Area of shaded portion = $\pi R^2 - \pi r^2 = \pi(\frac{3}{2})^2 - \pi(1)^2 = 3.9270$.
3.9270 sq.in. Ans.

3. $a = \frac{1}{2} \times 4 = 2$ (§136, Ex. 4, page 455).
$x = \sqrt{4^2 - 2^2} = \sqrt{12}$.
$S = \frac{1}{2}ap = \frac{1}{2} \times 2 \times 6\sqrt{12} = 12\sqrt{3}$, or 20.784 (§505a).



4. Diagonal of square = $2\sqrt{2}$. Area of square = $\frac{1}{2}(2\sqrt{2})^2 = 4$ (§273g).

5. $\frac{80}{360}$ of 180 sq.ft. = 40 sq.ft.

6. $6^2 - 3^2 = x^2$ (§430). Apothem = $x = 3\sqrt{3}$ in., or 5.196 in.

7. 90°. Ans.

8. $2\pi r = 2\pi(8) = 50.2656$. 50.266 in. Ans.

9. $4 + 6 + 8 + 9 + 10.5 = 37.5$. $\frac{1}{2} \times 37.5 \times 5 = 93.75$ (Ex. 1, page 459).

10. $\frac{6\frac{1}{4}}{1} = \frac{s^2}{s_1^2} = \frac{p^2}{p_1^2} = \frac{25}{4}$; $\frac{p}{p_1} = \frac{5}{2}$.

11. $\frac{1}{2} \times 20^2 = 200$ (area of square). $\frac{1}{4} \times 200 = 50$ (area of △ of sector). $\pi r^2 = 3.1416 \times 100 = 314.16$ (area of ⊙). $\frac{1}{4} \times 314.16 = 78.540$ (area of sector). $78.540 - 50 = 28.540$ (sq.in.).

12. 180° - 105° = 75° (the central ∠ of the ⊙).
$2\pi r = 2 \times 3.1416 \times 10$ in. = 62.832 in. (circumference of ⊙).
$\frac{75}{360} \times 62.832$ in. = 13.090 in. (length of minor arc).

Chapter 15. Elements of Trigonometry

PAGES 494-496

1. In $\triangle DEF$:

$\sin D = \frac{d}{e}$
$\cos D = \frac{f}{e}$
$\tan D = \frac{d}{e}$

$\sin F = \frac{f}{e}$
$\cos F = \frac{d}{e}$
$\tan F = \frac{f}{d}$

In $\triangle GHK$:

$\sin G = \frac{g}{k}$
$\cos G = \frac{h}{k}$
$\tan G = \frac{g}{h}$

$\sin H = \frac{h}{k}$
$\cos H = \frac{g}{k}$
$\tan H = \frac{h}{g}$

In $\triangle RST$:

$\sin T = \frac{t}{r}$
$\cos T = \frac{s}{r}$
$\tan T = \frac{t}{s}$

$\sin S = \frac{s}{r}$
$\cos S = \frac{t}{r}$
$\tan S = \frac{s}{t}$

2. In $\triangle ABC$,
$36 + 64 = 100.$

$\sin A = \frac{6}{10} = .60$
$\cos A = \frac{8}{10} = .80$
$\tan A = \frac{6}{8} = .75$

$\sin B = \frac{8}{10} = .80$
$\cos B = \frac{6}{10} = .60$
$\tan B = \frac{8}{6} = 1.33$

In $\triangle MNP$,
$64 + 225 = 289.$

$\sin M = \frac{15}{17} = .88$
$\cos M = \frac{8}{17} = .47$
$\tan M = \frac{15}{8} = 1.88$

$\sin P = \frac{8}{17} = .47$
$\cos P = \frac{15}{17} = .88$
$\tan P = \frac{8}{15} = .53$

In $\triangle XYZ$,
$25 + 144 = 169.$

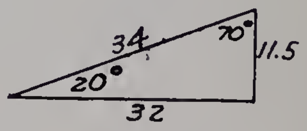
$\sin Z = \frac{12}{13} = .92$
$\cos Z = \frac{5}{13} = .38$
$\tan Z = \frac{12}{5} = 2.40$

$\sin X = \frac{5}{13} = .38$
$\cos X = \frac{12}{13} = .92$
$\tan X = \frac{5}{12} = .42$

3. (1) $\sin T = \frac{RS}{TR}$. $\cos T = \frac{ST}{TR}$. $\tan T = \frac{RS}{ST}$.
(2) $\sin T = \frac{SW}{ST}$. $\cos T = \frac{TW}{ST}$. $\tan T = \frac{SW}{TW}$.

4. (1) $\sin R = \frac{ST}{TR}$. $\cos R = \frac{RS}{TR}$. $\tan R = \frac{ST}{RS}$.
(2) $\sin R = \frac{SW}{RS}$. $\cos R = \frac{RW}{RS}$. $\tan R = \frac{SW}{RW}$.

*5. (1) $\sin 20° = \frac{11.5}{34} = .34$. $\cos 20° = \frac{32}{34} = .94$. $\tan 20° = \frac{11.5}{32} = .36$. (2) $\sin 70° = \frac{32}{34} = .94$. $\cos 70° = \frac{11.5}{34} = .34$. $\tan 70° = \frac{32}{11.5} = 2.78$.**



*The values taken for the sides of the $\triangle$ in Ex. 5 are arbitrary, but the student's results should be approximately the same as those found here.

**It may be noted that this result differs from the more accurate value given in the table, 2.7475.

PAGE 497

1. .3420. 2. .9703. 3. .9703. 4. .3420. 5. .3057. 6. 1.9626. 7. .2079. 8. .5878. 9. 11.4301. 10. The sine and tangent increase as the angles increase, and the cosine decreases as the angles increase. 11. 13°. 12. 26°. 13. 15°. 14. 51°. 15. 82°. 16. 82°.

PAGE 498

1. .5925. 2. .7361. 3. .3411. 4. 3.2129. 5. 1.9053. 6. .9784. 7. .1621. 8. .8358. 9. 1.0000. 10. .6639. 11. .6006. 12. ∞. 13. 1.7405. 14. .2630. 15. 0. 16. .4726.

PAGE 499

1. 6°30', or 6.5°. 2. 17°15', or 17.3°. 3. 62°12', or 62.2°. 4. 27°48', or 27.8°. 5. 11°48', or 11.8°. 6. 72°2', or 72.0°. 7. 83°15', or 83.3°. 8. 81°50', or 81.8°. 9. 39°28', or 39.3°. 10. 34°11', or 34.2°. 11. 15°38', or 15.6°. 12. 8°32', or 8.5°. 13. 58°48', or 58.8°. 14. 61°0', or 61.0°. 15. 71°5', or 71.1°.

PAGES 500-503

1. $\frac{b}{c} = \cos 60^\circ$. $c = \frac{b}{\cos 60^\circ} = \frac{40}{.5} = 80$.

2. $\frac{a}{b} = \tan 12^\circ$. $a = b \tan 12^\circ = 14(.2126) = 2.9764$, or 3.0 approximately.

3. $\angle A = 90^\circ - 55^\circ = 35^\circ$. $\frac{a}{b} = \tan 35^\circ$. $a = b \tan 35^\circ$ $= 10(.7002) = 7.002$, or 7.0 approximately.

4. $\frac{a}{c} = \sin 20.5^\circ$. $a = c \sin 20.5^\circ = 80(.3502) = 28.016$, or 28 approximately.

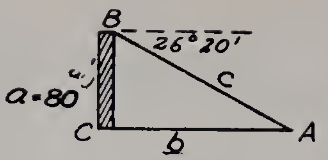
5. $\frac{a}{c} = \cos 56^\circ 10'$. $a = c \cos 56^\circ 10' = 16(.5568) = 8.9088$, or 8.91 approximately.

6. $\tan A = \frac{a}{b} = \frac{4.84}{3.63} = 1.3333$. $A = 53^\circ 8'$, or $53^\circ 10'$, approximately.

7. $\cos B = \frac{a}{c} = \frac{6.5}{9.8} = .6633$. $B = 48^\circ 27'$, or 48°, approximately.

8. $\frac{a}{120} = \tan 51.3^\circ$. $a = 120(1.2484) = 149.8080$.

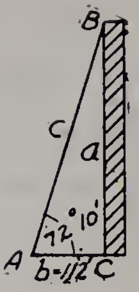
149.8 ft., or 150 ft. Ans.



9. $B = 90° - 26°20' = 63°40'$.

$\frac{b}{80} = \tan 63°40'$.

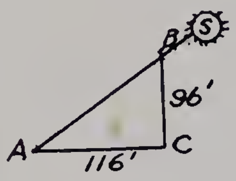
$b = 160.2(2.0211) = 323.78$. 323.9 ft. Ans.



10. $\frac{a}{112} = \tan 72°10'$.

$a = 112(3.1099) = 348.3088$. 348.3 ft. Ans.

11.

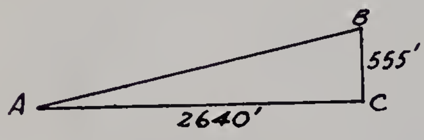


$\tan A = \frac{96}{116} = .8276$.

$A = 39°36'$,

or 40° approx.

13.

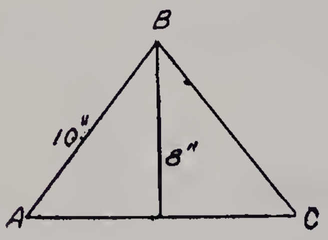


$\tan A = \frac{555}{2640} = .2102$.

$A = 11°52'$,

or 11°50′ approx.

15.

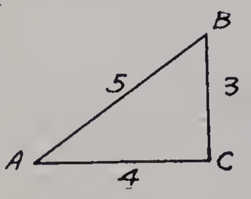


$\sin A = \frac{8}{10} = .8000$.

$A = 53°8'$.

Each base ∠ is 53°8′ and the vertex ∠ is 73°44′, or 74° approx.

17.

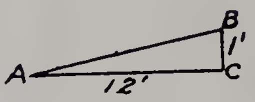


$\sin A = \frac{3}{5} = .6000$.

$A = 36°52'$.

$B = 90° - 36°52' = 53°8'$, if the 3, 4, and 5 are exact numbers.

12.

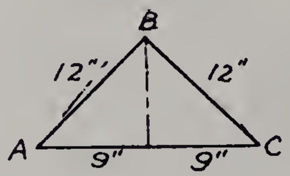


$\tan A = \frac{1}{12} = .0833$.

$A = 4°46'$,

or 5° approx.

14.

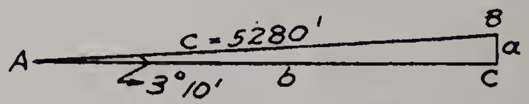


$\cos A = \frac{9}{12} = .7500$.

$A = 41°24'$.

The base ∠s are each 41°24′ and the vertex ∠ = 180° − 2 × 41°24′ = 97°12′, or 97° approx.

16.

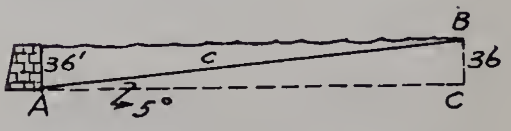


$\frac{a}{5280} = \sin 3°10'$.

$a = 5280(.0552) = 291.4560$.

291.5 ft., if the mile is measured to nearest foot and the angle to the nearest minute.

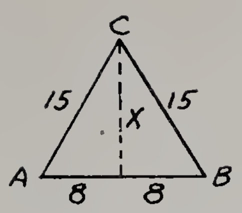
18.



$\sin 5° = \frac{36}{c}$. $c = \frac{36}{.0872}$

$= 412.84$. 412.8 ft. Ans.

19.



$\cos A = \frac{8}{15} = .5333.$

$A = 57°46'.$

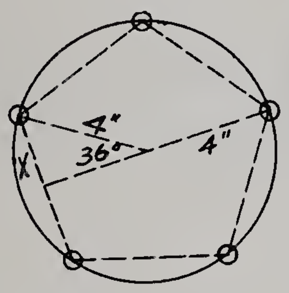
$x = 8 \tan A$

$= 8 \times 1.5862$

$= 12.6896.$

Area $\triangle$ ABC $= 8 \times 12.6896$ $= 101.5168.$ 101.5 sq.ft., or 100 sq.ft., approx. Ans.

21.



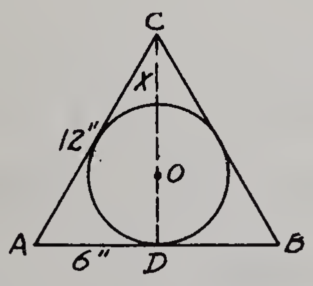
$\frac{x}{4} = \sin 36°$

$x = 4(.5878).$

$2x = 4.7024.$

4.702 in. Ans. (if data are exact.)

23.



$\sin x = \frac{6}{12} = .5.$

$x = 30°.$

$\frac{CD}{12} = \cos 30°.$

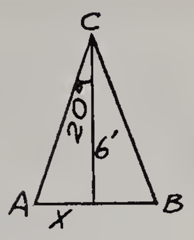
$CD = 12(.8660).$

$CD = 10.392.$

$OD = \frac{1}{3}CD = 3.464.$

3.464 in., or 3.5 in., approx. Ans.

20.

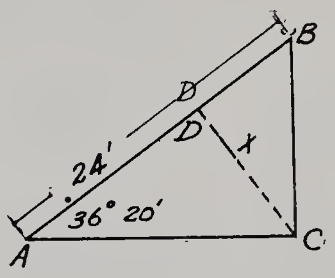


$\frac{x}{6} = \tan 20°.$

$x = 6(.3640) = 2.1840.$

Area of $\triangle$ ABC $= 6(2.1840)$ $= 13.1040.$ 13.10 sq.ft. Ans. (if data are exact)

22.



$\frac{BC}{24} = \sin 36°20'.$

$BC = 24(.5925).$

$BC = 14.22.$

$B = 90° - 36°20' = 53°40'$

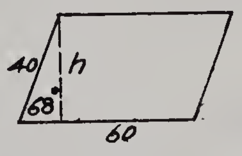
$\frac{x}{BC} = \sin 53°40'$

$x = 14.22(.8055).$

$x = 11.45421.$

11.45 ft., or 11 ft., approx. Ans.

24.



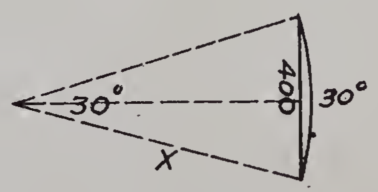
$\frac{h}{40} = \sin 68°.$

$h = 40(.9272).$

$h = 37.088.$

37.10. Ans. (if data are exact.)

25.



$\frac{200}{x} = \sin 15°.$ $x = \frac{200}{\sin 15°}$

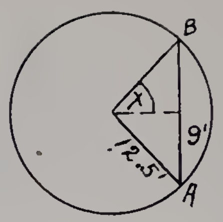
$x = \frac{200}{.2588} = 772.8$ (ft.).

772.8 ft., or 770 ft., approx.

26. $\sin x = \frac{9}{12.5} = .7200.$

$x = 46°3.5'. \quad 2x = 92°7'$

$\therefore \overset{\frown}{AB} = 92°7'$, or 92° if data are approximate.



27. (1) $\frac{h}{10 - 1} = \sin 20°$. $h = 9(.3420) = 3.078$. 3.1 ft. Ans. (2) $\frac{\frac{1}{2}w}{9} = \cos 20°$. $\frac{1}{2}w = 9(.9397) = 8.46573$. $w = 2 \times 8.4573$ $= 16.9146$ (ft.), or 17 ft. if data are approximate.

28. Proof: 1. $\sin A = \frac{a}{c}$ and $\sin B = \frac{b}{c}$ (§511). 2. $\sin^2 A = \frac{a^2}{c^2}$ and $\sin^2 B = \frac{b^2}{c^2}$ (Asmt. 8). 3. $\sin^2 A + \sin^2 B = \frac{a^2 + b^2}{c^2}$ (Asmt. 1). 4. $a^2 + b^2 = c^2$ (§255). 5. Then $\sin^2 A + \sin^2 B = \frac{c^2}{c^2}$ (Asmt. 6). 6. $\sin^2 A + \sin^2 B = 1$ (Asmt. 6).

29. Proof: 1. Using the figure of Ex. 28, $\sin A = \frac{a}{c}$ and $\cos A = \frac{b}{c}$ (§511). 2. $\frac{\sin A}{\cos A} = \frac{a}{c} \div \frac{b}{c} = \frac{a}{c} \times \frac{c}{b} = \frac{a}{b}$ (Asmt. 4). 3. $\tan A = \frac{a}{b}$ (§511). 4. $\tan A = \frac{\sin A}{\cos A}$ (Asmt. 7).

30. Proof: 1. When ∠A is acute, $a^2 = b^2 + c^2 - 2b'c$ (§444). 2. $\cos A = \frac{b'}{b}$ (§511). 3. $b' = b \cos A$ (Asmt. 3). 4. $a^2 = b^2 + c^2 - 2bc \cos A$ (Asmt. 6). 5. When ∠A is obtuse, $a^2 = b^2 + c^2 + 2b'c$ (§445). 6. $\cos A = \frac{-b'}{b}$. 7. $b' = -b \cos A$ (Asmt. 3). 8. $a^2 = b^2 + c^2 - 2bc \cos A$ (Asmt. 6). 9. When ∠A is a rt. ∠, the formula is true because $\cos A = 0$, and the formula becomes $a^2 = b^2 + c^2$.

31. Proof: 1. In either triangle, $\frac{h}{b} = \sin A$ and $\frac{h}{a} = \sin B$ (§511). (The sine of obtuse ∠B = the sine of its supplement). 2. Then $\frac{a}{b} = \frac{\sin A}{\sin B}$ (Asmt. 4). 3. $\frac{a}{\sin A} = \frac{b}{\sin B}$ (§391). 4. By drawing the altitude from A, we can prove that $\frac{b}{\sin B} = \frac{c}{\sin C}$ (Statements 1-3). 5. Then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Asmt. 6).

32. $x^2 = b^2 + c^2 - 2bc \cos X$ (Ex. 30).

$x^2 = 20.40^2 + 17.30^2 - 2 \times 20.40 \times 17.30 \times .9075$

$x^2 = 74.90$

$x = 8.66.$ 8.66. Ans.

33. $\frac{17}{\sin y} = \frac{24}{\sin 60°}$ (Ex. 31). $\sin y = \frac{17 \times \sin 60°}{24}$.

$\sin y = \frac{17 \times .866}{24}$. $\sin y = .6134$. $y = 37°51'$. 38° Ans. if data are approximate.

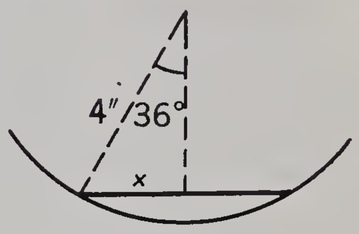
PAGE 503 (Test 38)

1. F. 2. T. 3. F. 4. F. 5. T. 6. T. 7. F. (Ex. 28, page 502). 8. T (§273b, 511).

PAGE 503 (Test 39)

1. $\sin A = \frac{8}{17} = .4706$. $\cos A = \frac{15}{17} = .8824$.

2. $\tan B = \frac{15}{8} = 1.875$.



3. $\frac{x}{4} = \sin 36°$. $x = 4(.5878)$.

$x = 2.3512$. $2x = 4.7024$

4.702 in. Ans. (if data are exact.)

Chapter 16. Inequalities

PAGE 505

1. 5 is less than x; x is greater than 5; AB plus BC is greater than AC. 2. b and d are in reverse order.

PAGE 506

1. $\angle y > \angle s$ (Asmt. 55). 2. $AC > DF$ (Asmt. 55). 3. Asmt. 54. 4. Asmt. 6. 5. Asmt. 54.

6. (1) Reverse order.
(2) Given $\angle x > \angle y$.
To prove the comp. of $\angle x <$ the comp. of $\angle y$.
Proof: 1. $\angle x > \angle y$ (Given). 2. 1 rt. $\angle$ = 1 rt. $\angle$ (Asmt. 19). 3. 1 rt. $\angle - \angle x <$ 1 rt. $\angle - \angle y$ (Asmt. 55). 4. But 1 rt. $\angle - \angle x$ is the comp. of $\angle x$ and 1 rt. $\angle - \angle y$ is the comp. of $\angle y$ (§20). 5. $\therefore$ The comp. of $\angle x <$ the comp. of $\angle y$ (Asmt. 6).

7. m is positive.

8. 1. $\angle A + \angle B + \angle C =$ 1 st. $\angle$ and $\angle A' + \angle B' + \angle C' =$ 1 st. $\angle$ (§120). 2. $\angle A + \angle B + \angle C = \angle A' + \angle B' + \angle C'$ (Asmt. 19). 3. $\angle A = \angle A'$ and $\angle B < \angle B'$ (Given). 4. $\angle A + \angle B < \angle A' + \angle B'$ (Asmt. 54). 5. From statements 2 and 4, $\angle C > \angle C'$ (Asmt. 55).

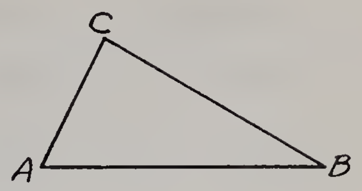
9. $a^2 < b^2$.

PAGE 507

1. $\angle A$ is the smallest $\angle$ and $\angle B$ is the largest $\angle$ (§520).
2. If two angles of a triangle are unequal, the sides opposite these angles are unequal.
3. If two sides of a triangle are equal, the angles opposite these sides are equal.

PAGES 508-509

1. MP is the shortest side and NP is the longest side (§521).
2. AC is the longest side and BC is the shortest side (§521).
3. If two angles of a triangle are unequal, the sides opposite these angles are unequal.



4. Given $\triangle ABC$.

To prove that $AB - BC < AC$.

Planning the Proof: 1. We can prove line segments unequal by Asmt. 9, 10, 14, 53-57, §§519, 521-523. 2. We shall use §519, and Asmt. 54.

Proof: 1. $AB < AC + BC$ (§519). 2. $AB - BC < AC$ (Asmt. 54).

5. Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §521.

Proof: 1. $\angle x = \angle y$ (Given). 2. $\angle z > \angle y$ (§93). 3. $\angle z > \angle x$ (Asmt. 6). 4. $\therefore AC > AD$ (§521).

6. The side opposite the 30° angle (§521).

7. Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §519.

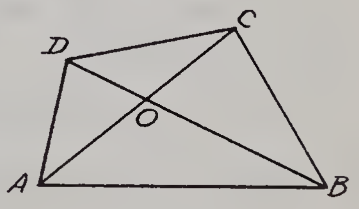
Proof: 1. $EBC < EF + FC$ (§519). 2. FD is the $\perp$ bisector of EA (Ex. 1, page 168). 3. $EB = AB$ and $EF = AF$ (§378c). 4. $\therefore AB + BC < AF + FC$ (Asmt. 6).

8. Planning the Proof: 1. We can prove line segments unequal as in Ex. 4; (b) $\angle$s unequal by Asmt. 9, 10, 53-57, §§93, 520. 2. We shall use (a) §519, Asmt. 9, 54, 56; (b) §93, Asmt. 53.

(a) Proof: 1. Extend AD to meet BC at E (Asmt. 13). 2. $AC + CE > AD + DE$ (§519, Asmt. 9). 3. $DE + EB > DB$ (§519). 4. $AC + CE + DE + EB > AD + DE + DB$ (Asmt. 56). 5. $AC + CE + EB > AD + DB$ (Asmt. 54). 6. $CE + EB = CB$ (Asmt. 9). 7. $\therefore AC + CB > AD + DB$ (Asmt. 6).

(b) Proof: 1. $\angle DEB > \angle C$ (§93). 2. $\angle ADB > \angle DEB$ (§93). 3. $\therefore \angle ADB > \angle C$ (Asmt. 53).

9. Given the quad. ABCD with the diagonals AC and BD.



To prove that $AB + BC + CD + DA > AC + BD$.

Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §519, Asmt. 54, 56.

Proof: 1. $AB + BC > AC$, $CD + DA > AC$, $AB + DA > BD$, and $BC + CD > BD$ (§519). 2. $2AB + 2BC + 2CD + 2DA > 2AC + 2BD$ (Asmt. 56). 3. $\therefore AB + BC + CD + DA > AC + BD$ (Asmt. 54).

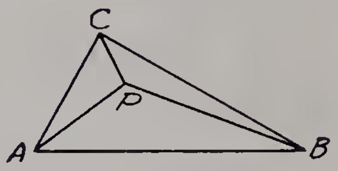
10. Given $PC \perp AB$ and PD any other line segment from P to AB.

To prove that $PC < PD$.

Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §519, Asmt. 9, 54.

Proof: 1. Extend PC to P' making $CP' = CP$ (Asmt. 13 and §9). 2. Draw DP' (Asmt. 11). 3. $PC + CP' < PD + DP'$ (§519, Asmt. 9). 4. $PC = CP'$ (Const.). 5. $PC \perp AB$ (Given). 6. $PD = DP'$ (§378c). 7. $2PC < 2PD$ (Asmt. 6). 8. $\therefore PC < PD$ (Asmt. 54).

11. Given $\triangle ABC$ and PA, PB, and PC, line segments joining any point P within the $\triangle$ to A, B, and C.

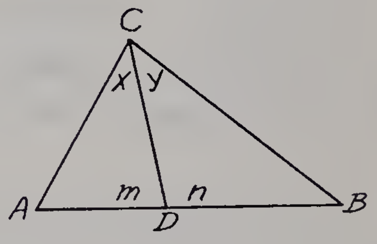


To prove that $PA + PB + PC > \frac{1}{2}(AB + BC + CA)$.

Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §519, Asmt. 54, 56.

Proof: 1. $PA + PB > AB$, $PB + PC > BC$, $PC + PA > CA$ (§519). 2. $2PA + 2PB + 2PC > AB + BC + CA$ (Asmt. 56). 3. $\therefore PA + PB + PC > \frac{1}{2}(AB + BC + CA)$ [Asmt. 54].

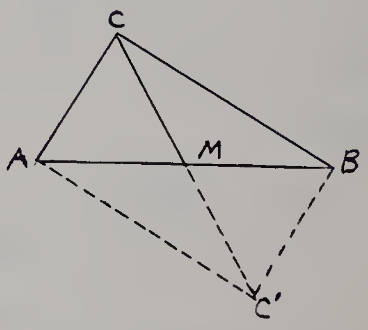
12. Given $\triangle ABC$ with $\angle A > \angle B$, and CD bisecting $\angle ACB$.



To prove that CD is oblique to AB.

Planning the Proof: 1. We can prove $\triangle$s unequal by Asmt. 9, 10, 53-57, §§93, 520. 2. We shall use Asmt.

Proof: 1. $\angle A + \angle x + \angle m =$ a st. $\angle$ and $\angle B + \angle y + \angle n =$ a st. $\angle$ (§120). 2. $\angle A + \angle x + \angle m = \angle B + \angle y + \angle n$ (Asmt. 7). 3. $\angle x = \angle y$ (Given). 4. $\angle A + \angle m = \angle B + \angle n$ (Asmt. 2). 5. $\angle A > \angle B$ (Given). 6. $\angle m < \angle n$ (Asmt. 55). 7. CD is not $\perp$ AB (§14). 8. $\therefore$ CD is oblique to AB (§16).



13. Given $\triangle ABC$ with CM the median to AB.

To prove that $CM < \frac{1}{2}(AC + CB)$.

Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §519, Asmt. 54.

Proof: 1. Extend CM to C' so that $MC' = CM$ (Asmt. 13 and §9). 2. Draw AC' and BC' (Asmt. 11). 3. $CM = MC'$ (Const.). 4. $AM = MB$ (Given). 5. AC'BC is a ▱ (§223d). 6. $AC = C'B$ (§217a). 7. $CM + MC' < BC' + CB$ (§519, Asmt. 9). 8. $2CM < AC + CB$ (Asmt. 6). 9. $\therefore CM < \frac{1}{2}(AC + CB)$ [Asmt. 54].

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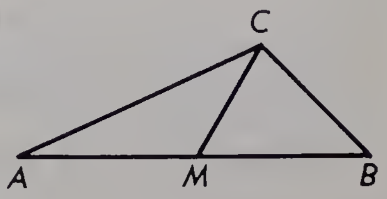
Planning the Proof: 1. We can prove line segments unequal by Asmt. 9, 10, 14, 53-57, §§519, 521-524. 2. We shall use §524.

Proof: 1. $\angle AMC > \angle BMC$ (Given).



2. CM is a median of the triangle (Given). 3. AM = BM (§22). 4. CM = CM (Iden.). 5. AC > BC (§524).

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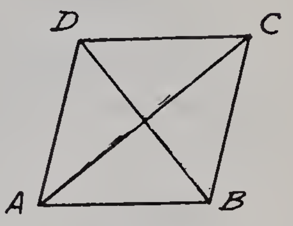
1. AB > DF (§524).

2. Given rhombus ABCD with diagonals AC and BD; ∠s A and C acute and ∠s B and D obtuse.



To prove that AC > BD.

Planning the Proof: 1. We can prove line segments unequal by §§519, 521-524, and Asmt. 9, 10, 14, 53-57. 2. We shall use §524.

Proof: 1. AB = CD (§217a). 2. BC = BC (Iden.). 3. ∠B is obtuse and ∠C is acute (Given). 4. ∠B > 1 rt. ∠ and ∠C < 1 rt. ∠ (§16). 5. ∠B > ∠C (Asmt. 53). 6. ∴ AC > BD (§524).

3. Planning the Proof: 1. We can prove angles unequal by Asmt. 9, 10, 53-57, §§93, 520, 525. 2. We shall use §525.

Proof: 1. ABC is isosceles (Given). 2. P is a point of the base AB such that AP < PB (Given). 3. AC = BC (§58). 4. CP = CP (Iden.). 5. ∠ACP < ∠PCB (§525).

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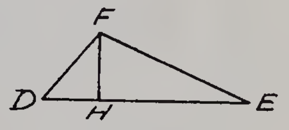
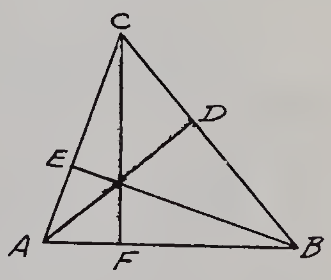
1. (1) DH. (2) EF > DF (§526).



2. Given △ABC with altitudes AD, BE, and CF.

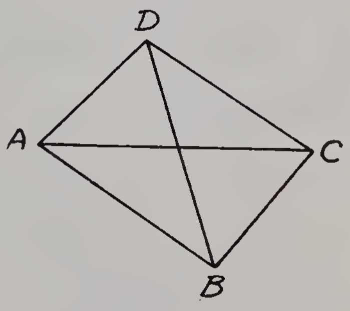
To prove that AD + BE + CF < AB + BC + AC.



Planning the Proof: 1. We can prove line segments unequal by Asmt. 9, 10, 53-57, §§519, 521-524, 526. 2. We shall use §522, Asmt. 56.

Proof: 1. AD ⊥ BC, BE ⊥ AC, CF ⊥ AB (Given). 2. AD < AB, BE < BC, CF < AC (§522). 3. ∴ AD + BE + CF < AB + BC + AC (Asmt. 56).

3. Given quad. ABCD with diagonals AC and BD.



To prove that AC < $\frac{1}{2}$(AB + BC + CD + DA) and BD < $\frac{1}{2}$(AB + BC + CD + DA).

Planning the Proof: 1. We can prove line segments unequal by §§519, 521-524, 526, Asmt. 9, 10, 53-57. 2. We shall use §519, Asmt. 54-56.

Proof: 1. $AC < AB + BC$ (§519). 2. $AC < CD + DA$ (§519). 3. $2AC < AB + BC + CD + DA$ (Asmt. 56). 4. $AC < \frac{1}{2}(AB + BC + CD + DA)$ [Asmt. 54]. 5. In like manner, $BD < \frac{1}{2}(AB + BC + CD + DA)$ [Statements 1-4].

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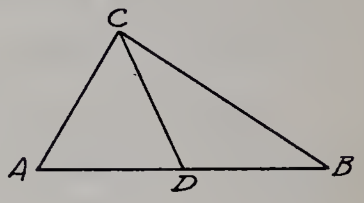
1. Given $\triangle ABC$ with median CD on base AB and $\angle ADC$ acute.

To prove that $BC \neq AC$.



Planning the Proof: 1. We can prove line segments unequal by §§519, 521-524, 526, 527, and Asmt. 2. We shall use §524.

Proof: 1. $\angle ADB$ is a st. $\angle$ (§16). 2. $\angle ADC <$ a rt. $\angle$ (Given). 3. $\angle ADB - \angle ADC > \angle ADB -$ a rt. $\angle$ (Asmt. 55). 4. $\angle CDB >$ a rt. $\angle$ (Asmt. 6, §16). 5. $\angle CDB > \angle ADC$ (Asmt. 53). 6. $AD = DB$ (Given). 7. $CD = CD$ (Iden.). 8. $\therefore BC > AC$ (§524), that is, $BC \neq AC$.

2. Given ▱ABCD with diagonals AC and BD, $\angle ABC$ obtuse and $\angle DAB$ acute.

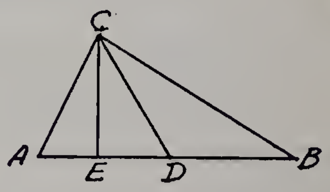
To prove that $AC \neq BD$.

Planning the Proof: (Same as Ex. 1).

Proof: 1. $\angle ABC >$ a rt. $\angle$ (Given). 2. $\angle DAB <$ a rt. $\angle$ (Given). 3. $\angle ABC > \angle DAB$ (Asmt. 53). 4. $AB = AB$ (Iden.). 5. $BC = AD$ (§217a). 6. $\therefore AC > BD$ (§524); that is, $AC \neq BD$.

3. Given $\triangle ABC$ with $AB > BC > AC$, median CD to side AB, and altitude CE on AB.

To prove that $CD > CE$.



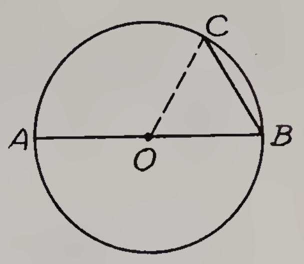
Planning the Proof: 1. We can prove line segments unequal as in Ex. 1. 2. We shall use §521.

Proof: 1. $BC > AC$ (Given). 2. $AD = DB$ (Given). 3. $CD = CD$ (Iden.). 4. $\angle BDC > \angle ADC$ (§525). 5. $\angle BDC + \angle ADC =$ a st. $\angle$ (Asmt. 24). 6. $\angle BDC =$ a st. $\angle - \angle ADC$ (Asmt. 2). 7. A st. $\angle - \angle ADC > \angle ADC$ (Asmt. 6). 8. A st. $\angle > 2\angle ADC$ (Asmt. 54). 9. A rt. $\angle > \angle ADC$ (Asmt. 54, §16). 10. $\angle CED =$ a rt. $\angle$ (Given). 11. $\angle CED > \angle ADC$ (Asmt. 6). 12. $\therefore CD > CE$ (§521).

PAGES 514-515

1. Yes (§333a, Asmt. 5).

2. Given $\odot O$ with diameter AB the chord of $\overset{\frown}{ACB}$ of 180° and BC the chord of $\overset{\frown}{BC}$ of 60°.

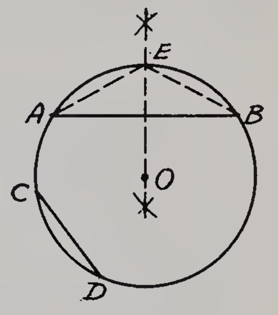


To prove that $AB = 2BC$.

Planning the Proof: 1. We can prove line segments equal by §§97, 157, 217, 308, 309, 378. 2. We shall use Asmt.

Proof: 1. Draw OC (Asmt. 11). 2. $\angle BOC \overset{\circ}{=} \overset{\frown}{BC}$ (§333a). 3. $\overset{\frown}{BC} = 60°$ (Given). 4. $\angle BOC = 60°$ (Asmt. 7). 5. $OC = OB$ (Asmt. 26). 6. $\angle B = \angle C$ (§98a). 7. $\angle BOC + \angle B + \angle C = 180°$ (§120). 8. $\angle B + \angle C = 120°$ (Asmt. 2). 9. $2\angle C = 120°$ (Asmt. 6). 10. $\angle C = 60°$ (Asmt. 5). 11. $\angle C = \angle BOC$ (Asmt. 7). 12. $CB = OB$ (§98a). 13. $AO + OB = AB$ (Asmt. 9). 14. $AO = OB$ (Asmt. 26). 15. $2BC = AB$ (Asmt. 6).

3. Chord AB < 2 chord CD.



Given $\odot O$ with $\overset{\frown}{AB} = 2\overset{\frown}{CD}$.

To prove that $AB < 2CD$.

Planning the Proof: 1. We can prove line segments unequal by §§519, 521-524, 526, 527, and Asmt. 2. We shall use §519.

Proof: 1. $\overset{\frown}{AB} = 2\overset{\frown}{CD}$ (Given). 2. $\frac{1}{2}\overset{\frown}{AB} = \overset{\frown}{CD}$ (Asmt. 4). 3. Let E be the midpoint of $\overset{\frown}{AB}$ and draw AE and BE (Asmt. 11). 4. Then $\overset{\frown}{CD} = \overset{\frown}{AE} = \overset{\frown}{BE}$ (Asmt. 7). 5. $CD = AE = BE$ (§308b). 6. $AB < AE + BE$ (§519). 7. $\therefore AB < 2CD$ (Asmt. 6).

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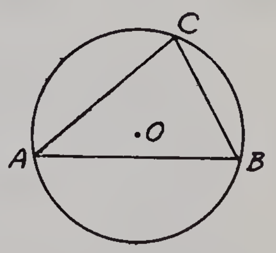
1. Given $\triangle ABC$ inscribed in $\odot O$, with $\angle B > \angle A$.



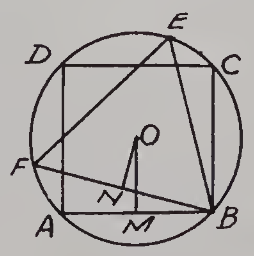
To prove that $\overset{\frown}{AC} > \overset{\frown}{BC}$.

Planning the Proof: 1. We can prove arcs unequal by §§528, 530. 2. We shall use §530.

Proof: 1. $\angle B > \angle A$ (Given). 2. $AC > BC$ (§521). 3. $\therefore \overset{\frown}{AC} > \overset{\frown}{BC}$ (§530).

2. $\angle A > \angle C > \angle B$.

3. Given square ABCD and equilateral $\triangle BEF$ inscribed in $\odot O$; $OM \perp AB$ and $ON \perp FB$.



To prove that $OM > ON$.

Planning the Proof: 1. We can prove line segments unequal by §§519, 521-524, 526, 531-534, and Asmt. 2. We shall use §532.

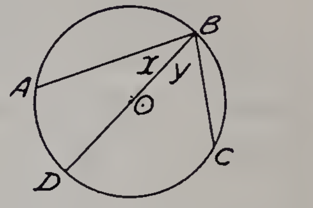
Proof: 1. $AB = BC = CD = DA$ (Given). 2. $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DA}$

(§312b). 3. $\overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CD} + \overset{\frown}{DA} = 360°$ (§314). 4. $4\overset{\frown}{AB} = 360°$ (Asmt. 6). 5. $\overset{\frown}{AB} = 90°$ (Asmt. 4). 6. In like manner, $\overset{\frown}{FB} = 120°$ (Statements 1-5). 7. Since $120° > 90°$, $\overset{\frown}{FB} > \overset{\frown}{AB}$ (Asmt. 6). 8. $FB > AB$ (§531). 9. $OM > ON$ (§532).

4. Given $\odot O$ with diameter BD and chord $AB >$ chord BC.

To prove that $\angle x < \angle y$.

Planning the Proof: 1. We can prove $\angle$s unequal by §§93, 520, 525, 529 and Asmt. 2. We shall use Asmt.

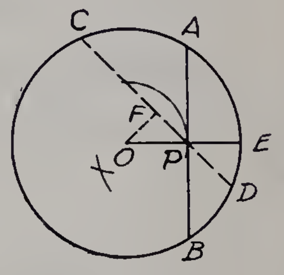


Proof: 1. $AB > BC$ (Given). 2. $\overset{\frown}{AB} > \overset{\frown}{BC}$ (§530). 3. $\overset{\frown}{BAD} = \overset{\frown}{BCD}$ (§283). 4. $\overset{\frown}{AD} < \overset{\frown}{DC}$ (Asmt. 55). 5. $\frac{1}{2}\overset{\frown}{AD} < \frac{1}{2}\overset{\frown}{DC}$ (Asmt. 54). 6. $\angle x \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AD}$ and $\angle y \overset{\circ}{=} \frac{1}{2}\overset{\frown}{DC}$ (§333b). 7. $\therefore \angle x < \angle y$ (Asmt. 6).

5. Given $\odot O$, radius OE through P, and chord $AB \perp OE$ at P.

To prove that AB is the shortest chord of $\odot O$ through P.

Planning the Proof: 1. Same as in Ex. 3. 2. We shall use §533.

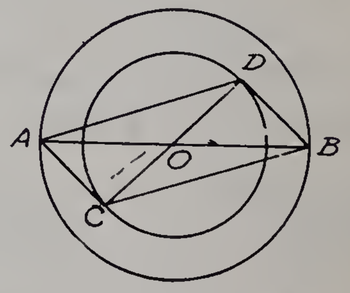


Proof: 1. Draw any other chord CD of $\odot O$ through P (Asmt. 11). 2. Construct $OF \perp CD$ (§178a). 3. $\angle OFP$ is a rt. $\angle$ (§14). 4. $\triangle OFP$ is a rt. $\triangle$ (§57). 5. $OP > OF$ (§523). 6. $OP \perp AB$ (Given). 7. $CD > AB$ (§533); that is, AB is less than any other chord through P.

6. Given two concentric ⓢ with center O, AB a diameter of the larger $\odot$, CD a diameter of the smaller $\odot$, and quad. ACBD.

To prove that ACBD is an oblique ▱.

Planning the Proof: 1. Same as in Ex. 4. 2. We shall use §525.



Proof: 1. $CO = OD$ and $AO = OB$ (Asmt. 26). 2. ACBD is a ▱ (§223d). 3. $AB > CD$ (Given). 4. $AC = DB$ (§217a). 5. $AD = AD$ (Iden.). 6. $\angle ADB > \angle CAD$ (§525). 7. $\angle ADB + \angle CAD =$ a st. $\angle$ (§222). 8. $\angle CAD <$ a st. $\angle - \angle CAD$ (Asmt. 55). 9. $2\angle CAD <$ a st. $\angle$ (Asmt. 54). 10. $\angle CAD <$ a rt. $\angle$ (Asmt. 54, §16). 11. Similarly, $\angle ADB >$ a rt. $\angle$ (Statements 7-10). 12. $\angle ADB = \angle ACB$ and $\angle CAD = \angle CBD$ (§219). 13. $\angle ACB >$ a rt. $\angle$ and $\angle CBD <$ a rt. $\angle$ (Asmt. 6). 14. ▱ACBD is not a rectangle (§185); that is, it is oblique.

PAGES 516-517 (Test 40)

1. F. 2. T. 3. T. 4. F. 5. F. 6. F. 7. T. (Ex. 13, page 509). 8. F. (§§518, 519).

PAGE 517 (Test 41)

1. unequal. 2. greater than. 3. any other chord. 4. chords. 5. increased. 6. greater. 7. less than (Ex. 4, page 509). 8. hypotenuse. 9. perpendicular (Ex. 5, page 516). 10. 400.

Chapter 17. Coordinate Geometry

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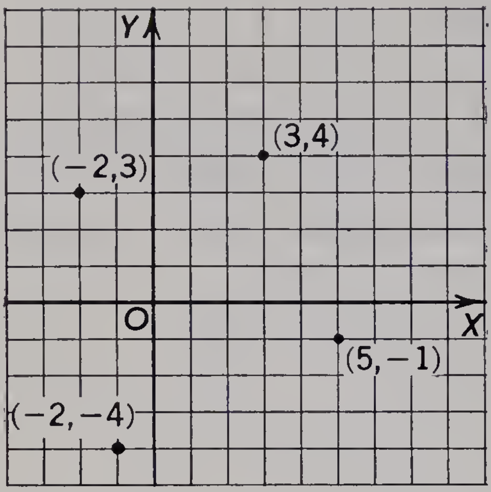
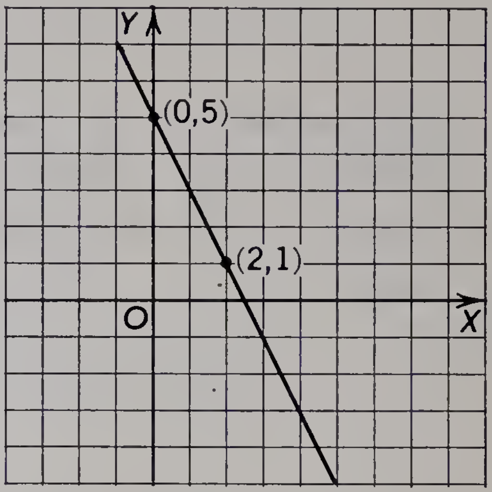
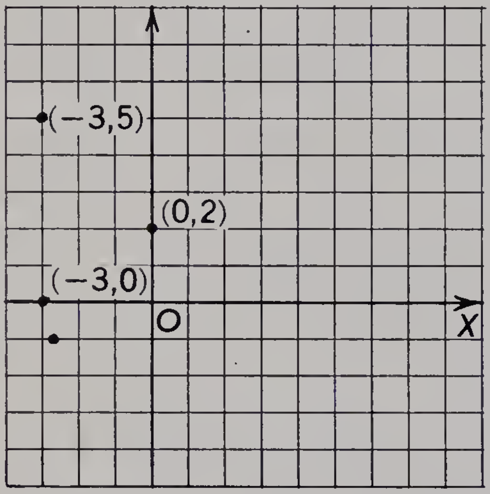
1. a.



2. a.



1. b.



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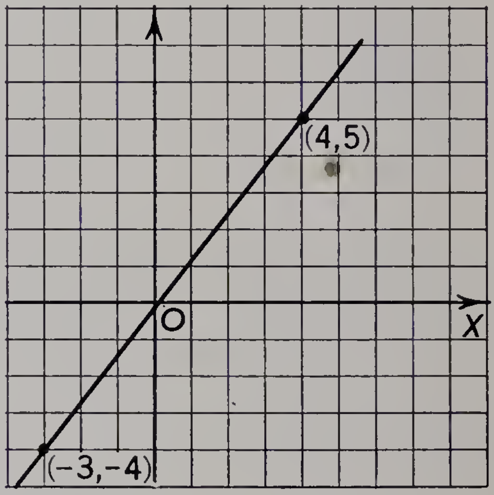
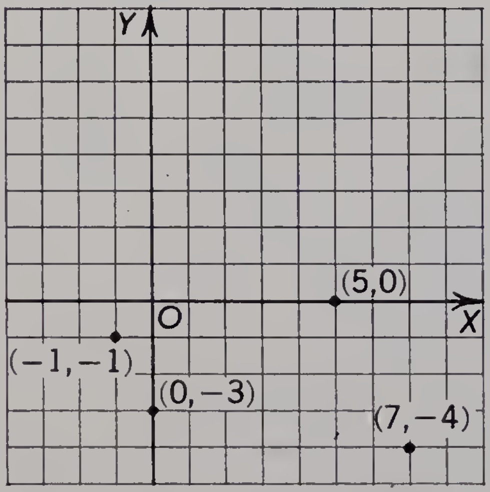


1. c.



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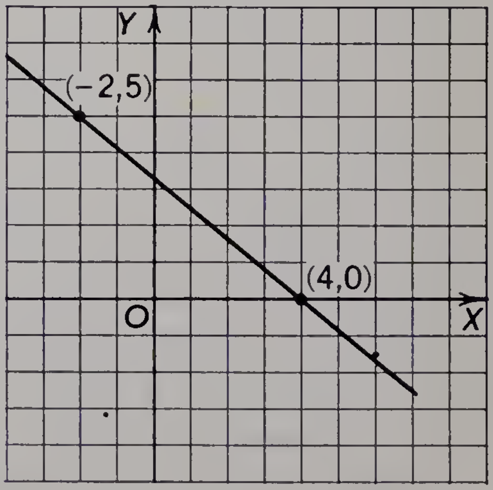

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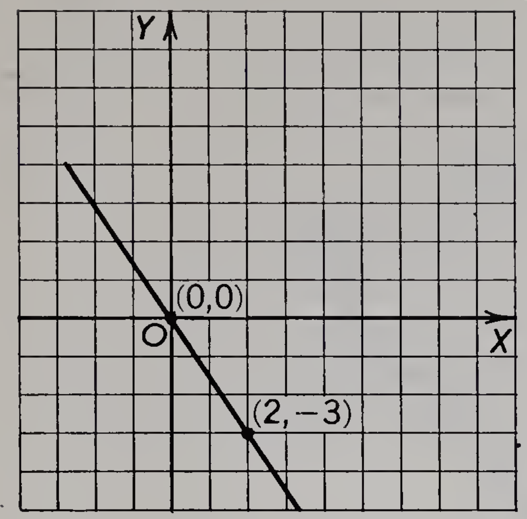
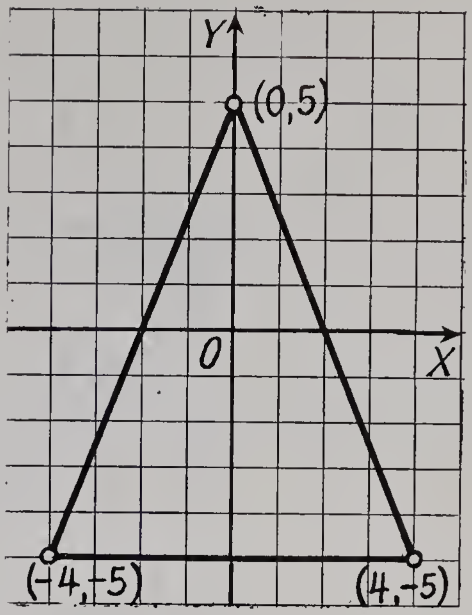
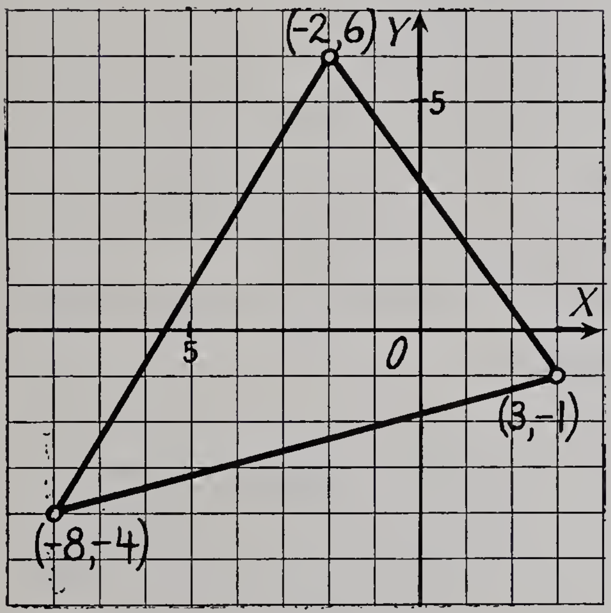


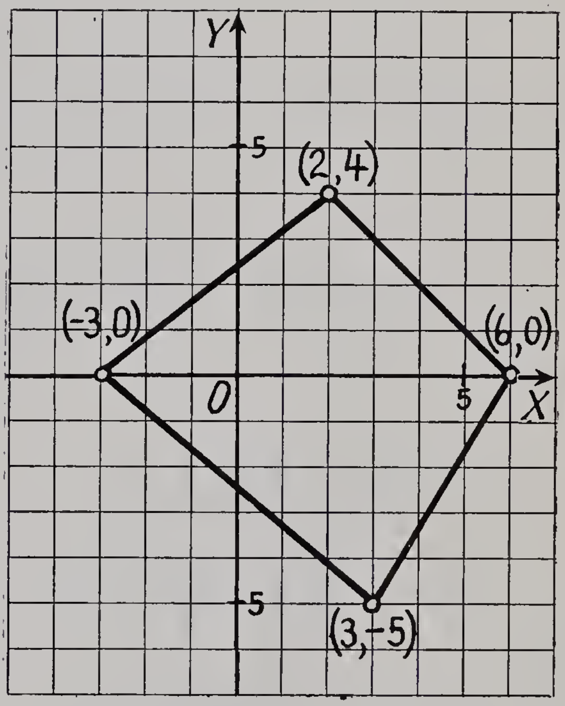
3. c.



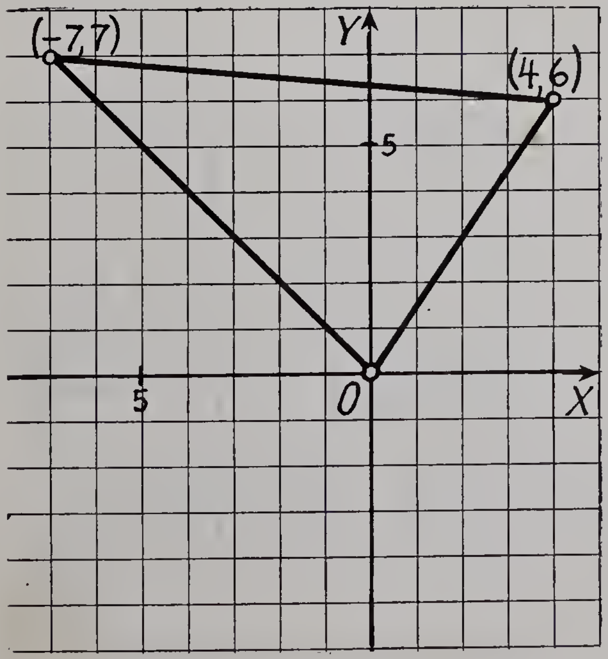
3. a.



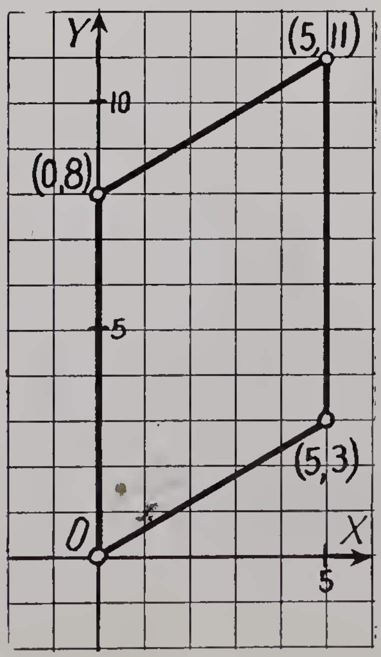
4. a.



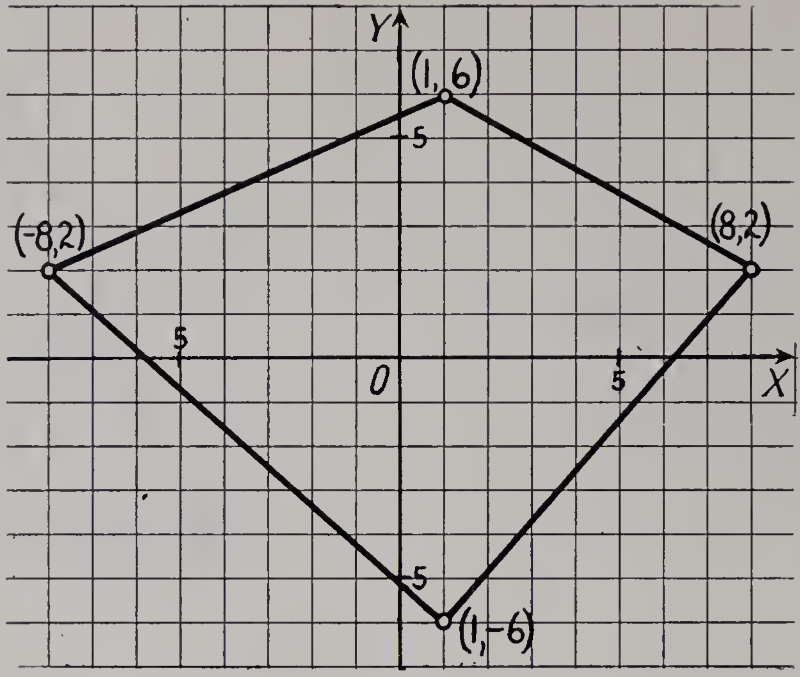
3. b.



4. b.



4. c.



PAGE 523

1. The slope of AB is $\frac{3}{5}$; the slope of EF is $\frac{1}{2}$; the slope of RS is $-\frac{2}{3}$.

2. a. $m = \frac{0 - 4}{0 - 4} = 1$

b. $m = \frac{5 - 0}{0 - 5} = -1.$

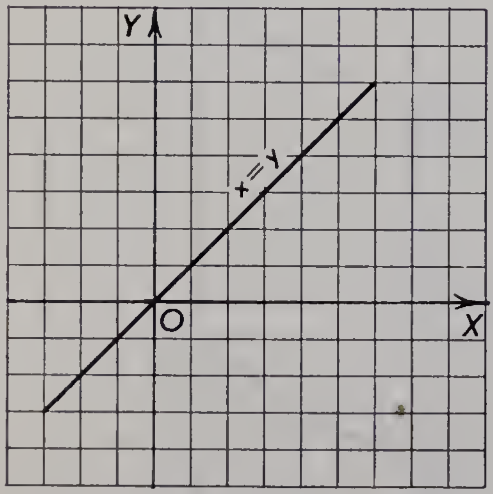
c. $m = \frac{-2 - 3}{3 + 2} = -1.$

d. $m = \frac{-3 - 1}{-5 - 2} = \frac{4}{7}.$

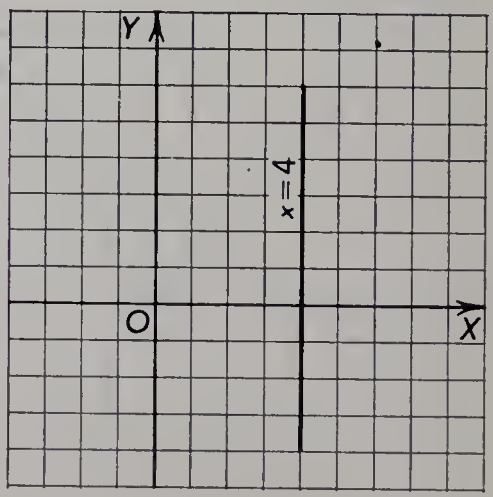
e. $m = \frac{0 - 0}{4 + 4} = 0.$

f. $m = \frac{7 - 5}{7 - 1} = \frac{1}{3}.$

3.

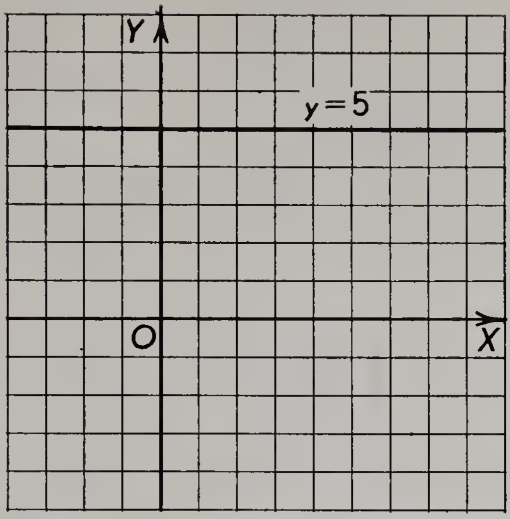


4.

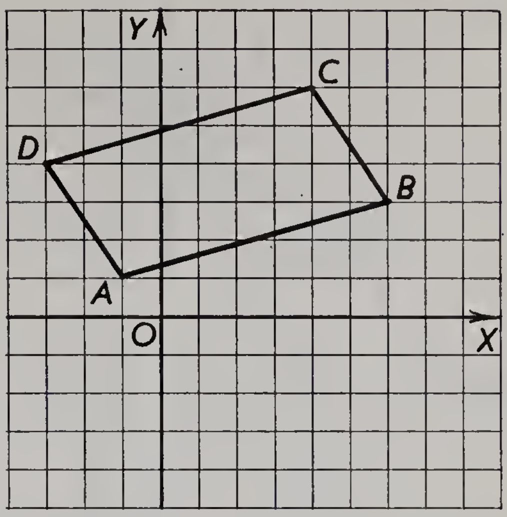


The points satisfy x = 4

5.



6.



6. (cont'd). The slope of AB = $\frac{2}{7}$.

The slope of DC = $\frac{2}{7}$.

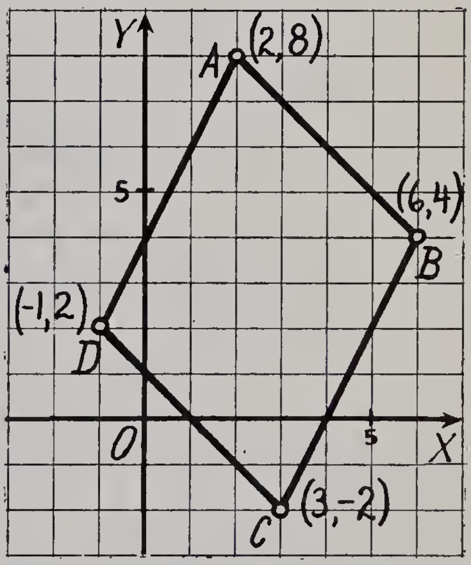
The slope of CB = $-\frac{3}{2}$.

The slope of DA = $-\frac{3}{2}$.

AB and DC have the same slope. CB and DA have the same slope.

7. ABCD is a ▱.

PAGES 525-526



1. For AB, $m = \frac{y_1 - y_2}{x_1 - x_2}$, $m = \frac{8 - 4}{2 - 6} = -1$.

For CD, $m = \frac{2 + 2}{-1 - 3} = -1$.

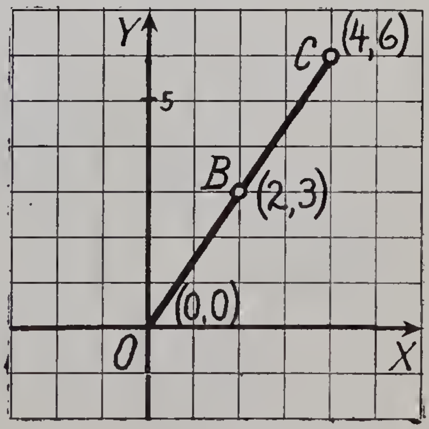
∴ AB ∥ CD (§538).

For AD, $m = \frac{8 - 2}{2 + 1} = 2$.

For BC, $m = \frac{4 + 2}{6 - 3} = 2$.

∴ AD ∥ BC (§538).

∴ ABCD is a ▱ (§185).

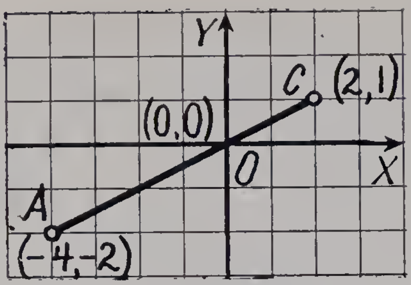


2. For OB, $m = \frac{3}{2}$.

For BC, $m = \frac{6 - 3}{4 - 2} = \frac{3}{2}$.

∴ OBC is a st. line (§538 and Asmt. 37).

2. (b)

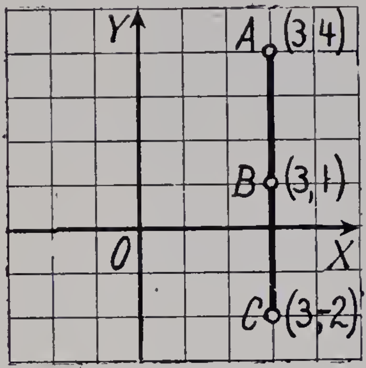


For AO, $m = \frac{-2}{-4} = \frac{1}{2}$

For OC, $m = \frac{1}{2}$.

∴ AOC is a st. line (§538 and Asmt. 37).

2. (c)

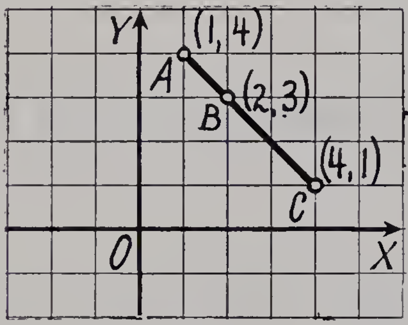


For AB, $m = \frac{4 - 1}{3 - 3} = \infty$.

For BC, $m = \frac{1 + 2}{3 - 3} = \infty$.

It is readily seen that all points for which $x = 3$ lie on a st. line passing through (3, 0) and ⊥ to the x-axis.

2. (d)

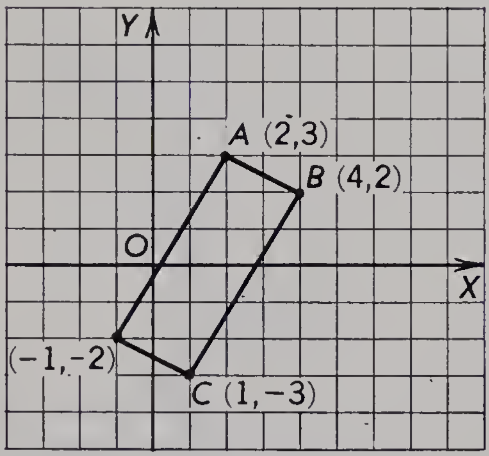


For AB, $m = \frac{4 - 3}{1 - 2} = -1$.

For BC, $m = \frac{3 - 1}{2 - 4} = -1$.

∴ ABC is a st. line (§538 and Asmt. 37).

3\.



For DA, $m = \frac{3 + 2}{2 + 1} = \frac{5}{3}$.

For CB, $m = \frac{2 + 3}{4 - 1} = \frac{5}{3}$.

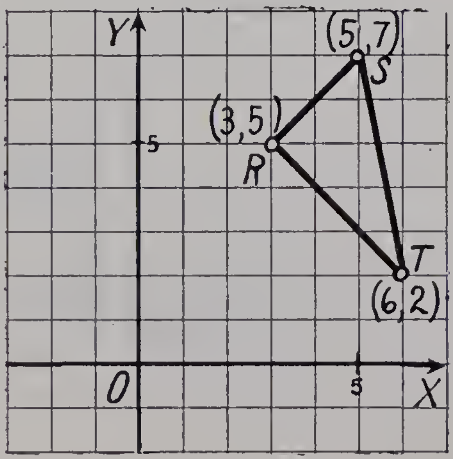
For DC, $m = \frac{-2 + 3}{-1 - 1} = -\frac{1}{2}$.

For AB, $m = \frac{3 - 2}{2 - 4} = -\frac{1}{2}$.

Then DA ∥ CB and DC ∥ AB (§538).

Then ABCD is a ▱ (§185).

4\.



For RS, $m_1 = \frac{7 - 5}{5 - 3} = 1$.

For RT, $m_2 = \frac{5 - 2}{3 - 6} = -1$.

$m_1 m_2 = -1$. ∴ RS ⊥ RT (§538).

∴ △RST is a rt. △ (§57).

5. For AB, $m = \frac{11 - 10}{-3 + 5} = \frac{1}{2}$.

For CD, $m = \frac{3 - 2}{1 + 1} = \frac{1}{2}$.

∴ AB ∥ CD (§538).

For AD, $m = \frac{10 - 2}{-5 + 1} = -2$.

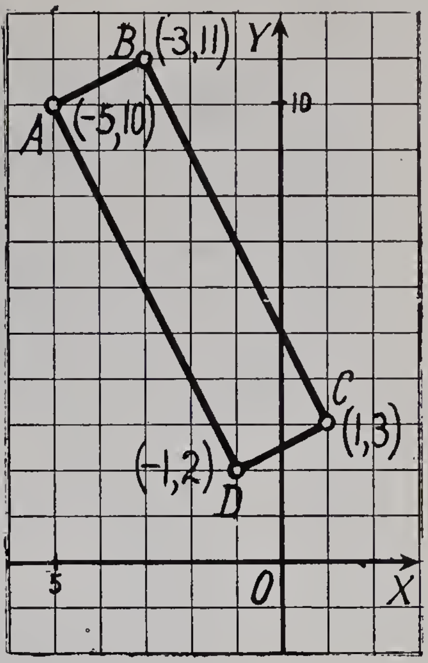
For BC, $m = \frac{11 - 3}{-3 - 1} = -2$.

∴ AD ∥ BC (§538).

(Slope of AB) x (Slope of AD) = $(\frac{1}{2})(-2) = -1$.

∴ AB ⊥ AD (§538).

∴ ABCD is a rectangle (§185).



6. For AB, $m = \frac{1 - 0}{2 - 0} = \frac{1}{2}$.

For DC, $m = \frac{-3 + 4}{4 - 2} = \frac{1}{2}$.

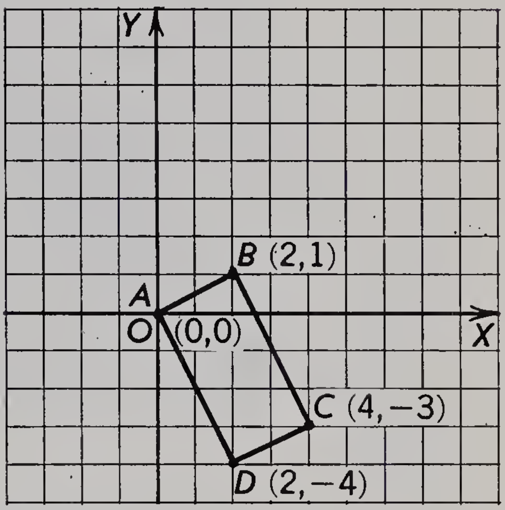
∴ AB ∥ DC (§538).

For AD, $m = \frac{-4 - 0}{2 - 0} = -2$.

For BC, $m = \frac{1 + 3}{2 - 4} = -2$.

∴ AD ∥ BC (§538).

ABCD is a ▱ (§223a).



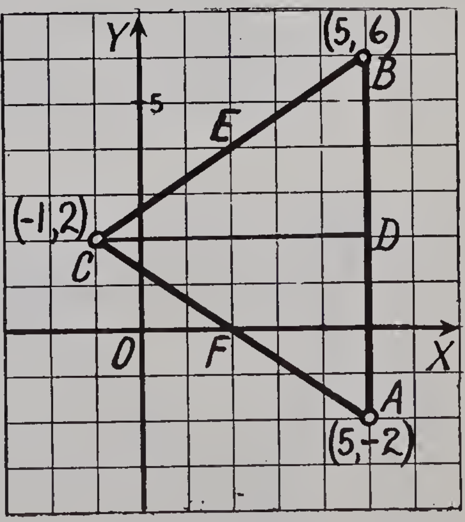
(The slope of AD) x (the slope of DC) = $(-2)(\frac{1}{2}) = -1$. Then ∠ ADC is a rt. ∠ (§538). Then ABCD is a rectangle (§185).

PAGE 527

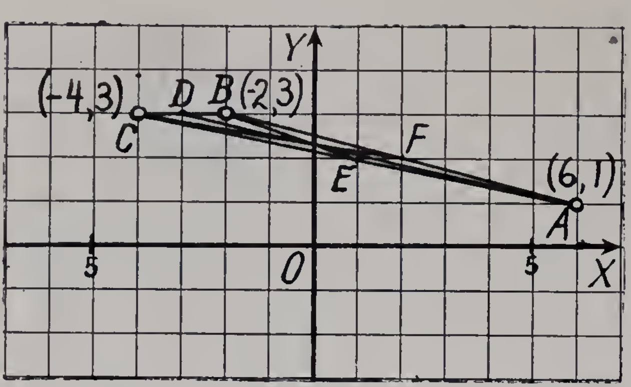
1. (a) For D, $x = \frac{5 + 5}{2} = 5$, $y = \frac{6 - 2}{2} = 2$.

For E, $x = \frac{5 - 1}{2} = 2$, $y = \frac{6 + 2}{2} = 4$.

For F, $x = \frac{5 - 1}{2} = 2$, $y = \frac{-2 + 2}{2} = 0$.



1. (b)



For D, $x = \frac{-2 - 4}{2} = -3$,

$y = \frac{3 + 3}{2} = 3$.

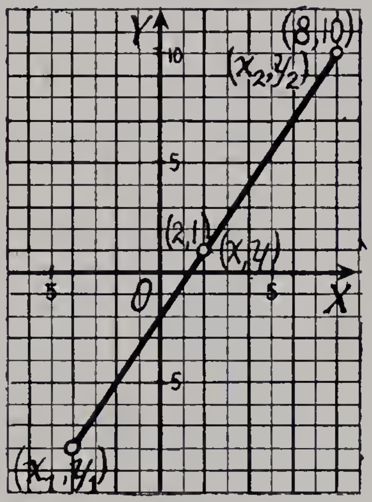
For E, $x = \frac{6 - 4}{2} = 1$,

$y = \frac{1 + 3}{2} = 2$.

For F, $x = \frac{6 - 2}{2} = 2$,

$y = \frac{1 + 3}{2} = 2$.

2.



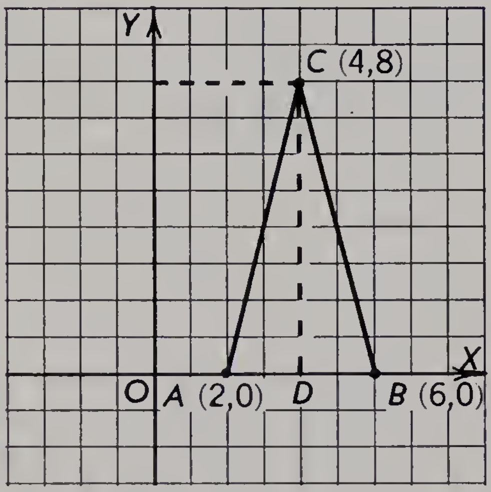
$x = \frac{x_1 + x_2}{2}$, $2 = \frac{x_1 + 8}{2}$,

$4 = x_1 + 8$, $x_1 = -4$.

$y = \frac{y_1 + y_2}{2}$, $1 = \frac{y_1 + 10}{2}$,

$2 = y_1 + 10$, and $y_1 = -8$.

3.



The value of x at D the midpoint of AB $= \frac{6 + 2}{2} = 4$.

Draw CD.

For CD, $\frac{\Delta y}{\Delta x} = \frac{8}{0}$.

Since CD has no slope, it is ∥ OY and ⊥ OX (§538).

The area of $\triangle ABC = \frac{1}{2}AB \times CD$.

$= \frac{1}{2} \times 4 \times 8 = 16$.

PAGE 529

1. (a) $AC = \sqrt{(1 - 5)^2 + (-2 - 6)^2}$

$AC = \sqrt{16 + 64}$

$AC = 4\sqrt{5}$

$AB = \sqrt{(1 + 2)^2 + (-2 - 3)^2}$

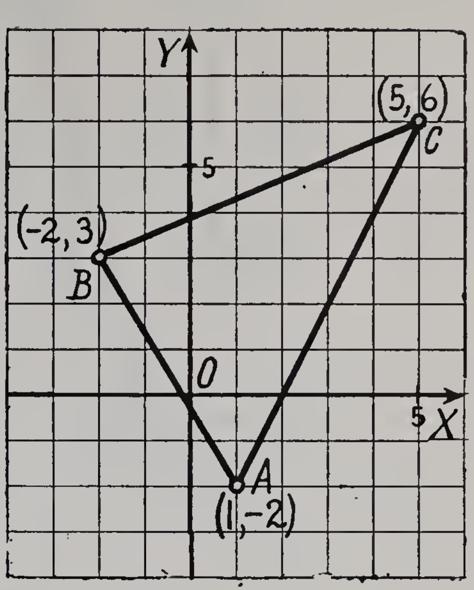
$AB = \sqrt{9 + 25}$

$AB = \sqrt{34}$

$BC = \sqrt{(-2 - 5)^2 + (3 - 6)^2}$

$BC = \sqrt{49 + 9}$

$BC = \sqrt{58}$



1. (b) $AB = \sqrt{(5 + 2)^2 + (-3 - 2)^2}$

$AB = \sqrt{49 + 25}$

$AB = \sqrt{74}$

$BC = \sqrt{(-2 - 3)^2 + (2 - 4)^2}$

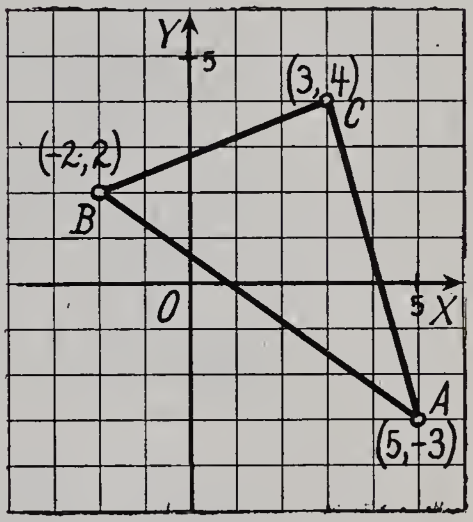
$BC = \sqrt{25 + 4}$

$BC = \sqrt{29}$

$AC = \sqrt{(5 - 3)^2 + (-3 - 4)^2}$

$AC = \sqrt{4 + 49}$

$AC = \sqrt{53}$



2. (a) $AB = \sqrt{(5 + 1)^2 + (-1 - 3)^2}$

$AB = \sqrt{36 + 16}$

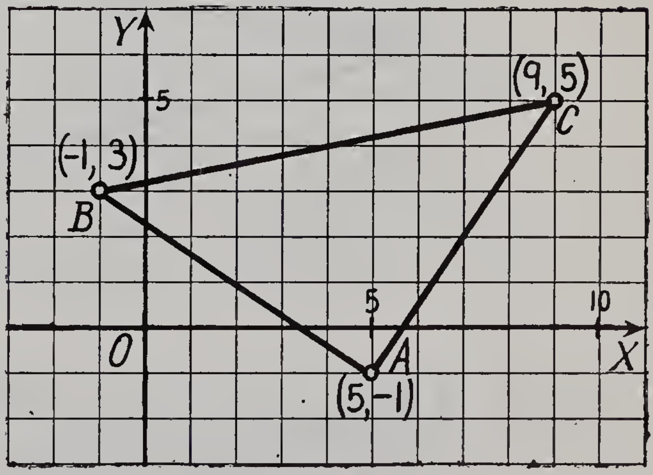
$AB = 2\sqrt{13}$

$AC = \sqrt{(5 - 9)^2 + (-1 - 5)^2}$

$AC = \sqrt{16 + 36}$

$AC = 2\sqrt{13}$

Since $AB = AC$, $\triangle ABC$ is isosceles.



2. (b) $AB = \sqrt{(2 + 3)^2 + (-1 - 4)^2}$

$AB = \sqrt{25 + 25}$; $AB = 5\sqrt{2}$

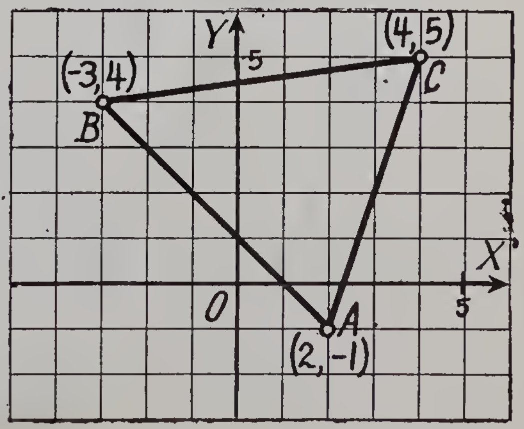
$AC = \sqrt{(2 - 4)^2 + (-1 - 5)^2}$

$AC = \sqrt{4 + 36}$; $AC = 2\sqrt{10}$

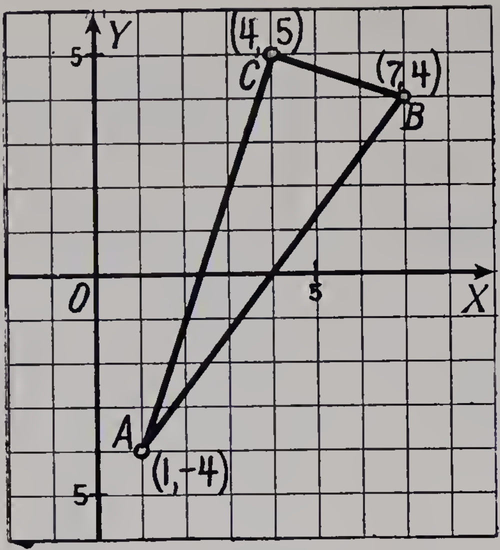
$BC = \sqrt{(-3 - 4)^2 + (4 - 5)^2}$

$BC = \sqrt{49 + 1}$; $BC = 5\sqrt{2}$

Since $AB = BC$, the $\triangle ABC$ is isosceles.



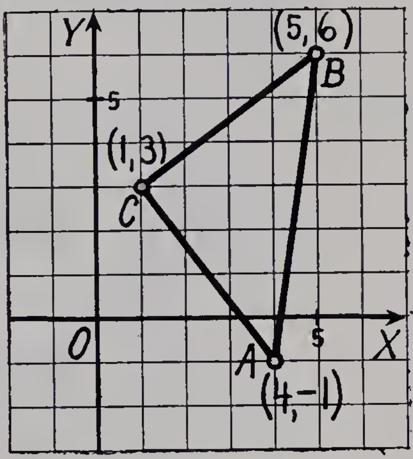
3. (a)



$\overline{AB}^2 = (1 - 7)^2 + (-4 - 4)^2$
$\overline{AB}^2 = 36 + 64$
$\overline{AB}^2 = 100$
$\overline{AC}^2 = (1 - 4)^2 + (-4 - 5)^2$
$\overline{AC}^2 = 9 + 81$
$\overline{AC}^2 = 90$
$\overline{BC}^2 = (7 - 4)^2 + (4 - 5)^2$
$\overline{BC}^2 = 9 + 1$
$\overline{BC}^2 = 10$

Since $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$,
△ABC is a rt. △ (§255).

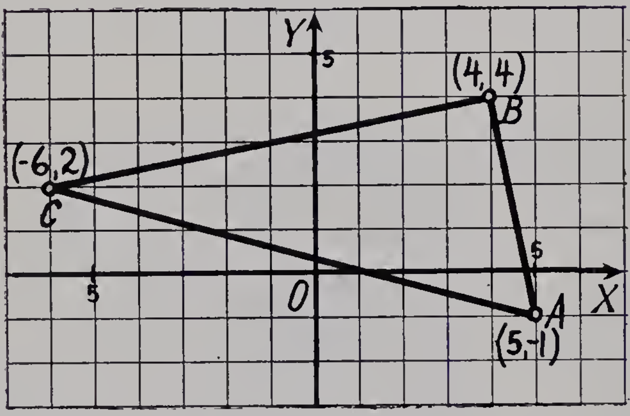
(b)



$\overline{AB}^2 = (4 - 5)^2 + (-1 - 6)^2$
$\overline{AB}^2 = 1 + 49$
$\overline{AB}^2 = 50$
$\overline{AC}^2 = (4 - 1)^2 + (-1 - 3)^2$
$\overline{AC}^2 = 9 + 16$
$\overline{AC}^2 = 25$
$\overline{BC}^2 = (5 - 1)^2 + (6 - 3)^2$
$\overline{BC}^2 = 16 + 9$
$\overline{BC}^2 = 25$

Since $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$,
△ABC is a rt. △ (§255).

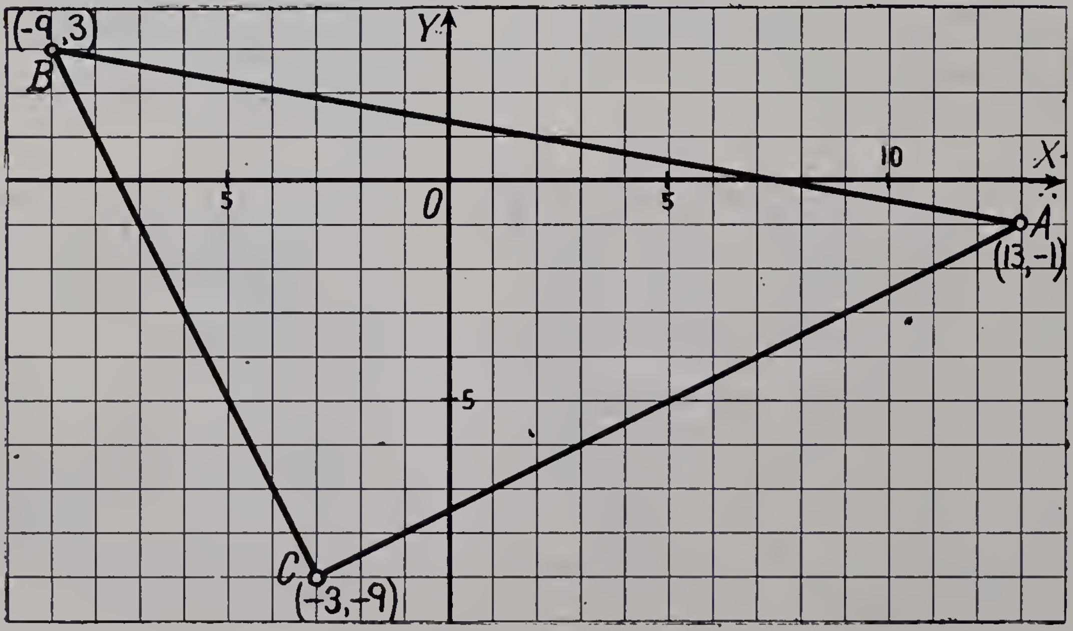
(c)



$\overline{AB}^2 = (5 - 4)^2 + (-1 - 4)^2$
$\overline{AB}^2 = 1 + 25$
$\overline{AB}^2 = 26$
$\overline{BC}^2 = (4 + 6)^2 + (4 - 2)^2$
$\overline{BC}^2 = 100 + 4$
$\overline{BC}^2 = 104$
$\overline{AC}^2 = (5 + 6)^2 + (-1 - 2)^2$
$\overline{AC}^2 = 121 + 9$
$\overline{AC}^2 = 130$

Since $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$,
△ABC is a rt. △ (§255).

(d)



$\overline{AB}^2 = (13 + 9)^2 + (-1 - 3)^2$
$\overline{AB}^2 = 484 + 16$
$\overline{AB}^2 = 500$
$\overline{AC}^2 = (13 + 3)^2 + (-1 + 9)^2$
$\overline{AC}^2 = 256 + 64$
$\overline{AC}^2 = 320$

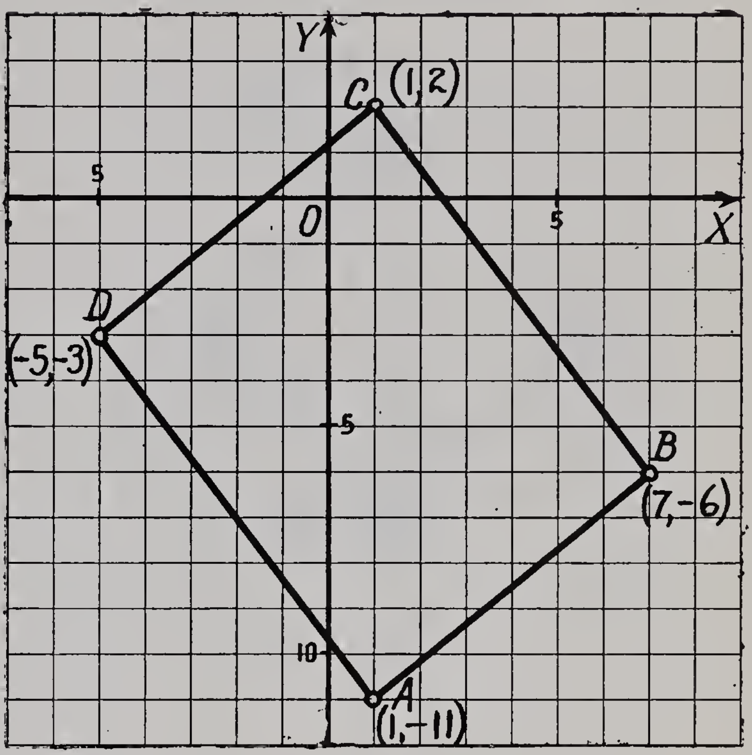
$\overline{BC}^2 = (-9 + 3)^2 + (3 + 9)^2$
$\overline{BC}^2 = 36 + 144$
$\overline{BC}^2 = 180$

Since $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$, $\triangle ABC$ is a rt. $\triangle$ (§255).

4. (a)

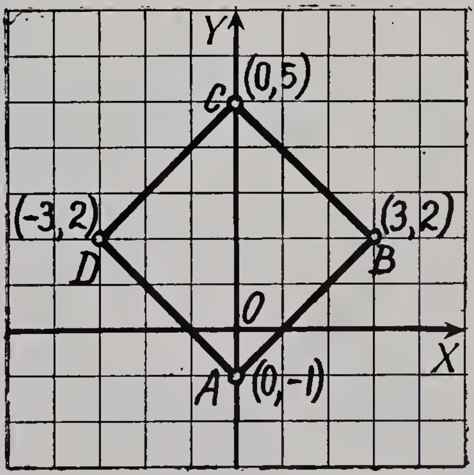
$AB = \sqrt{(1 - 7)^2 + (-11 + 6)^2}$
$AB = \sqrt{36 + 25}$
$AB = \sqrt{61}$
$CD = \sqrt{(1 + 5)^2 + (2 + 3)^2}$
$CD = \sqrt{36 + 25}$
$CD = \sqrt{61}$
$BC = \sqrt{(7 - 1)^2 + (-6 - 2)^2}$
$BC = \sqrt{36 + 64}$
$BC = 10$
$AD = \sqrt{(1 + 5)^2 + (-11 + 3)^2}$
$AD = \sqrt{36 + 64}$
$AD = 10$

Since AB = CD and BC = AD, ABCD is a ▱ (§197)



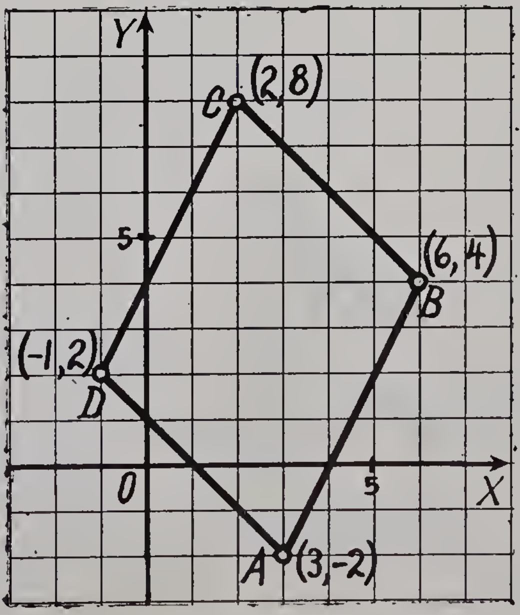
(b) $AB = \sqrt{(0 - 3)^2 + (-1 - 2)^2}$
$AB = \sqrt{9 + 9}$
$AB = 3\sqrt{2}$
$BC = \sqrt{(3 - 0)^2 + (2 - 5)^2}$
$BC = \sqrt{9 + 9}$
$BC = 3\sqrt{2}$
$CD = \sqrt{(0 + 3)^2 + (5 - 2)^2}$
$CD = \sqrt{9 + 9}$
$CD = 3\sqrt{2}$
$AD = \sqrt{(0 + 3)^2 + (-1 - 2)^2}$
$AD = \sqrt{9 + 9}$
$AD = 3\sqrt{2}$

Since AB = BC = CD = AD, ABCD is a ▱ (§197), and a rhombus (§185).

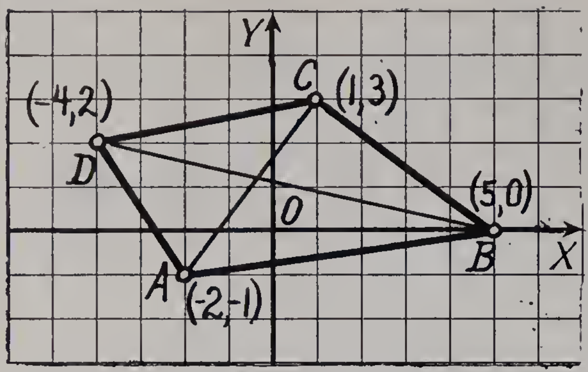


5. $AB = \sqrt{(3 - 6)^2 + (-2 - 4)^2}$
$AB = \sqrt{9 + 36}$
$AB = 3\sqrt{5}$
$BC = \sqrt{(6 - 2)^2 + (4 - 8)^2}$
$BC = \sqrt{16 + 16}$
$BC = 4\sqrt{2}$
$CD = \sqrt{(2 + 1)^2 + (8 - 2)^2}$
$CD = \sqrt{9 + 36}$
$CD = 3\sqrt{5}$
$AD = \sqrt{(3 + 1)^2 + (-2 - 2)^2}$
$AD = \sqrt{16 + 16}$
$AD = 4\sqrt{2}$

Since AB = CD and CD = AD, ABCD is a ▱ (§197).



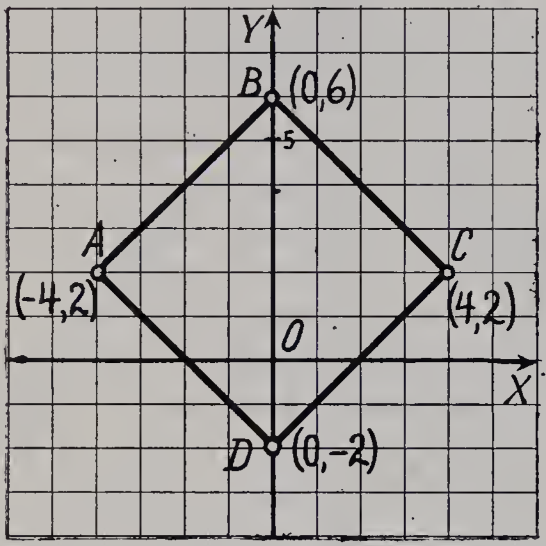
6.



$$AC = \sqrt{(-2 - 1)^2 + (-1 - 3)^2}$$
$$AC = \sqrt{9 + 16}$$
$$AC = 5$$
$$BD = \sqrt{(5 + 4)^2 + (0 - 2)^2}$$
$$BD = \sqrt{81 + 4}$$
$$BD = \sqrt{85}$$

$5, \sqrt{85}$. Ans.

7.



For BC, $m_1 = \frac{6 - 2}{-4} = \frac{4}{-4} = -1$

For AD, $m_2 = \frac{2 + 2}{-4} = \frac{4}{-4} = -1$

∴ BC ∥ AD (§538).

For AB, $m_3 = \frac{6 - 2}{4} = 1$

For CD, $m_4 = \frac{2 + 2}{4} = 1$

∴ AB ∥ CD (§538)

Then ABCD is a ▱ (§185).

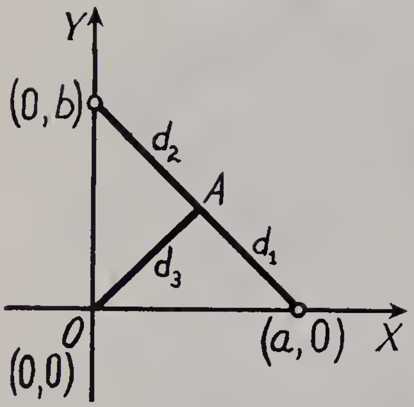
Since $m_1 m_3 = -1$, AB ⊥ CB (§538).

$$AB = \sqrt{(0 + 4)^2 + (6 - 2)^2}$$
$$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}.$$
$$BC = \sqrt{(0 - 4)^2 + (6 - 2)^2}$$
$$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}.$$

∴ ABCD is a square (§185).

PAGES 530-531

1.

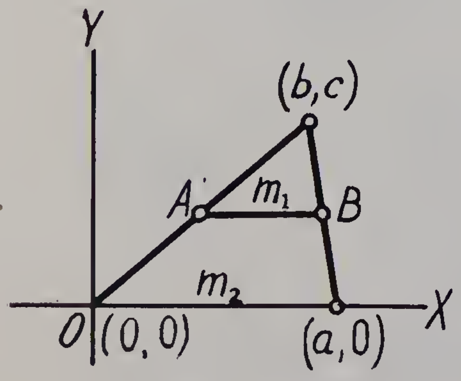


For A, $x = \frac{a}{2}$, $y = \frac{b}{2}$ (§539).

$$d_1 = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2}$$
$$d_2 = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2}$$
$$d_3 = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2}$$

$\therefore d_1 = d_2 = d_3$.

2.



For A, $x = \frac{b}{2}$, $y = \frac{c}{2}$ (§539).

For B, $x = \frac{a + b}{2}$, $y = \frac{c}{2}$ (§539).

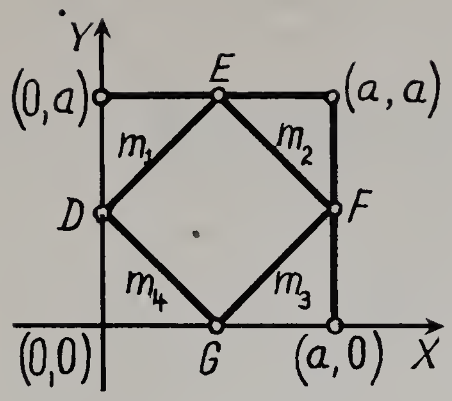
$$m_1 = \frac{\frac{c}{2} - \frac{c}{2}}{\frac{b}{2} - \frac{a + b}{2}} = \frac{0}{-\frac{a}{2}} = 0. \qquad m_2 = \frac{0}{a} = 0.$$

Since $m_1 = m_2$, AB ∥ base (§538).

$$AB = \sqrt{\left(\frac{b}{2} - \frac{a + b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2} = \sqrt{\frac{a^2}{4}} = \frac{a}{2}$$ and the base = a.

∴ AB = $\frac{1}{2}$ the base.

3.



Place the square in the position shown, with vertices (0,0), (a,0), (a,a) and (0,a).

For D, $x = 0$, $y = \frac{a}{2}$ (§539).

For E, $x = \frac{a}{2}$, $y = a$ (§539).

For F, $x = a$, $y = \frac{a}{2}$ (§539).

For G, $x = \frac{a}{2}$, $y = 0$ (§539).

$m_1 = \dfrac{a - \frac{a}{2}}{\frac{a}{2} - 0} = 1.$ $m_2 = \dfrac{a - \frac{a}{2}}{\frac{a}{2} - a} = -1.$ $m_3 = \dfrac{\frac{a}{2} - 0}{a - \frac{a}{2}} = 1.$

$m_4 = \dfrac{\frac{a}{2} - 0}{0 - \frac{a}{2}} = -1.$

Since $m_1 = m_3$ and $m_2 = m_4$, DEFG is a ▱ (§197).

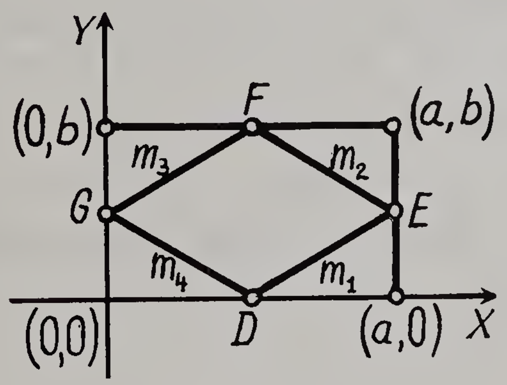
Since $m_1m_2 = -1$, DE ⊥ EF (§538) and DEFG is a rectangle (§185).

$DE = \sqrt{(\frac{a}{2} - 0)^2 + (a - \frac{a}{2})^2}$, $DE = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}}$, $DE = \frac{a}{2}\sqrt{2}.$

$DG = \sqrt{(0 - \frac{a}{2})^2 + (\frac{a}{2} - 0)^2}$, $DG = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}}$, $DG = \frac{a}{2}\sqrt{2}.$

Since DE = DG, DEFG is a square (§185).

4.



Place the rectangle in the position shown with vertices (0,0), (a,0), (a,b), and (0,b).

For D, $x = \frac{a}{2}$, $y = 0$ (§539).

For E, $x = a$, $y = \frac{b}{2}$ (§539).

For F, $x = \frac{a}{2}$, $y = b$ (§539).

For G, $x = 0$, $y = \frac{b}{2}$ (§539).

$m_1 = \dfrac{\frac{b}{2} - 0}{a - \frac{a}{2}} = \dfrac{b}{a}.$ $m_2 = \dfrac{b - \frac{b}{2}}{\frac{a}{2} - a} = -\dfrac{b}{a}.$ $m_3 = \dfrac{b - \frac{b}{2}}{\frac{a}{2} - 0} = \dfrac{b}{a}.$

$m_4 = \dfrac{\frac{b}{2} - 0}{0 - \frac{a}{2}} = -\dfrac{b}{a}.$

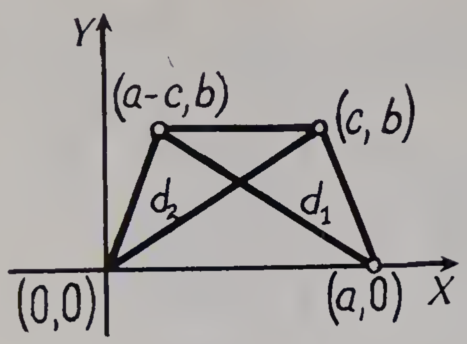
Since $m_1 = m_3$ and $m_2 = m_4$, DEFG is a ▱ (§197).

$DE = \sqrt{(\frac{a}{2} - a)^2 + (0 - \frac{b}{2})^2}$, $DE = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$, $DE = \frac{1}{2}\sqrt{a^2 + b^2}.$

$EF = \sqrt{(\frac{a}{2} - a)^2 + (b - \frac{b}{2})^2}$, $EF = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$, $EF = \frac{1}{2}\sqrt{a^2 + b^2}.$

Since DE = EF, DEFG is a rhombus (§185).

5.



$d_1 = \sqrt{(a - c - a)^2 + (b - 0)^2}$,

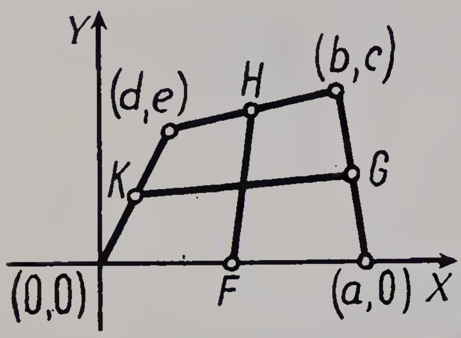
$d_1 = \sqrt{(c^2 + b^2}$.

$d_2 = \sqrt{(c - 0)^2 + (b - 0)^2}$,

$d_2 = \sqrt{c^2 + b^2}$.

$\therefore d_1 = d_2$.

6.



For F, $x = \frac{a}{2}$, $y = 0$ (§539).

For G, $x = \frac{a + b}{2}$, $y = \frac{c}{2}$ (§539).

For H, $x = \frac{b + d}{2}$,

$y = \frac{c + e}{2}$ (§539).

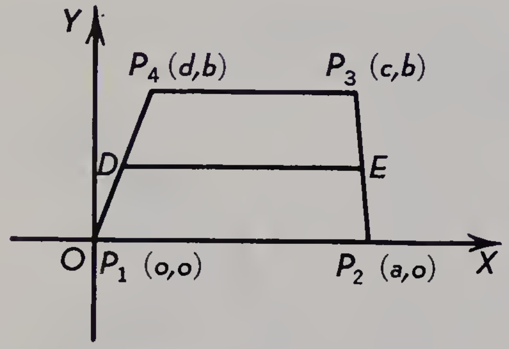
For K, $x = \frac{d}{2}$, $y = \frac{e}{2}$ (§539).

Midpoint of KG is $x = \frac{a + b + d}{4}$, $y = \frac{c + e}{4}$.

Midpoint of HF is $x = \frac{a + b + d}{4}$, $y = \frac{c + e}{4}$.

Since KG and HF have the same midpoint they bisect each other (Asmt. 17).

7.



Represent the trapezoid $P_1P_2P_3P_4$ by placing one vertex at the origin and one base along the x-axis. Represent P_2 by (a,0), P_3 by (c,b), and P_4 by (d,b). DE is the median of $P_1P_2P_3P_4$.

By §539, D is $(\frac{d}{2}, \frac{b}{2})$ and E is $(\frac{a + c}{2}, \frac{b}{2})$.

$DE = \sqrt{(\frac{a + c}{2} - \frac{d}{2})^2 + (\frac{b}{2} - \frac{b}{2})^2}$ (§540). $DE = \frac{a + c - d}{2}$.

$P_1P_2 = a$ and $P_4P_3 = c - d$. $P_1P_2 + P_4P_3 = a + c - d$ (Asmt. 1).

Then $DE = \frac{1}{2}(P_1P_2 + P_4P_3)$ (Asmt. 6).

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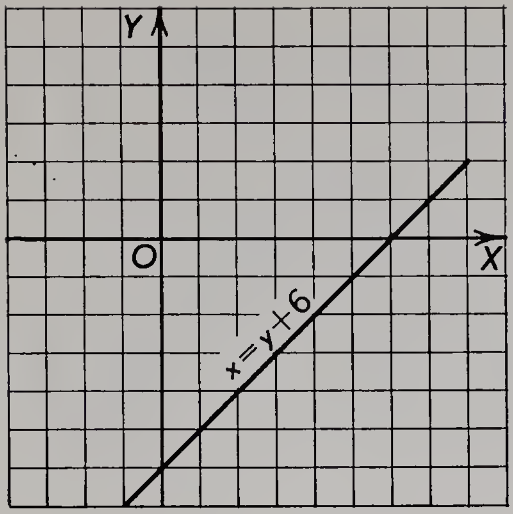
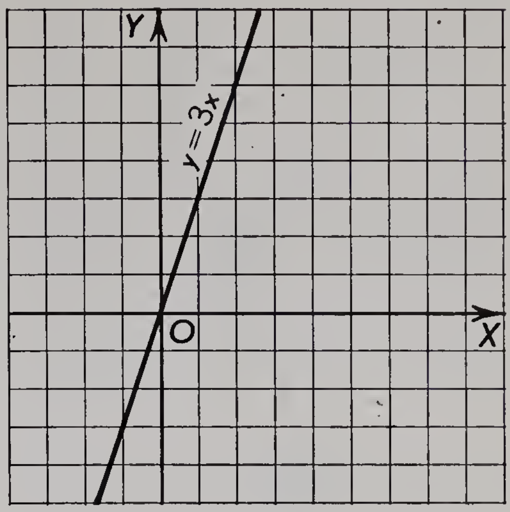
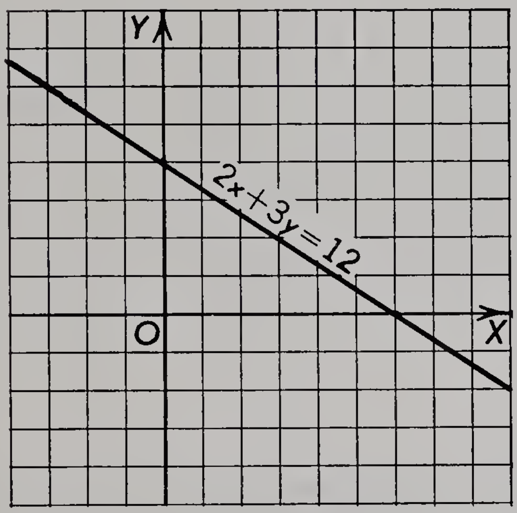
1.



2.



3.



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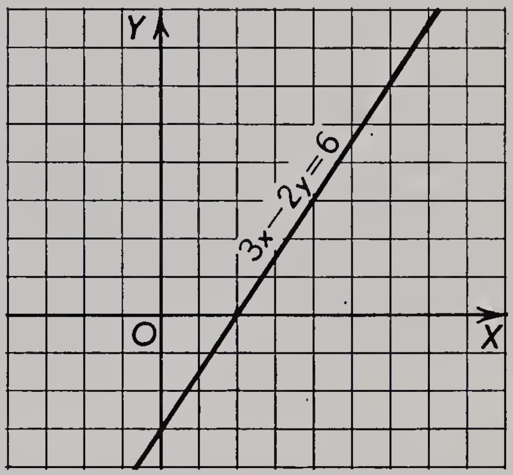
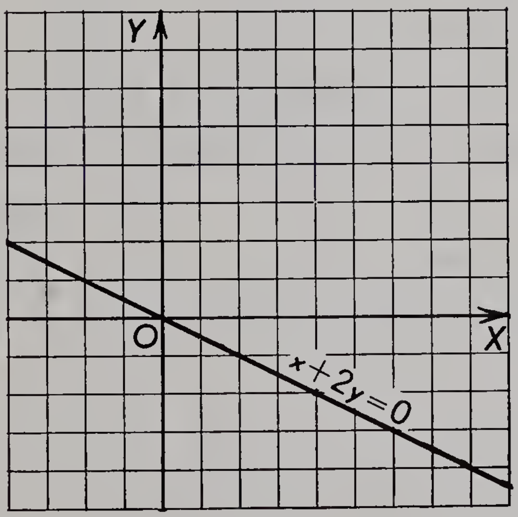


5.



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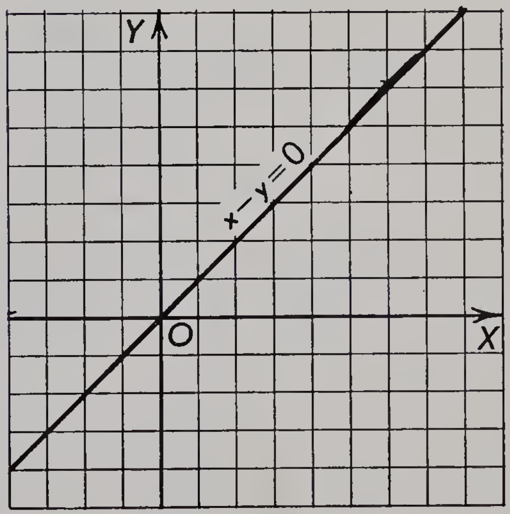
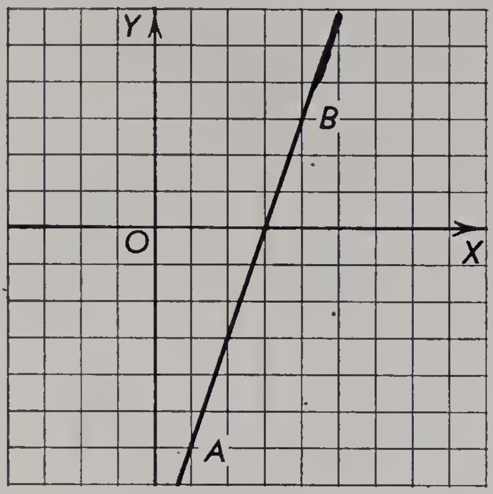


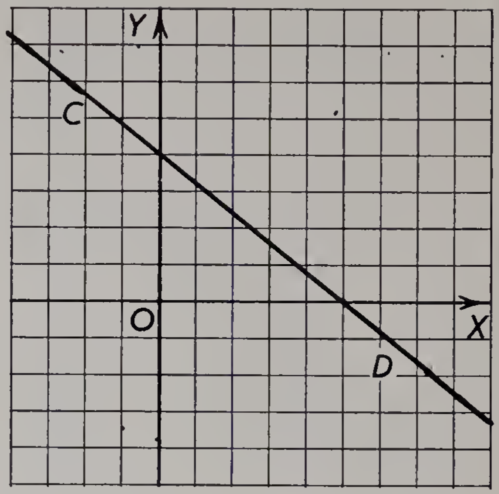
PAGES 532-533

1.



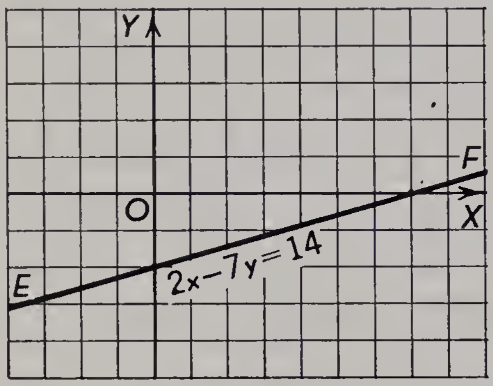
AB is the required locus.

2.



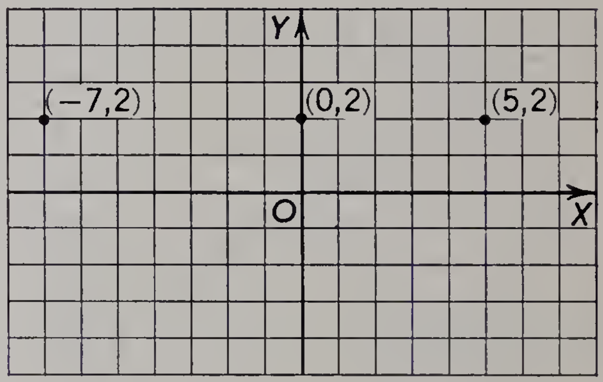
CD is the required locus

3.



EF is the required locus.

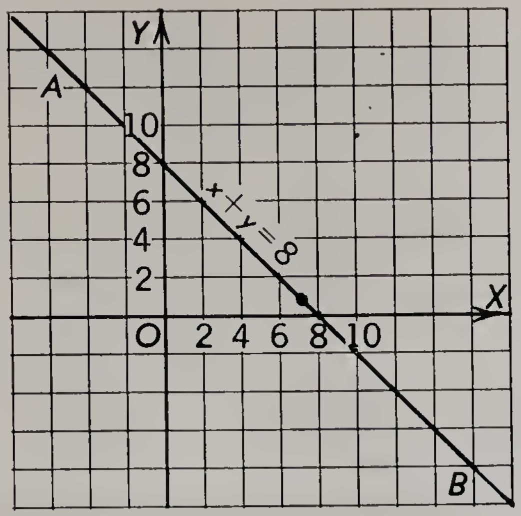
4.



The coordinates of the points do not satisfy the equation 0y + x = 2; but they do satisfy the equation y + 0x = 2.

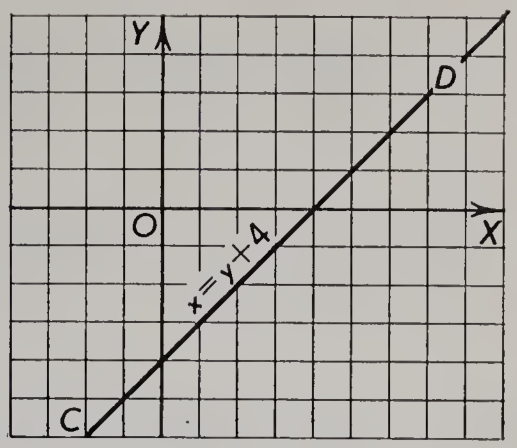
5. a. The line parallel to the y-axis and 3 units to the right of it. b. The line parallel to the y-axis and 4 units to the left of it. c. The line parallel to the x-axis and 2 units above it. d. The line parallel to the x-axis and 5 units below it. e. The line parallel to the y-axis and c units to the right of it. f. The line parallel to the x-axis and k units below it.

6.



AB is the required locus.

7.

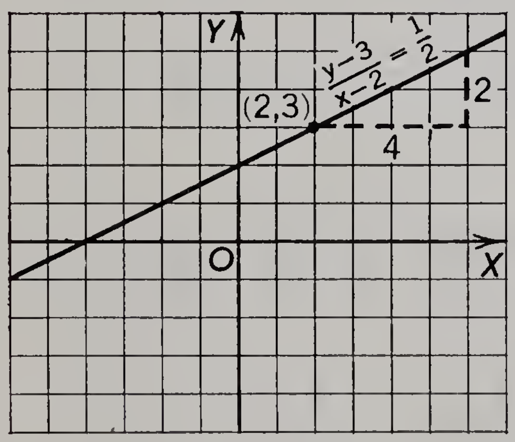


CD is the required locus.

PAGE 533

1. a. $y - y_1 = m(x - x_1)$; $y - 3 = \frac{5}{2}(x - 2)$; $2y - 6 = 5x - 10$; $5x - 2y = 4$.
b. $y - y_1 = m(x - x_1)$; $y - 0 = 1(x - 3)$; $y = x - 3$; $x - y = 3$.
c. $y - y_1 = m(x - x_1)$; $y + 4 = -\frac{1}{2}(x - 2)$; $2y + 8 = -x + 2$; $x + 2y = -6$.
d. $y - y_1 = m(x - x_1)$; $y - 7 = 0(x - 2)$; $y - 7 = 0$.
e. $y - y_1 = m(x - x_1)$; $y - 2 = \frac{1}{2}(x + 5)$; $2y - 4 = x + 5$; $x - 2y = -9$.
f. $y - y_1 = m(x - x_1)$; $y - 6 = -2(x - 0)$; $y - 6 = -2x$; $2x + y = 6$.

2.



$y - y_1 = m(x - x_1)$ (§544).
Multiplying $\frac{y - 3}{x - 2} = \frac{1}{2}$ by $(x - 2)$, the equation becomes $y - 3 = \frac{1}{2}(x - 2)$.

$\therefore M = \frac{1}{2}$ and one point on the line is (2,3).

PAGE 534

1. a. $y = mx + b$; $y = 2x + 5$; $2x - y = -5$.
b. $y = mx + b$; $y = \frac{1}{3}x - 3$; $3y = x - 9$; $x - 3y = 9$.
c. $y = mx + b$; $y = 5x - 2$; $5x - y = 2$.
d. $y = mx + b$; $y = \frac{1}{2}x + 3$; $2y = x + 6$; $x - 2y = -6$.

2. a. $2x + 3y = 12$
$3y = -2x + 12$
$y = -\frac{2}{3}x + 4$
The slope is $-\frac{2}{3}$
The y-intercept $= +4$.

b. $5x - y = 8$
$-y = -5x + 8$
$y = 5x - 8$
The slope is 5.
The y-intercept $= -8$.

c. $x = y + 6$
$y = x - 6$
The slope is 1.
The y-intercept $= -6$.

3. a. $x - 3y = 9$
$-3y = -x + 9$
$y = \frac{1}{3}x - 3$

b. $2x + y = 10$
$y = -2x + 10$

c. $x + 2y = 6$
$2y = -x + 6$
$y = -\frac{1}{2}x + 3$

d. $y - 3x = 12$
$y = 3x + 12$

e. $3x - 9y = 18$
$-9y = -3x + 18$
$y = \frac{1}{3}x - 2$

f. $4x - 2y = 4$
$-2y = -4x + 4$
$y = 2x - 2$

Slopes: a. $\frac{1}{3}$. b. -2. c. $-\frac{1}{2}$. d. 3. e. $\frac{1}{3}$. f. 2.

Then the graphs of a and e are parallel and the graphs of c and f are perpendicular.

PAGE 535

1. a. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 3}{x - 4} = \frac{7 - 3}{-2 - 4}$; $\frac{y - 3}{x - 4} = -\frac{2}{3}$;
$3y - 9 = -2x + 8$; $2x + 3y = 17$.

b. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y + 1}{x - 3} = \frac{3 + 1}{5 - 3}$; $\frac{y + 1}{x - 3} = 2$;
$y + 1 = 2x - 6$; $2x - y = 7$.

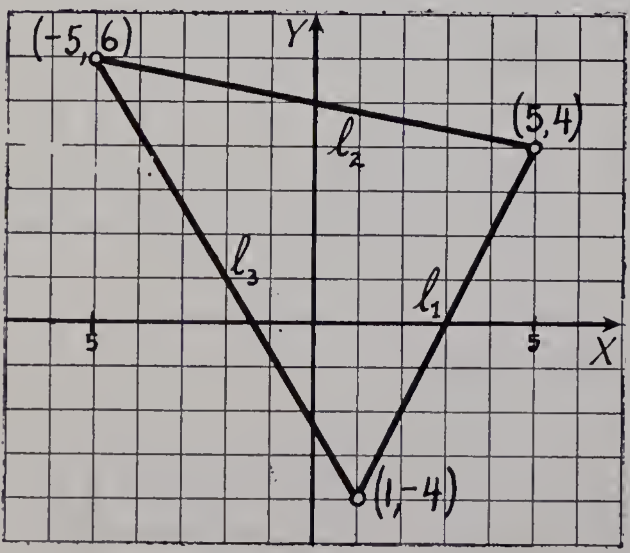
c. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y + 4}{x - 6} = \frac{-6 + 4}{2 - 6}$; $\frac{y + 4}{x - 6} = \frac{1}{2}$;
$2y + 8 = x - 6$; $x - 2y = 14$.

d. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 3}{x} = \frac{5 - 3}{-7}$; $\frac{y - 3}{x} = -\frac{2}{7}$;
$7y - 21 = -2x$; $2x + 7y = 21$.

e. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 2}{x - 4} = \frac{-3 - 2}{-5 - 4}$; $\frac{y - 2}{x - 4} = \frac{5}{9}$;
$9y - 18 = 5x - 20$; $5x - 9y = 2$.

f. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y}{x - 6} = \frac{-2}{-1 - 6}$; $\frac{y}{x - 6} = \frac{2}{7}$;
$7y = 2x - 12$; $2x - 7y = 12$.

2.

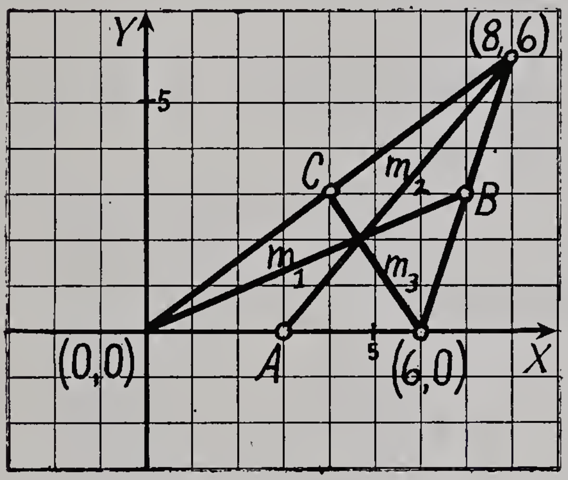


For ℓ_1, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$;
$\frac{y + 4}{x - 1} = \frac{4 + 4}{5 - 1}$; $\frac{y + 4}{x - 1} = 2$;
$y + 4 = 2x - 2$; $2x - y = 6$.

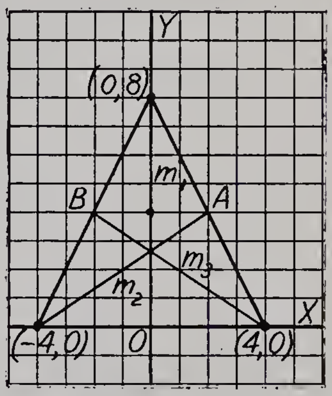
For ℓ_2, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$;
$\frac{y - 6}{x + 5} = \frac{4 - 6}{5 + 5}$; $\frac{y - 6}{x + 5} = -\frac{1}{5}$.
$5y - 30 = -x - 5$;
$x + 5y = 25$.

For ℓ_3, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 6}{x + 5} = \frac{-4 - 6}{1 + 5}$; $\frac{y - 6}{x + 5} = -\frac{5}{3}$;
$3y - 18 = -5x - 25$; $5x + 3y = -7$.

3. For A, $x = 3$, $y = 0$.
For B, $x = 7$, $y = 3$.
For C, $x = 4$, $y = 3$.
For m_1, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y}{x} = \frac{3}{7}$. $3x - 7y = 0$.
For m_2, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 6}{x - 8} = \frac{0 - 6}{3 - 8}$; $\frac{y - 6}{x - 8} = \frac{6}{5}$;
$5y - 30 = 6x - 48$; $6x - 5y = 18$.
For m_3, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 3}{x - 4} = \frac{0 - 3}{6 - 4}$; $\frac{y - 3}{x - 4} = -\frac{3}{2}$;
$2y - 6 = -3x + 12$; $3x + 2y = 18$.

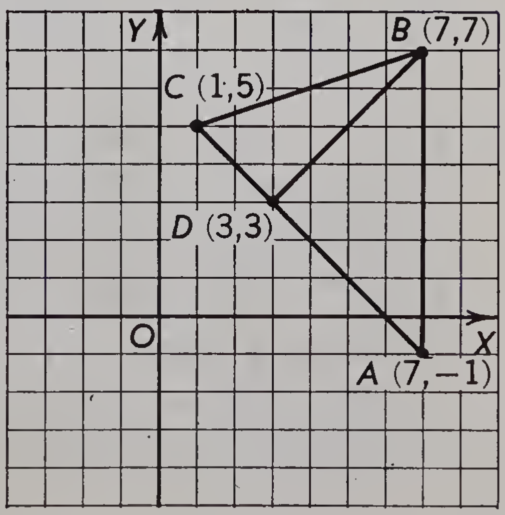


(Ex. 3)



(Ex. 4)

4. For O, $x = 0$, $y = 0$.
For A, $x = 2$, $y = 4$.
For B, $x = -2$, $y = 4$. For m_1, $x = 0$.
For m_2, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{x - 4}{x - 2} = -\frac{4}{6}$; $2x - 3y + 8 = 0$.
For m_3, $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y}{x - 4} = -\frac{4}{6}$; $2x + 3y - 8 = 0$.



5. a. The slope of CA = -1. Then the slope of the altitude from B = $+1$ (§538). The equation of the altitude is $y - 7 = 1(x - 7)$ (§544), or $x - y = 0$.
The equation of CA is $\frac{y - 5}{x - 1} = \frac{-1 - 5}{7 - 1}$, which simplified is $x + y = 6$. Solving $x - y = 0$ and $x + y = 6$, we get $x = 3$, $y = 3$, which is the point of intersection

of the altitude from B and line CA. The length of the altitude BD = $\sqrt{(7 - 3)^2 + (7 - 3)^2} = 4\sqrt{2}$.

b. The length of AC = $\sqrt{(7 - 1)^2 + (-1 - 5)^2} = 6\sqrt{2}$. The area of $\triangle ABC = \frac{1}{2}CA \times BD = \frac{1}{2} \times 6\sqrt{2} \times 4\sqrt{2} = 24$.

PAGE 536

1. a. $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{x}{3} + \frac{y}{4} = 1$; $4x + 3y = 12$.
   b. $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{x}{5} - \frac{y}{1} = 1$; $x - 5y = 5$.
   c. $\frac{x}{a} + \frac{y}{b} = 1$; $-\frac{x}{5} + \frac{y}{2} = 1$; $-2x + 5y = 10$; $2x - 5y = -10$.
   d. $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{x}{3} - \frac{y}{7} = 1$; $7x - 3y = 21$.

2. a. $3x + 5y = 15$; $\frac{3x}{15} + \frac{5y}{15} = 1$; $\frac{x}{5} + \frac{y}{3} = 1$.
   b. $3x + 4y = 12$; $\frac{3x}{12} + \frac{4y}{12} = 1$; $\frac{x}{4} + \frac{y}{3} = 1$.
   c. $x - 5y = 5$; $\frac{x}{5} - \frac{5y}{5} = \frac{5}{5}$; $\frac{x}{5} - \frac{y}{1} = 1$.

PAGES 537-538

1. a. $x^2 + y^2 = 25$.
   b. $x^2 + (y - 2)^2 = 9$; $x^2 + y^2 - 4y - 5 = 0$.
   c. $(x + 3)^2 + y^2 = 16$; $x^2 + y^2 + 6x - 7 = 0$.
   d. $(x + 1)^2 + (y - 3)^2 = 4$; $x^2 + y^2 + 2x - 6y + 6 = 0$.
   e. $(x - 1)^2 + (y + 4)^2 = 64$; $x^2 + y^2 - 2x + 8y - 47 = 0$.
   f. $(x - 2)^2 + (y - 5)^2 = 36$; $x^2 + y^2 - 4x - 10y - 7 = 0$.

2. $r^2 = (-4 - 0)^2 + (3 - 0)^2 = 16 + 9 = 25$.
   $(x + 4)^2 + (y - 3)^2 = 25$; $x^2 + y^2 + 8x - 6y = 0$.

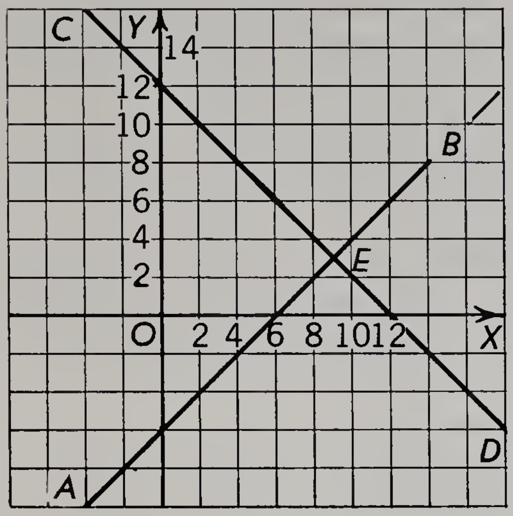
3. $x = \frac{5 - 1}{2} = 2$. $y = \frac{2 + 8}{2} = 5$.
   $r^2 = (5 - 2)^2 + (2 - 5)^2$; $r^2 = 9 + 9 = 18$.
   $(x - 2)^2 + (y - 5)^2 = 18$; $x^2 + y^2 - 4x - 10y + 11 = 0$.

4. a. $x^2 + y^2 = 100$; $(x + 0)^2 + (y + 0)^2 = 100$.
   Center (0, 0), radius 10.
   To graph the locus, use (0, 0) as the center and a radius = 10.
   b. $x^2 + 10x + 25 + y^2 + 8y + 16 = 16$; $(x + 5)^2 + (y + 4)^2 = 16$.
   Center (-5, -4), radius 4.
   c. $x^2 + 10x + 25 + y^2 - 24y + 144 = 169$; $(x + 5)^2 + (y - 12)^2 = 169$.
   Center (-5, 12), radius 13.

d. $x^2 - 6x + 9 + y^2 = 16$; $(x - 3)^2 + y^2 = 16$.
Center (3, 0), radius 4.

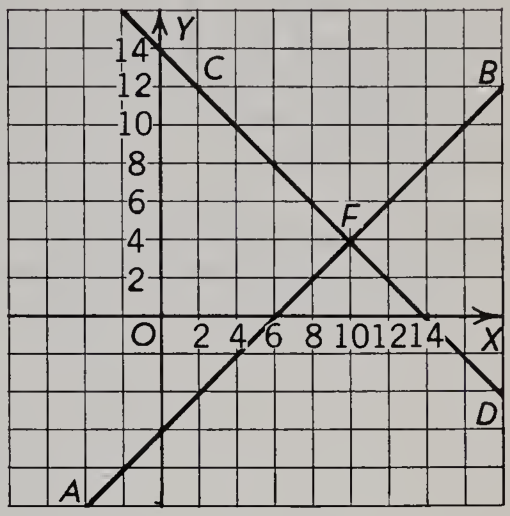
PAGE 539

1.



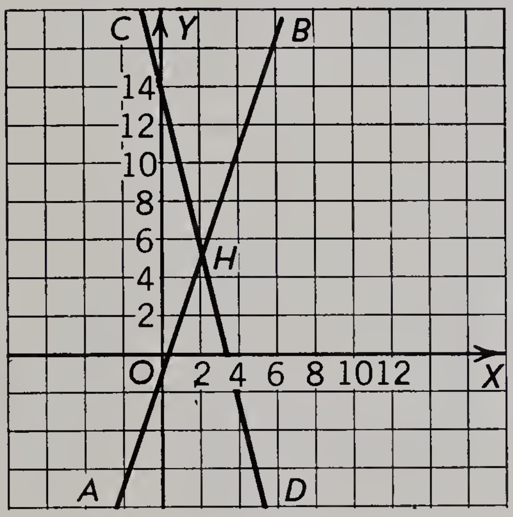
The locus is the point E (9, 3).

2.



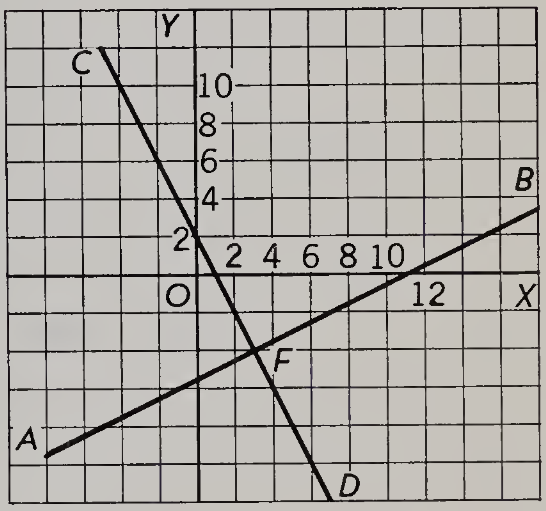
The locus is the point F (10, 4).

3.



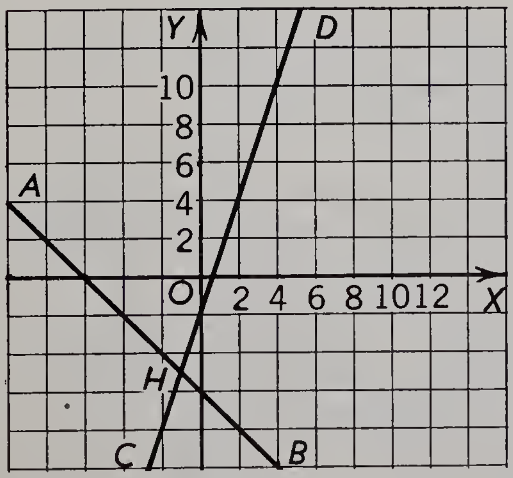
The locus is the point H approx. (2, $5\frac{1}{2}$)

4.



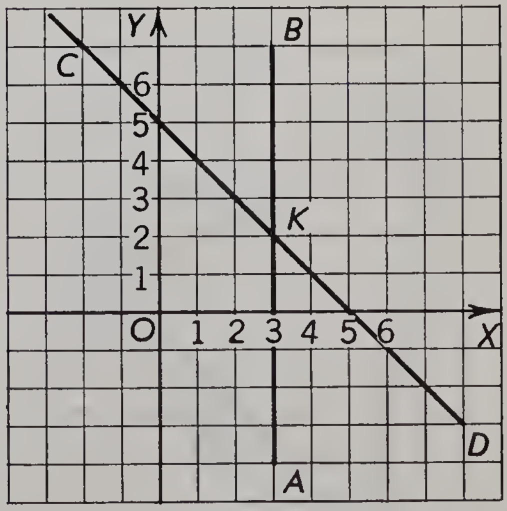
The locus is the point F (3, -4)

5.



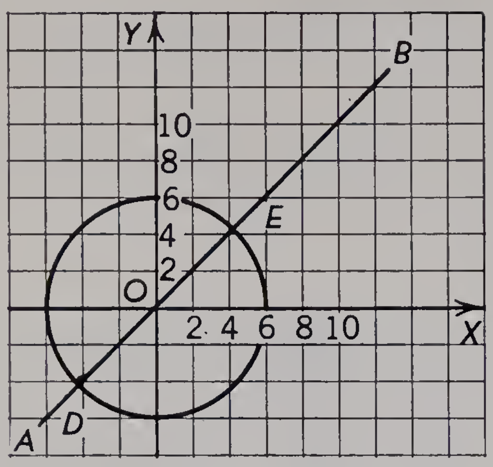
The locus is the point H (-1, -5)

6.



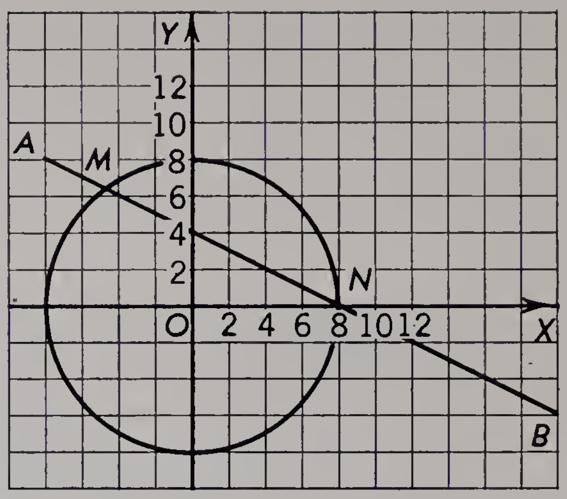
The locus is the point K (3, 2)

7.



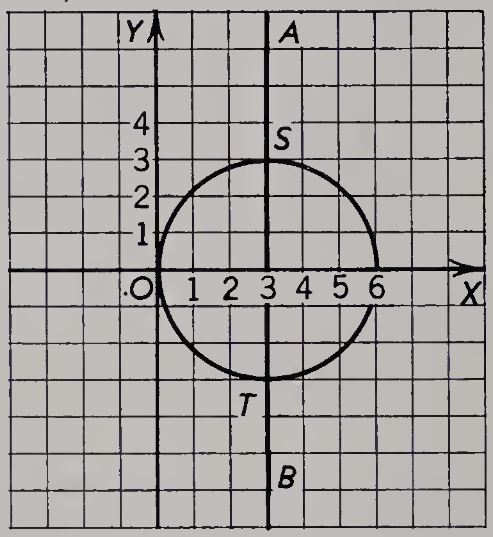
The locus consists of point D, approx. (-4, -4) and point E, approx. (4, 4).

8.



The locus consists of point M, approx. $(-5, 6\frac{1}{2})$ and point N (8, 0)

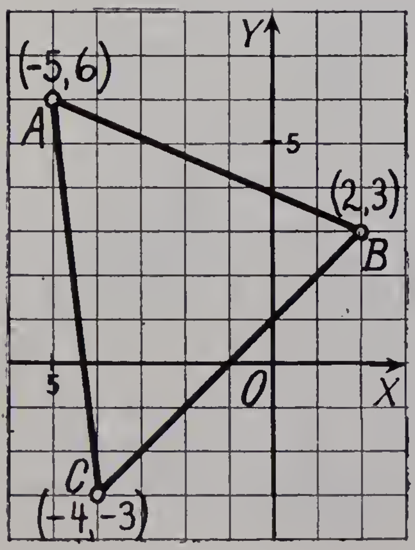
9.



The locus consists of the point T (3, -3) and the point S (3, 3).

PAGE 539 (Test 42)

1.



$AB = \sqrt{(-5 - 2)^2 + (6 - 3)^2}$

$AB = \sqrt{49 + 9}$

$AB = \sqrt{58}$

$BC = \sqrt{(2 + 4)^2 + (3 + 3)^2}$

$BC = \sqrt{36 + 36}$

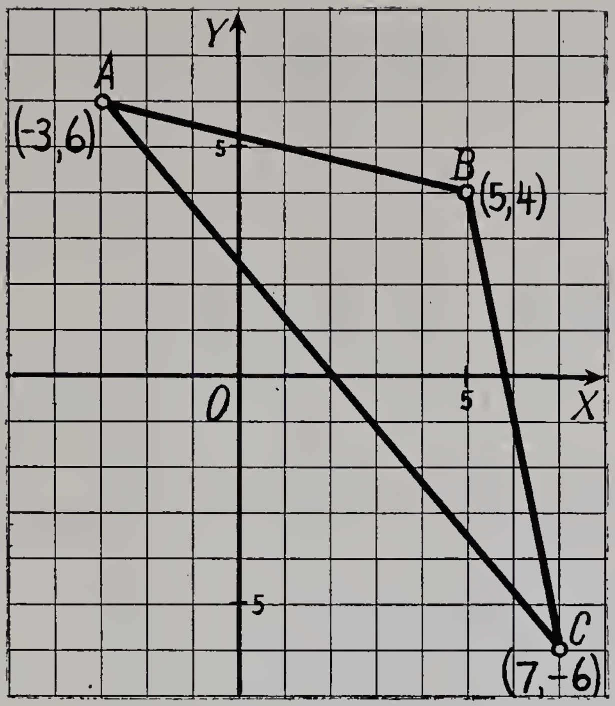
$BC = 6\sqrt{2}$

$AC = \sqrt{(-5 + 4)^2 + (6 + 3)^2}$

$AC = \sqrt{1 + 81}$

$AC = \sqrt{82}$

2.



For midpoint of AC, $x = \frac{-3 + 7}{2} = 2,$

$y = \frac{6 - 6}{2} = 0.$

For midpoint of BC, $x = \frac{5 + 7}{2} = 6,$

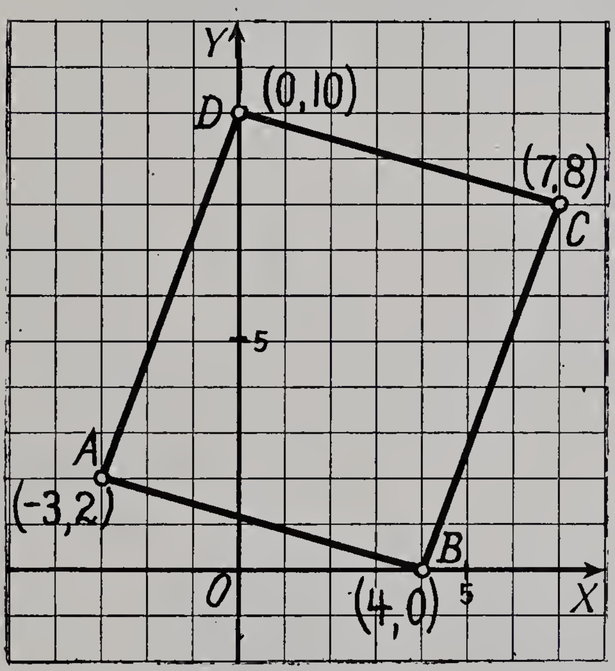
$y = \frac{4 - 6}{2} = -1.$

For midpoint of AB, $x = \frac{-3 + 5}{2} = 1,$

$y = \frac{6 + 4}{2} = 5.$

3. $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - 4}{2 - 5} = 1.$

4.



For AB, $m = \frac{2 - 0}{-3 - 4} = -\frac{2}{7}.$

For DC, $m = \frac{10 - 8}{0 - 7} = -\frac{2}{7}.$

∴ AB ∥ DC (§538).

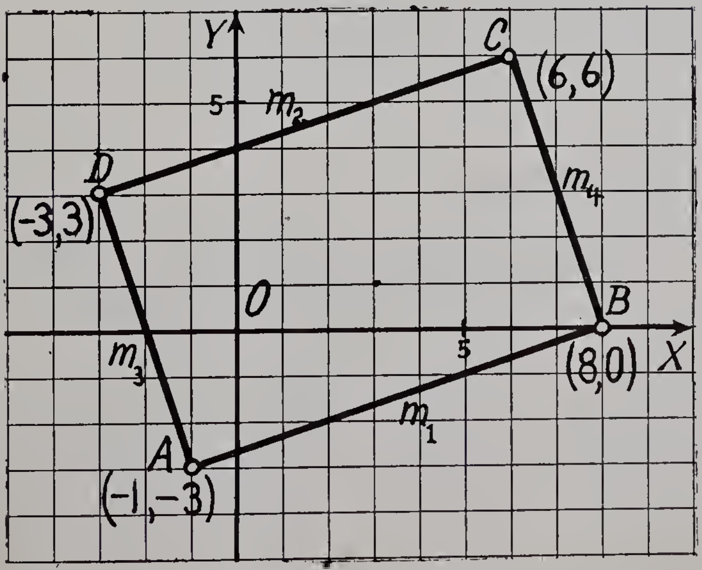
For AD, $m = \frac{10 - 2}{0 + 3} = \frac{8}{3}.$

For BC, $m = \frac{8}{7 - 4} = \frac{8}{3}.$

∴ AC ∥ BC (§538).

∴ ABCD is a ▱ (§185).

5.



$m_1 = \frac{-3}{-1 - 8} = \frac{1}{3}.$

$m_2 = \frac{6 - 3}{6 + 3} = \frac{1}{3}.$

∴ AB ∥ DC (§538).

$m_3 = \frac{3 + 3}{-3 + 1} = -3.$

$m_4 = \frac{6 - 0}{6 - 8} = -3.$

∴ AD ∥ BC (§538).

∴ ABCD is a ▱ (§185).

Since $m_2 m_3 = -1$, DC ⊥ AD (§538).

∴ ABCD is a rectangle (§185).

6. $y - y_1 = m(x - x_1)$; $y - 5 = \frac{2}{3}(x - 2)$; $3y - 15 = 2x - 4$; $2x - 3y = -11$.

7. $y = mx + b$; $y = 2x + 5$; $2x - y = -5$.

8. $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$; $\frac{y - 5}{x - 3} = \frac{-1 - 5}{7 - 3}$; $\frac{y - 5}{x - 3} = -\frac{3}{2}$. $2y - 10 = -3x + 9$; $3x + 2y = 19$.

9. $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{x}{3} = \frac{y}{-4} = 1$, or $\frac{x}{3} - \frac{y}{4} = 1$; $4x - 3y = 12$.

10. $x^2 + y^2 = 36$.

11. $r^2 = 16 + 9 = 25$. $(x - 4)^2 + (y + 3)^2 = 25$; $x^2 - 8x + 16 + y^2 + 6y + 9 = 25$; $x^2 + y^2 - 8x + 6y = 0$.

12. $(x - 6)^2 + (y - 2)^2 = 16$; $x^2 - 12x + 36 + y^2 - 4y + 4 = 16$; $x^2 + y^2 - 12x - 4y + 24 = 0$.

Chapter 18. Additional Topics

I. AERONAUTICS

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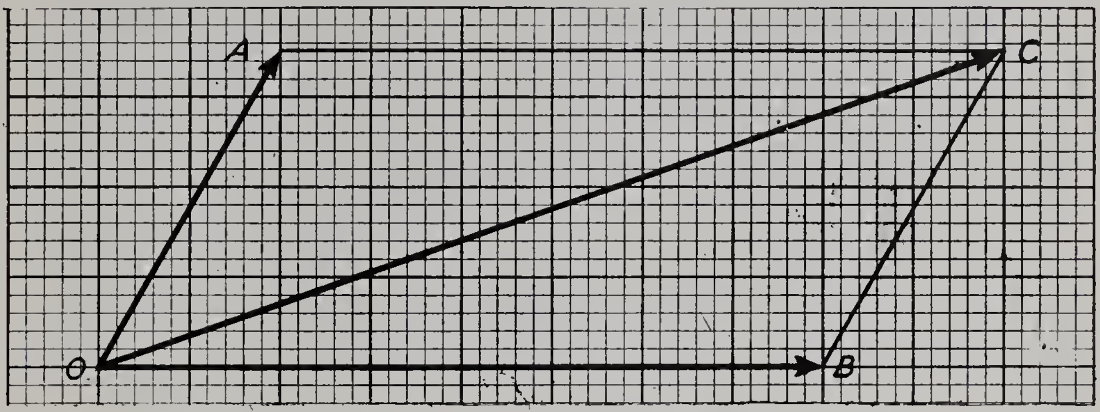
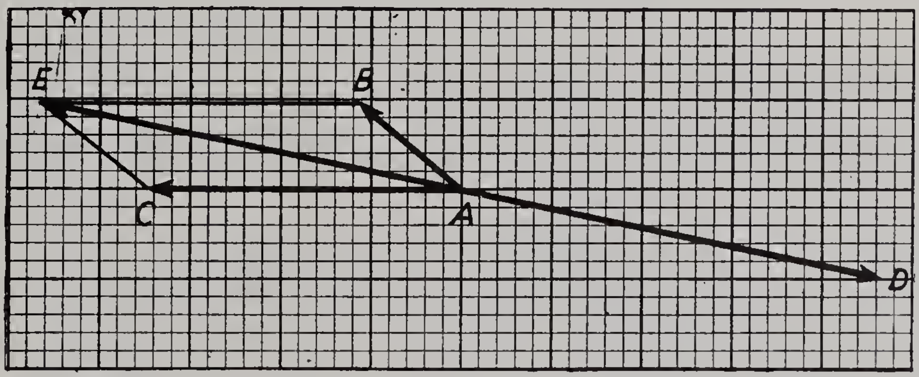
1. Let 1 cm stand for 10 lb. Then 4 cm. will stand for 40 lb. and 8 cm. will stand for 80 lb. Construct ▱OBCA with OB = 8 cm. and OA = 4 cm. Measure OC. OC = 10.6 cm. 10.6 x 10 = 106, the number of pounds in the resultant. ∠BOC = 19°.



2. 5 pounds in direction of AB.

3. AB = 1.5 cm. If 1.5 cm. represents 30 lb., 1 cm. will represent 20 lb. Then AC will represent 70 lb. ∠BAC = 40°. Construct ▱ABEC and draw diagonal AE. Measure AE, which is 4.7 cm. Then AE represents 20 x 4.7 lb., or 94 lb. Extend the resultant AE through A so that AD = AE. The force AD is 94 lb. ∠BAD = 152°.



PAGES 543-544

1. 262 sq.ft. = 37,728 sq.in. 12.8 - 12.1 = .7. .7 x 37,728 = 26,409.6. The total lift is 26,410 pounds.

2. (a) $\frac{V_1}{V_2} = \sqrt{\frac{W_1}{W_2}}$. Squaring both members of the equation,

$$\frac{V_1^2}{V_2^2} = \frac{W_1}{W_2}.$$

(b) $\frac{V_1^2}{V_2^2} = \frac{W_1}{W_2}$. Then $\frac{100^2}{V_2^2} = \frac{2400}{3000}$.

Solving, $V_2 = 111.8$. Its velocity is 111.8 m.p.h.

3. Let x = the number of miles east the plane can go.

	Time	Rate	Distance
Going	$\frac{x}{120}$	120	x
Returning	$\frac{x}{80}$	80	x

$\frac{x}{120} + \frac{x}{80} = \frac{15}{2}$. Multiplying by 240, $2x + 3x = 1800$. Then $5x = 1800$, and $x = 360$, the number of miles.

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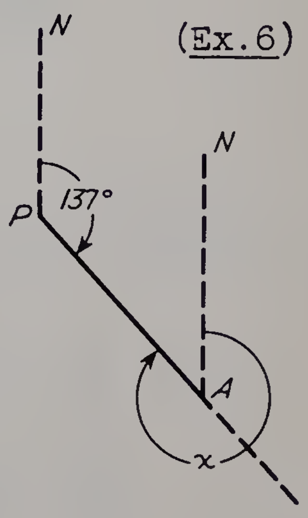
1. *a*. 0418 *c*. 1815 *e*. 0700
b. 1200 *d*. 2332 *f*. 2400, or 0000

2. 0815 = 8 hr. 15 min. 17 hr. 4 min. + 8 hr. 15 min. = 25 hr. 19 min. 25 hr. 19 min. - 24 hr. = 1 hr. 19 min. The time is 0119, or 1:19 A.M., Thursday.

3. 1345 = 13 hr. 45 min. 24 hr. + 13 hr. 45 min. = 37 hr. 45 min. 37 hr. 45 min. = 36 hr. 105 min. 36 hr. 105 min. - 16 hr. 50 min. = 20 hr. 55 min. The time was 2055, or 8:55 P.M., Monday.

4. *a*. $\angle m = 27^\circ 18' 34'' + 18^\circ 37' 52'' = 45^\circ 55' 86'' = 45^\circ 56' 26''$.
b. $\angle n = 27^\circ 18' 34'' - 18^\circ 37' 52'' = 26^\circ 77' 84'' - 18^\circ 37' 52''$ $= 8^\circ 40' 42''$.

5. The bearing of the lighthouse from the ship = 180° + 52°18' = 232°18'.



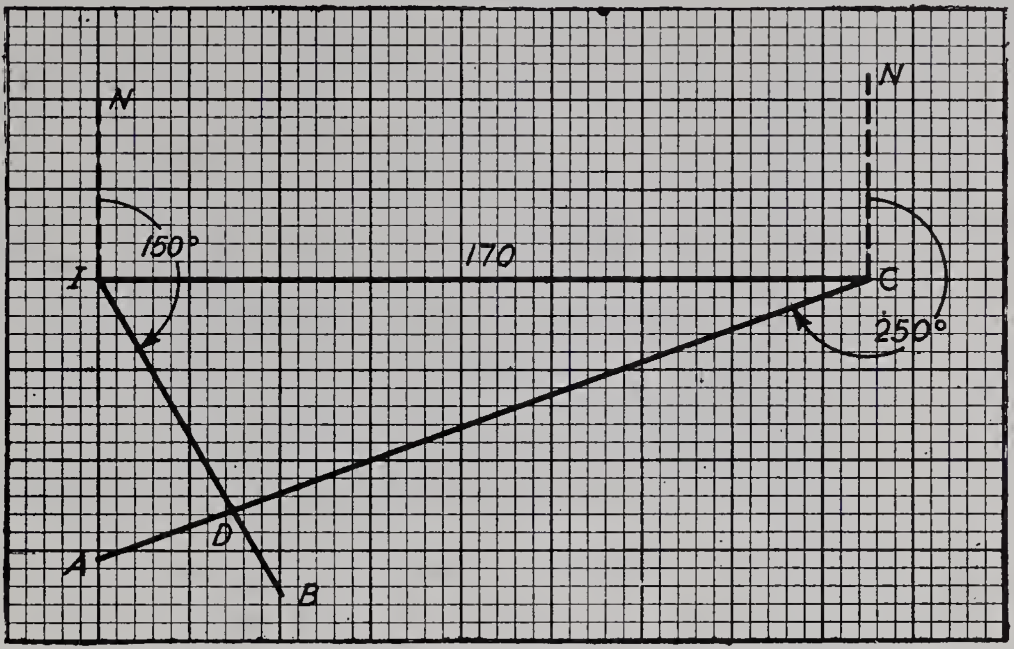
(*Ex.6*)

6. Let x = bearing of his plane from the airport. Then x = 137° + 180° = 317°.

7. Zero degrees.

8. If the bearing of Columbus from his plane was 70°, the bearing of his plane from Columbus was 180° + 70° = 250°. The bearing of his plane from Indianapolis was 330° - 180° = 150°.

Let 1 cm. represent 20 mi. Then $8\frac{1}{2}$ cm. will represent 170 mi. Draw IC in direction 90° and 8.5 cm. long. Draw CA in direction 250° and IB in direction 150°, intersecting in D. DC by measurement = 7.4 cm. Then his distance to Columbus was 7.4 x 20 = 148 (mi.), approx. (See diagram, below).



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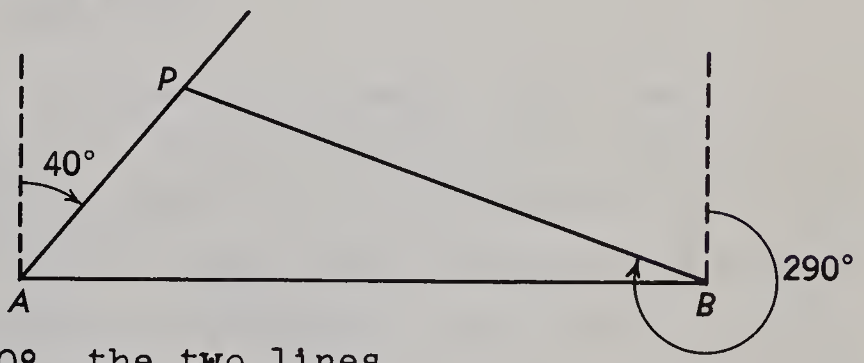
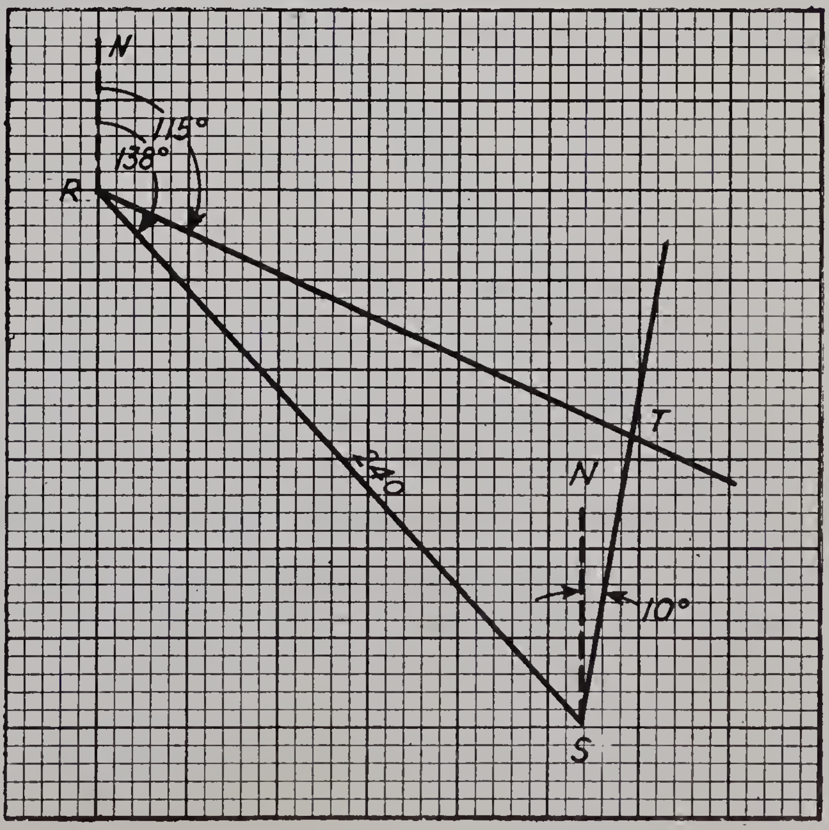
1. From A draw a line in direction 90°. Let 1 cm. = 10 mi. Then 6 cm. = 60 mi. Make AB = 6 cm. From A draw a line in direction 40° and from B draw a line in direction 290°, the two lines intersecting in P.

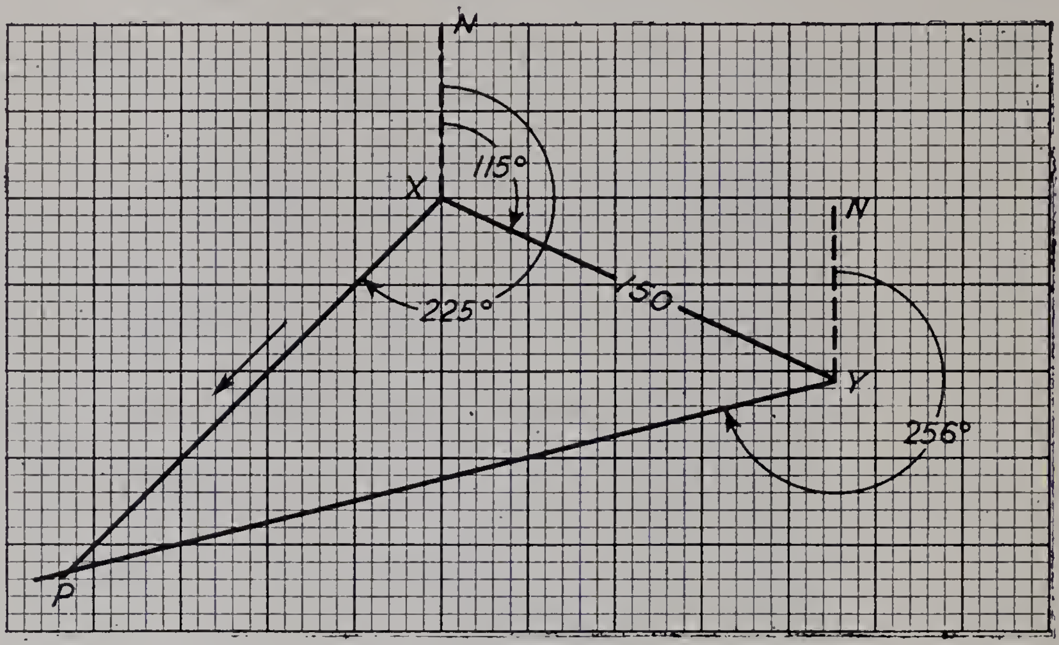


By measurement AP = 2.1 cm. Then AP = 2.1 x 20 = 42 (mi.), approx.

2. From R draw a line in direction 138°. Let 1 cm. represent 30 mi. Then 8 cm. will represent 240 mi. Make RS = 8 cm. From S draw a line in direction 10° and from R draw a line in direction 115°, the two rays intersecting in T.



RT measures 6.5 cm. and ST measures 3.2 cm. Then RT = 6.5 x 30 = 195 (mi.), approx., and ST = 3.2 x 30 = 96 (mi.), approx.



3. Use scale 1 cm. = 30 mi. If the bearing of station Y from the plane is 76°, the bearing of the plane from Y is 256°. Locate point Y 5 cm. from X and in direction 115° from X. From X draw a line in direction 225° and from Y draw a line in direction 256°. The lines intersect in point P, the fix of the plane. PX is 6.1 cm. 6.1 x 30 = 183. Then the pilot is approximately 183 mi. from X.

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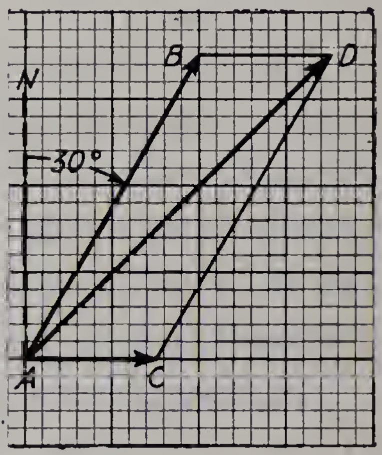
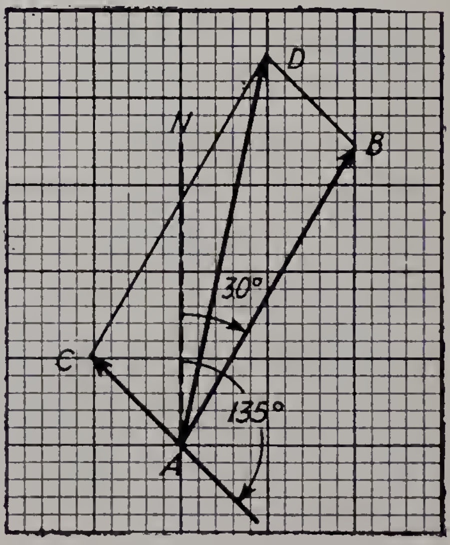
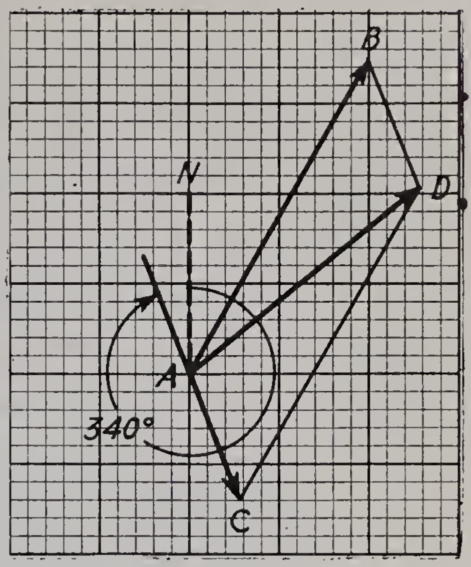
1. Let 1 cm. represent 30 m.p.h. Draw AB 4 cm. long and in direction 30°. Draw AC $1\frac{1}{2}$ cm. long in direction 90°. Complete ▱ACDB. AD gives the ground speed and direction of course. AD = 5 cm. The ground speed is about 150 m.p.h. in direction 45°. The drift angle BAD is about 15°.



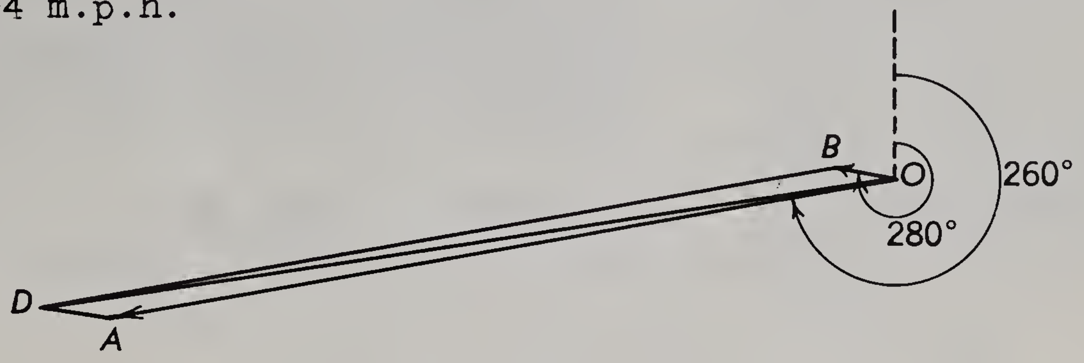
2. Let 1 cm. represent 30 m.p.h. Draw AB in direction 30° and 4 cm. long. Draw AC in direction 315° and $1\frac{1}{2}$ cm. long. Complete ▱ACDB. The course AD is in direction 12°, approx. The drift angle = ∠BAD = 18°, approx. The ground speed represented by the length of AD is about 139 m.p.h.



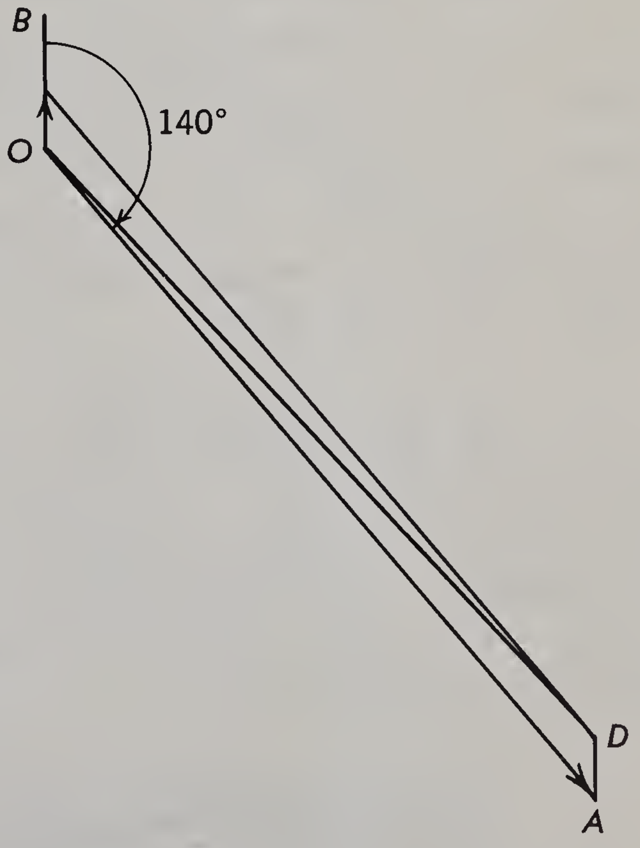
3. Let 1 cm. represent 30 m.p.h. Draw AB 4 cm. long and in direction 30°. Draw AC in direction 160° and 1.5 cm. long. Complete ▱ACDB. The course AD is in direction 51°, approx. The ground speed is about 98 m.p.h. The drift angle = ∠BAD = 21°, approx.



4. Let 1 cm. represent 40 m.p.h. Draw OA 7 cm. long and in direction 260°. Draw OB .5 cm. long and in direction 280°. Complete ▱AOBD. The ground speed, represented by OD, is approximately 304 m.p.h.



5. Wind speed = 20 m.p.h. Draw OB .5 cm. long and in direction 0°. Draw OA 7.5 cm. long and in direction 140°. Complete ▱AOBD. The ground speed, represented by OD, is approximately 288 m.p.h.



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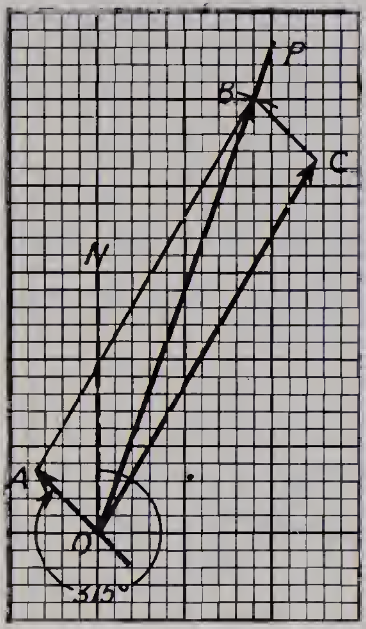
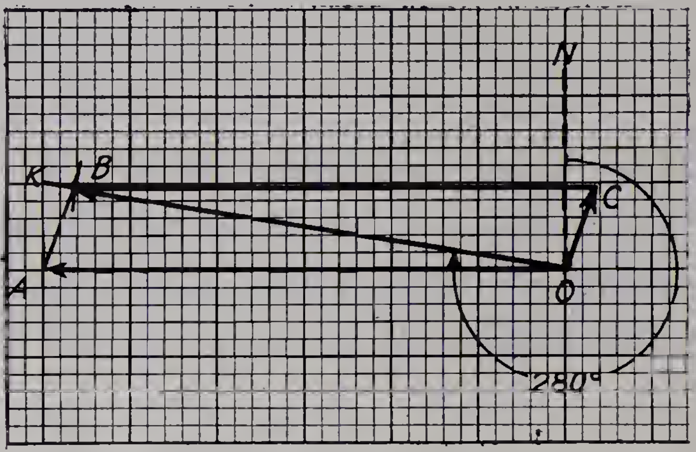
1. Draw line OP in direction 20°. Using the scale 1 cm. = 20 m.p.h., draw OA in direction 315° and 1 cm. long. With A as a center and radius 5 cm. long, describe an arc intersecting OP in B. Complete ▱OABC. The plane should be headed along OC in direction 30°. The wind-correction angle BOC is 10°. The ground speed represented by OP is about 106 m.p.h.



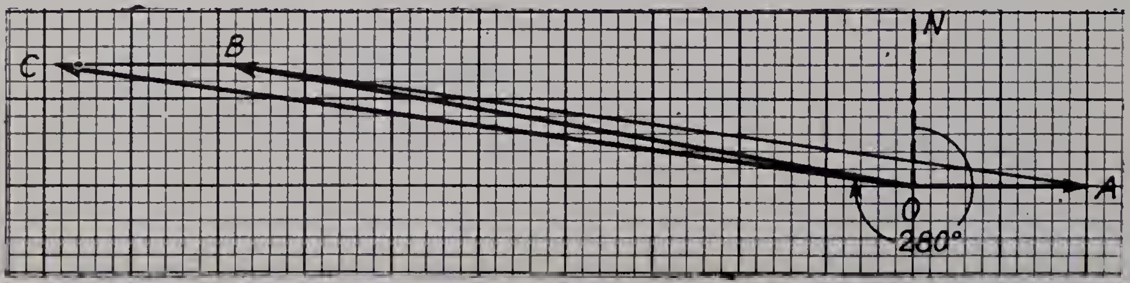
(Ex. 1)



(Ex. 2)

2. Draw line OK in direction 280°. Using a scale of 1 cm. = 40 m.p.h., draw OC in direction (200° - 180°), or 20°, and 1 cm. long. With C as a center and a radius = 6 cm., draw an arc intersecting OK in B. Complete ▱OABC. Diagonal OB = 5.7 cm. and ∠AOB = 9°. The ground speed is about 225 m.p.h. and the wind-correction angle is about 9°. The heading is about (280° - 9°), or 271°.

3. Use the scale 1 cm. = 20 m.p.h. Draw OB, the diagonal of the ▱, 8 cm. long and in direction 280°. Draw OA in direction 90° and 2 cm. long. Complete ▱OABC. Then OC represents the air speed. Since OC = about 9.9 cm., the air speed is about 198 m.p.h. The heading of theplane is along OC, which is in direction about 278°.



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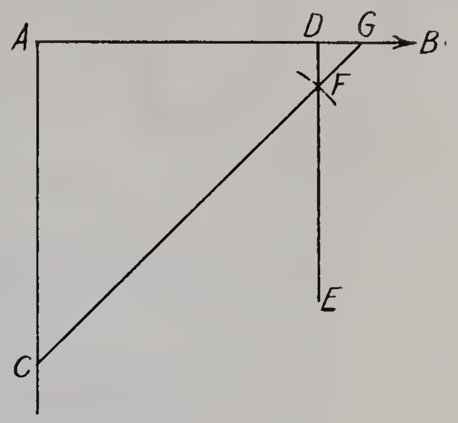
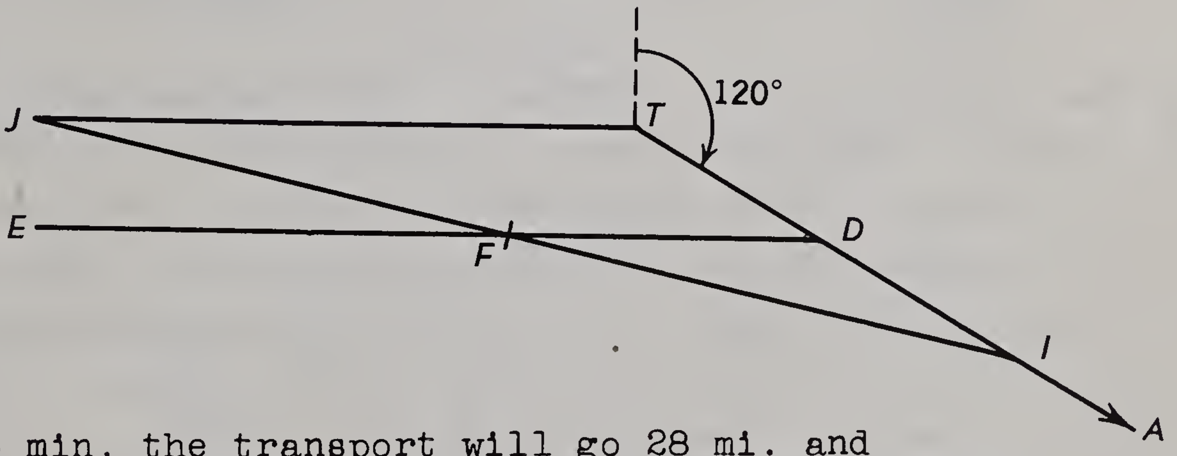


1. 1. Draw AB to denote the course of the transport plane. 2. Draw AC in direction 180° six centimeters long. 3. In 10 minutes the transport plane will go 53.3 mi. and the pursuit plane will go 75 mi. 4. Mark off AD equal 5.3 cm. to denote the position of the transport plane in 10 minutes (scale 1 cm. = 10 mi.). 5. Through D draw DE ∥ AC. 6. With C as the center and a radius = 7.5 cm., describe an arc intersecting DE in F. 7. Draw CF intersecting AB in G. 8. G is the point of interception. 9. CG, the course, is in direction $45\frac{1}{2}^\circ$, approx. 10. $\frac{AG}{AD} = \frac{t}{10}$; substituting the values of AG and AD and solving for t, we have 11.4. Then the time of interception is about 11.4 minutes.

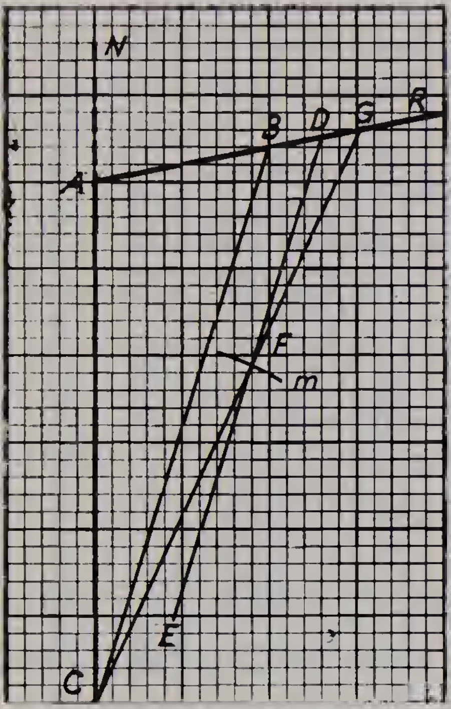
2. Draw JT 8 cm. long to represent 80 mi. (the original distance from the jet to the transport). Draw TA in the direction 120° to represent the course of the transport.



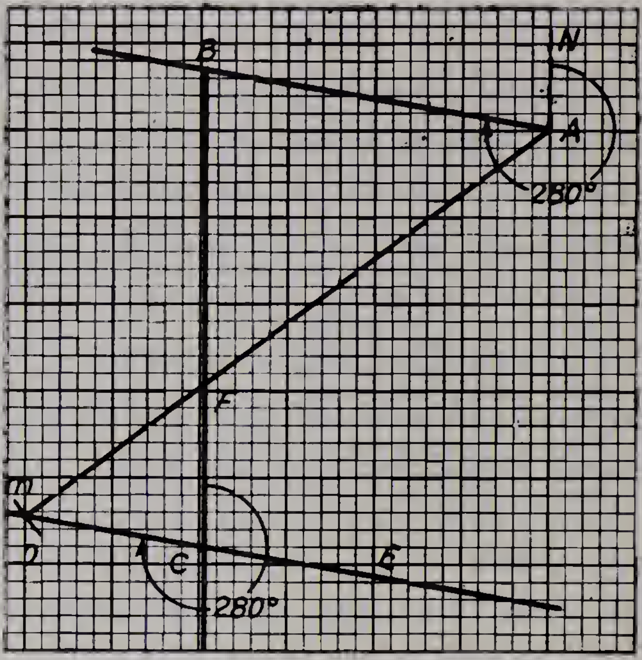
In 6 min. the transport will go 28 mi. and the jet 65 mi. On TA measure TD 2.8 cm. to represent the distance the transport goes in 6 min. Draw DE ∥ JT. With J as center and radius 6.5 cm. describe an arc intersecting DE in F. Draw JF and extend it to intersect TA in I. Let t represent the time elapsing before the transport is intercepted. Then $\frac{TD}{TI} = \frac{6}{t}$. Substituting the measured lengths of TD and TI in this equation gives $\frac{2.8}{6.5} = \frac{6}{t}$, whence t = 13.9. The jet will intercept the transport in approximately 13.9 minutes.

3. Let 1 cm. represent 25 mi. The ship sails along AR in direction 79°. At 8 A.M. the ship will be at B and the plane will be at C. Draw BC. In 36 minutes the ship will be at D. The plane will be in DE, which is ∥ BC and also in arc m, which

has its center at C and has a radius of 4.3 cm. Then the plane is at F. Draw CF which intersects AR in G, the point of interception. The plane's course is CG, which is approx. 187 mi. The distance of G from A is about 77 mi. The time for the plane to fly the 187 mi. is 1.04 hr., or 1 hr. $2\frac{1}{2}$ min., approx.



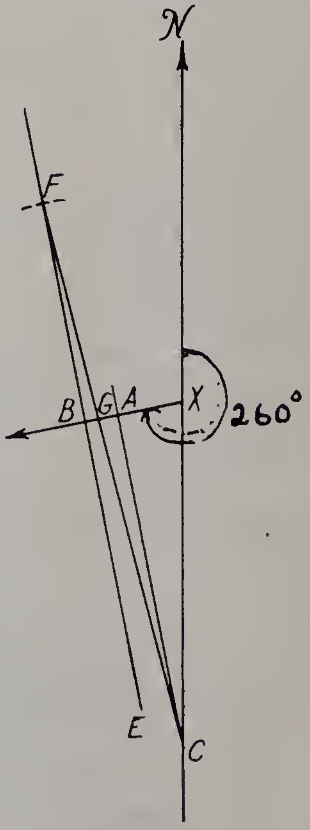
(Ex. 3)



(Ex. 4)

4. Use the scale 1 cm. = 20 mi. Let A be the position of the pursuit plane. Then the bomber is at B in direction 280° from A and 4 cm. from A. BC is the line of flight of the bomber. In 30 min. the bomber will be at C. In one half hour the pursuit plane must be in line CE. In one half hour the bomber must be in arc m, which has A for its center and 7.5 cm. for its radius. Draw AD, intersecting BC in F. Then F is the point of interception and AF is direction of flight of the pursuit plane. The course is in direction 233°, approx. The pursuit plane's distance to F is 20 x 4.9 mi., which is 98 mi., approx. The time for the interception is $\frac{98}{300}$ hr., or 19.6 min., approx.

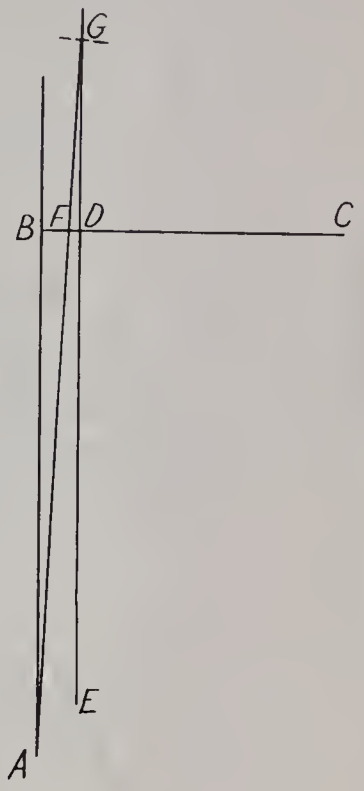
5. 1. Draw XN to give the north-south line. 2. Using the scale 1 cm. = 30 mi. extend NX to C so that XC represents 340 mi. 3. Draw the line through X in direction 260°. 4. On this line mark off XA to represent 64 miles, the distance the ship goes in 4 hours. 5. Draw AC. 6. In 2 hours the ship goes 32 miles and the plane



(Ex. 5)

560 miles. 7. Mark off AB to give the distance the ship goes in 2 hours. 8. Through B draw the line BE parallel to AC. 9. With C as the center and a radius to represent 560 miles describe an arc intersecting BE in F. 10. Draw CF intersecting XA in G. Then G is the point of interception. The course of the airplane is in direction CF which is 346°, approx. 11. The distance of G from x = 2.8 x 30 mi. = 84 mi., approx. 12. The time of ship = 5.2 hours, approx. 13. The time of plane = 1.2 hour, approx.

6. 1. Let the hunter be at A and the hare at B. 2. In $\frac{1}{10}$ of a second the hare goes 4 ft. and bullet goes 200 ft. 3. Draw BC in direction 90°. 4. On BC take BD to represent 4 ft. 5. Draw DE ∥ BA. 6. With A as the center and a radius equal to 10 cm., describe an arc intersecting DE in G. 7. Draw AG intersecting BC in F. 8. F is the point of intersection. 9. The line segment BF represents 2.4 ft. 10. The hare will go 2.4 ft. 11. The hunter shoots in direction 1°.



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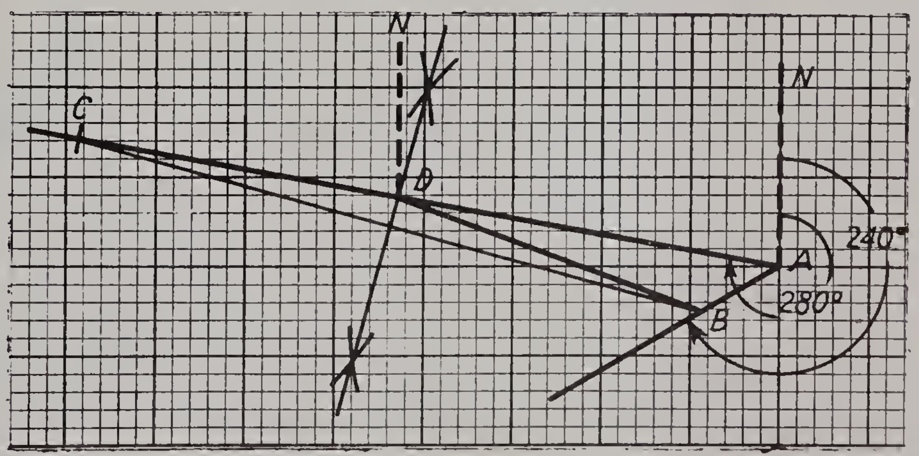
1. The time each way is 3 hours. Then the distance = 3 x 120 mi. = 360 mi.

2. Let 1 cm. represent 30 mi. Then d draw AC in direction 280° to represent 240 mi., the distance the plane would go in $1\frac{1}{2}$ hr. Draw AB in direction 240° to represent the distance the ship will go in $1\frac{1}{2}$ hr. Draw the ⊥ bisector of BC. It intersects AC in D, where the plane starts its return to the ship. Draw BD. AD represents 135 mi., approx. On the return course to the ship, DB, the heading is about 109°.



3. Let x = the number of miles the plane can go before starting the return. 120 m.p.h. = the rate going, 90 m.p.h. = the rate returning. Then $\frac{x}{90} + \frac{x}{130} = 4$. Solving, $x = 212\frac{8}{11}$ (mi.).

2. GEOMETRIC RELATIONS; ORTHOGRAPHIC PROJECTION

PAGES 557-558

1. I. 2. III. 3. I. 4. II. 5. I. 6. III. 7. II.

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1. a. 6. b. 4. c. 1. d. 0. e. 1. f. 0. 2. a. 16. b. 4. c. 4. d. 3. e. 4. f. 1. 3. a. 27. b. 0. c. 0. d. 8. e. 12. f. 6. 4. a. 24. b. 5. c. 12. d. 5. e. 2. f. 0. 5. a. 32. b. 2. c. 9. d. 18. e. 3. f. 0. 6. a. 10. b. 4. c. 4. d. 1. e. 0. f. 1.

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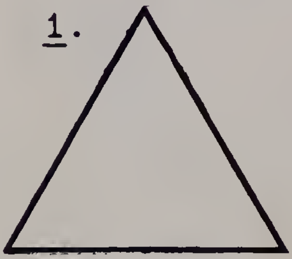

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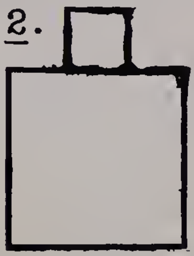

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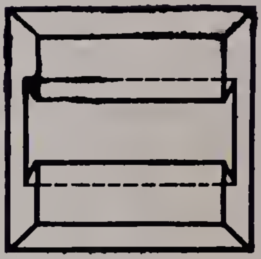

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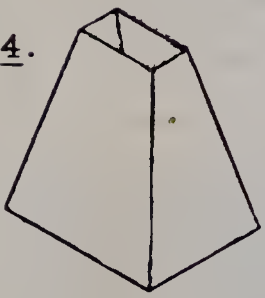

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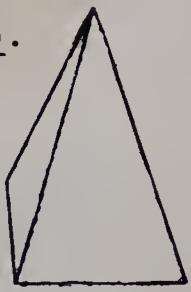

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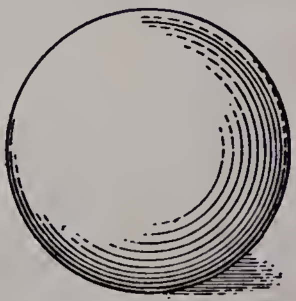
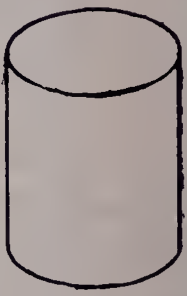


7.



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